Critique of the Vertical Gradient of Gravity

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Growing interest by Geodesists and Theoretical Physicists in high precision studies of the earth's gravitational field warrant a critical review of precision requirements to yield useful results. Several problems are now under consideration. All of these problems involve, more or less, the precise value of the vertical gradient of gravity.

Elevation corrections for gravity mapping.

The major present use of the so-called Free-Air Vertical Gravity Gradient is to calculate elevation corrections of gravity station data for gravity maps. It is standard practice to use the "normal" gradient value 0.3086 mgls/meter (0.09406 mgls/ft). This ignores the fact that published data demonstrate that the value of the earth's vertical gravity gradient varies at least plus or minus five percent (± 5%). In high topography (say 4000 meters – 12,000 feet) this produces sea-level anomaly values that may be in error more than fifty milligals (50 mgls).

Errors of this magnitude on official published gravity maps are not tolerable. The often-heard argument that this is not an error but a real part of the anomaly, is not valid. It produces inconsistent anomaly values for stations observed at different elevations - such as ground and airborne.

Vertical gradient measurement

The measurement of the vertical gradient of gravity (V) is expressed by the equation

$$V + e_V = (\Delta g + e_g)/(\Delta H - e_H)$$

where e_V, e_g and e_H represent errors in the data. The fractional errors in these factors are

$$e_V/V = (1 + e_g/\Delta g)/(1 - e_H/\Delta H) - 1$$

A plot of this equation is shown in Fig. 1.

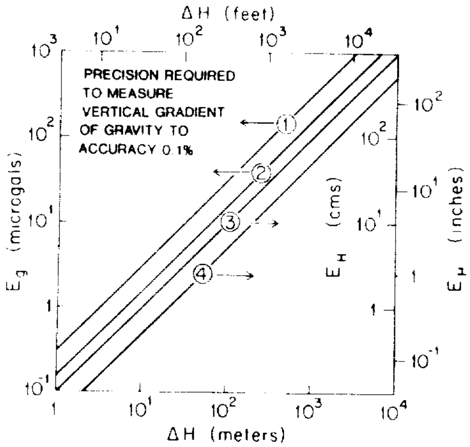

Fig. 1 Curve 1 $e_H/\Delta H = 0$
Curve 3 $e_g/\Delta g = 0$
Curves 2 & 4 $e_g/\Delta g = e_H/\Delta H$

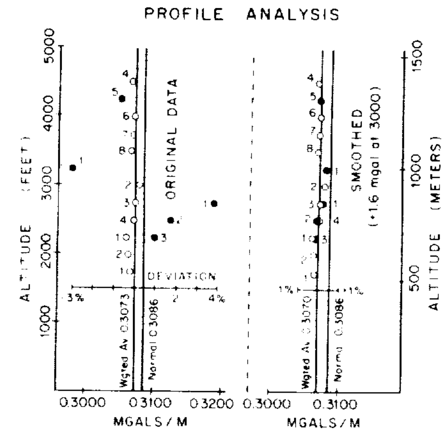

Fig. 2 Airborne Vertical Gradient (Discussed below)

The precision requirements versus elevation range of observation from Figure 1 are tabulated below

Table 1 Precision required to measure the vertical gradient to 0.1%

Method	ΔH Meters	e_g (μgal) (a)	e_g (μgal) (b)	e_H (cms) (c)	e_H (cms) (d)
TRIPOD	3	0.9	0.46	0.15	0.3
TOWER	100	31	15	5	10
AIRBORNE	1000	309	154	50	100

(a): Zero error in ΔH

(b) & (c): Equal errors $\frac{E_g}{\Delta g}$ and $\frac{E_H}{\Delta H}$

(d): Zero error in Δg

It is apparent that present instrumentation cannot achieve the necessary precision on a portable tripod with elevation range of, say, three meters

(10 feet). Tall buildings and towers, with elevation ranges up to several hundred meters can achieve the necessary precision but are seldom available where needed. Upward continuation of ground based gravity survey data are difficult to evaluate. An example of an airborne vertical gradient measurement, which achieves the desired accuracy, is shown in Fig. 2 above.

The data in Fig. 2 were observed at six elevations, up to 5500 feet, (1600+ meters) which provided fifteen (15) internal gradient values. Most of these data were in excellent agreement (open circles in the left hand section). Five discordant points (blackened circles) all involve data at a single level. A smoothing correction of +1.6 mgls to that value eliminates the scatter as shown in the right hand section. The RMS error of the smoothed gradient data is 0.1% (±3 E°).

<u>Borehole gravity</u>

Many borehole gravity surveys in oil and gas wells have been published. The borehole data which penetrated the Greenland ice sheet are of great interest. The possibility of deep ocean profiles also has been proposed. An active new purpose of these data is to improve the accepted value of the gravitational constant, usually designated as G or γ which is the least accurate of all fundamental physical constants. For this purpose the density of the formation penetrated, as well as the gravity gradient, and various corrections require accuracies of better than 0.1%. This needs to be reported in detail for each case.

The actual value of the vertical gradient at the borehole site is also involved. The observed gravity variation in a borehole is

$$\Delta g = (V - 4\Pi\ G\ \sigma)\Delta H - (TC_1 - TC_2)$$

where TC is the calculated correction for surface topography and nonuniform subsurface formation layering at the two end points for the gravity measurement, Δg. Solving for G we have

$$G = [V - \frac{\Delta g}{\Delta H} + \frac{\Delta TC}{\Delta H}]/4\Pi\sigma$$

The use of the "normal" value 0.3086 mgls/m (0.09406 mgls/ft) for the free-air gradient value (V) at the site may involve very large errors in the "observed" value of G.

CONCLUSION

The principal conclusion from this review is that the essential absence of Free Air Vertical Gravity Gradient control and actual values of gravimeter calibrations require serious attention. Large errors in high topography on official published gravity maps also cannot be ignored.

Post Script

Since oral delivery of this paper at the recent Chapman Gravity Conference in Fort Lauderdale, Florida, I have had access to a manuscript report on a related topic (Romaides et al 1988). This is a detailed report on gravity observations in a 600 meter television tower and procedures to calculate the comparative vertical gravity profile by upward continuation of ground based gravity survey data which were especially designed for the purpose. Precision of data and analysis is a major feature of the paper. Calibration of the LaCosts-Romberg gravimeter, which was used for the study, is also detailed. If and when published this report will provide a significant up-date for the present paper.

References

Romaides, Jekeli, Lazarewicz, Eckhardt and Sands, "A Detection of Non-Newtonian Gravity", Air Force Geophysics Laboratory, Hanscom AFB, MA 01731-5000 (No date).

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