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1.
One of the main purposes of geodesy is to determine the gravity field of the earth in the space outside its physical surface [13]. This purpose can be pursued without any particular knowledge of the internal density even if we don't know the exact shape of the physical surface of the earth, though this seems to entangle the two domains, as it was in the old Stoke's theory before the appearance of Molodensky's approach [10].
Nevertheless even when large, dense and homogeneous data sets are available, it has always been recognized that subtracting from the gravity field the effect of the outer layer of the masses (topographic effect) yields a much smoother field, which allows for computations with a much lower approximation error [16].
This is obviously more important when we have a sparse data set, so that any smoothing of the gravity field helps in interpolating between the data without raising the modelling error [3]: this approach is nowadays generally followed also because it has become very cheap in terms of computing times since the appearance of spectral techniques [14].

2.
As we know the mathematical description of the IGP is dominated mainly by two principles, which in loose terms can be formulated as follows:
1) the knowledge of the external gravity field determines mainly the "lateral" variations of the density;
2) the deeper is the density anomaly giving rise to a gravity anomaly, the more improperly posed is the problem of recovering the former from the latter.
For a sphere, of radius normalized to 1, the relation between harmonic coefficients of the external gravity field $u(P) = \Sigma \, u_{nm} Y_{nm}(\sigma_P)$ and of the internal density $\rho(Q) = \Sigma \, \rho_{nm}(r_Q) Y_{nm}(\sigma_Q)$ is described by the formula

$$u_{nm} = \frac{1}{2n+1} \int_0^1 \rho_{nm}(r) \, r^{n+2} \, dr \quad . \tag{1}$$

Several applications of (1) derived from fixing the radial variations of ρ: e.g. the single layer case is considered as well as harmonic or quasi-harmonic densities, which correspond to suitable variational problems.

3.
The statistical relation between ρ and n (and its inverse) is also investigated in its general form, proving that degree cross-covariances have to be introduced to describe the behaviour of ρ, i.e.

$$\sigma_{\rho,n}(r_1, r_2) = \frac{1}{2n+1} \sum_m \rho_{nm}(r_1) \, \rho_{nm}(r_2) \quad : \tag{2}$$

the general relation between such functions and the usual degree variances of the potential

$$\sigma^2_{u,n} = \frac{1}{2n+1} \Sigma_n u^2_{nm} \quad ,$$

is

$$\sigma^2_{n,m} = \frac{1}{2n+1} \int_0^1 \int dr_1 \, dr_2 \, \sigma_{\rho,n}(r_1,r_2) \, (r_1 \cdot r_2)^{n+2} \tag{3}$$

The meaning of Kaula's rule is investigated in this framework proving that it demonstrates, within a layered model, the presence of a white noise in the lateral variations of the outermost layer; this is interpreted as the effect mainly of the rough signal due to topographic masses and their compensation.

4.
Furtheron the problem of the simultaneous estimate of a spherical anomalous potential and of the external, topographic masses is addressed criticizing the choice of the mixed collocation approach, as presented in [12]. This approach in fact fixes the relation between internal covariance of ρ and n in such a way that it has been proved to be wrong in practical cases [5].
A reasonable improvement is found when the modelling of the density is constant on a scheme of overlapping blocks, since this allows the construction of suitable crosscovariance models. This approach is now undergoing a practical investigation.
The paper will be published in full length by the Danish Geodetic Institute in the book in honour of the 60th birthday of Torben Krarup.

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