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FFT—Local Gravimetric Geoid Computation

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Abstract

Model computations show that changes of sampling interval introduce only 0.3 cm changes, whereas zero padding provides an improvement of more than 5 cm in the FFT generated geoid. For the GPS survey of Franklin County, Ohio, the parameters selected as a result of model computations, allow large reduction in local data requirements while still retaining the *cm* accuracy when tapering and padding is applied. The results are shown in tables.

Introduction

The following is a brief description of computational modeling carried out in order to obtain optimal results from the use of the Fast Fourier Transform (FFT) technique for local geoid computation. These experiments were designed to find the most favorable parameters for local geoid computation using gravity data only. The availability of analytical expressions for the model, both the potential and the gravity, permits us to evaluate the effect of changing any of the parameters introduced when using FFT. It is recognized, that some of the parameters depend very much on the model. Thus these computational experiments are model related and can not be applied blindly for all practical work. Still, the model used in these studies provides the opportunity to test some interesting aspects of the FFT technique.

Model Description

A three-dimensional model of a granitic intrusion (Gibb and van Boeckel,1970), which consists of 64 prisms and covers an area of 80×75 km^2, with a change of about 60 mgal and 75 cm in gravity and geoidal height respectively, was used in these model computations. For details see Nagy(1988). The analytical expressions for the potential, U, and the gravity, Δg, for a single prism are given below (Nagy,1980) :

$$\begin{aligned} U &= k\rho\Big[x\,y\ln(z+r) + y\,z\ln(x+r) + z\,x\ln(y+r) \\ &\quad -\frac{1}{2}x^2\arctan\frac{y\,z}{x\,r} - \frac{1}{2}y^2\arctan\frac{z\,x}{y\,r} - \frac{1}{2}z^2\arctan\frac{x\,y}{z\,r}\Big] \end{aligned}$$

$$\Delta g = k\rho\Big[x\ln(y+r) + y\ln(x+r) - z\arctan\frac{x\,y}{z\,r}\Big]$$

$$\text{where} \qquad r = \sqrt{x^2+y^2+z^2}.$$

The negative of the potential divided by the normal gravity, γ, gives the geoidal height for a prism. Summing up the required quantities for all prisms of the model provide the *exact* reference values, with which the result of the various numerical computations can be compared. The difference is clearly the error of the numerical procedure. In this case, the error generated by the FFT method.

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Effect of Sampling Interval

As the transfer function, i.e. the function used to weigh the gravity anomalies to produce the geoidal height, is relatively flat as compared, for example, with the functions used in calculating the deflections of the vertical, or the vertical derivatives, one expects no large changes associated with the changes in the sampling interval. This has been confirmed with model computations. Different sampling intervals between 1 and 15 km covering the same area produced only 0.3 cm change in geoidal height. For this reason, the sampling interval does not seem to be of major concern in local geoid determinations.

Effect of Padding

The Fourier method assumes periodicity, i.e. the field given in a two-dimensional array is repeated in the frequency domain around the central part in both dimensions and introduces the so called *leakage* into the computations, causing unwanted errors. To partially compensate for this error, the technique known as *padding* is used. Padding consists of putting zeros around the values of the input matrix, practically doubling the dimensions. For the model using 5 km sampling interval and a 26×26 grid, the gravity was practically zero at all boundries. The model geoid over this grid has a span (difference between maximum and minimum) of 74.9 cm. The use of FFT on the corresponding gravity anomaly produced a geoid with the span of 67.3 cm i.e. an error of 7.6 cm. Carrying out the zero padding to generate a matrix of 50×50 resulted in a different geoid with a span of 73 cm. This means that doing only zero padding, the error was reduced from 7.6 cm to 1.9 cm. This is a far greater change then produced by varying the sampling interval. Here the great importance of modeling is stressed. The results of computations without and with padding *are different*. However without the knowledge of the *exact model values*, one would not be able to draw any conclusions. In the case of modeling, the comparison with the exact values makes it obvious which computation gives the better result.

Effect of Tapering

Normally the gravity anomalies at the boundaries are not zero. In order to have a smoother transition, the techique known *tapering* is used. The purpose of tapering is to bring down the non-zero gravity values at the boundary smoothly to zero. There are various ways of achieving this, but model computations show that the particular method used for tapering is not critical. Table 1 summarizes some numerical results with various combinations of tapering with zero padding. The input matrix was generated at a 1 km interval, consisting of an array of 62×62 (used as reference), covering the central part of the anomaly field, with reasonably large non-zero values at the boundaries. All geoidal height related quantities are given in *cm*.

Table 1

Array Size	Tapering %	Padding %	Geoidal Heights		Residual Errors	
			min	max	Span	RMS
62	0	0	-24.55	23.89	40.73	7.43
70	6	0	-22.31	26.74	27.30	4.87
90	6	16	-15.98	35.17	22.51	2.96
90	22	0	21.22	34.68	11.98	2.54
110	22	16	-17.38	42.76	8.54	1.77
110	38	0	-21.71	42.34	16.63	2.96
130	38	17	-17.13	49.69	10.97	2.16
110	6	32	-12.22	41.78	20.97	2.66
130	22	33	-13.17	48.99	7.76	1.64

Numerical Errors

It is well known that the computed values toward the border of the area become erroneous. Modeling provides again a unique opportunity to study this question by comparing the analytical and the FFT-derived values and, based upon the residuals, draw some conclusions. On the model used, there is a sharp drop in residual errors after reducing the array size by about 10%, thereafter no significant reduction in errors occur. Obviously, the results are again model dependent.

Practical Application

Based upon the results of model computations, the FFT method has been used to calculate relative geoidal heights for the Franklin County GPS survey. The calculation was done in two steps :

- the regional component was calculated from the **OSU86F** truncated to $n = 36$,
- the local component was derived by applying the FFT technique to the residual gravity field, which was gridded at $5'$ intervals resulting in an array of 192×192 providing the desired coverage for the area of interest.

The geoidal height difference is the sum of the global geoid and column [1]. This value will be used later in the comparisons listed in Table 3.

The results of some of the computations are shown in Table 2. Column [1] is the direct application of FFT; all other solutions are the changes with respect to this solution. The dramatic reduction of errors by the combined effect of tapering and padding (for example, solution [8] vs. solution [4]) is readily recognizable from Table 2.

Table 2

Base line	Global geoid	FFT [1]	Residual FFT Geoids [2]	[3]	[4]	[5]	[6]	[7]	[8]
Rhodes → Clark	-8	3.3	-.3	.5	2.4	1.1	.4	.6	-.4
18–83 → Clark	-7	-9.2	-.4	-.1	1.6	.9	.3	.2	-.5
18–83 → Rhodes	1	-12.5	-.1	-.6	-.8	-.2	-.1	-.4	-.1
Britton → 18–83	10	-30.9	-.1	-.8	-2.9	-1.0	-.1	-.7	.7
Hoover → Clark	3	9.0	.5	.1	-.8	-1.1	-1.1	-.3	-.5
18–83 → Shannahan	-17	4.1	-.6	1.1	5.0	2.7	.5	.8	-1.3
Jackson → Britton	-20	18.3	-.3	1.9	7.2	2.9	1.1	2.2	-.7
Smith → Jackson	-8	59.0	.4	.9	2.4	.5	-.5	.4	-1.2
Smith → Hoover	-28	28.2	-.9	1.8	9.1	4.4	1.9	2.4	-1.2

Legend :

No.	Array	Remarks
[1]	192	8.0° border around baselines
[2]	144	5.5° border around baselines
[3]	72	3.0° border around baselines
[4]	52	20 rows and columns removed at south and east (simulating lack of data at shore lines)
[5]	72	zero padding for removed data
[6]	144	50% zero padding on [5] all around
[7]	72	20% tapering on [4] all around
[8]	144	50% zero padding on [7]

The relative geoidal height computations were repeated next by truncating the **OSU86F** global model to $n = 180$ and both results (i.e. the $n = 36$ and $n = 180$) were then compared to values derived from GPS surveys and leveling (Table 3). The GPS survey used in these comparisons was reported earlier by Engelis et al. (1984) and Kearsley (1985), and are listed as **OSU** and **AUS** respectively in Table 3. Relative geoid heights (**GPS**) were derived at the National Geodetic Survey of USA (Fury,1985); the old values were given in the above cited references.

Table 3

Base line	Length	Relative Geoid Heights					
		GPS	old	OSU	AUS	36	180
Rhodes → Clark	10	-5	(-7)	-3	-6	-5	-3
18–83 → Clark	11	-18	(-19)	-14	-19	-16	-14
18–83 → Rhodes	4	-13	(-13)	-11	-13	-12	-12
Britton → 18–83	13	-21	(-19)	-21	-19	-21	-22
Hoover → Clark	10	19	(19)	12	15	12	12
18–83 → Shannahan	22	-18	(-25)	-11	-19	-13	-10
Jackson → Britton	24	-4	(1)	0	-5	-2	0
Smith → Jackson	14	58	(32)	50	63	51	52
Smith → Hoover	35	4	(-13)	3	5	0	4

Conclusions

The numerical experiments presented here confirm the effectiveness of the FFT method for local gravimetric computation and show some of the results which can be obtained from model computations for use as guidelines in practical applications to obtain the best result from the FFT technique.

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A Comparison between SEASAT, GEOSAT and Gravimetric Geoids Computed by FFT and Collocation in the Central Mediterranean Sea

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Abstract

Gravimetric geoids have been computed for the central Mediterranean Sea between latitude 32° and 36° and longitude 18° and 22° using FFT and collocation. A comparison with cross-over adjusted SEASAT and GEOSAT data in the area showed for both gravimetric geoids the standard deviation of the differences to be 0.20 m and 0.15 m, respectively. The mean and standard deviation of the difference between the FFT and the collocation geoid heights were -0.82 and 0.20 m, respectively. This quite large difference may be due to the different data sampling and noise weighting used by the two methods, but is not yet fully explained.

1. Introduction

In the early 1990'ties the ERS-1 and the Topex/Poseidon satellites will be launched, both equipped with a radar altimeter. The usefulness of the altimeter data for oceanographic purposes will be greatly improved, if we are able to compute a precise height reference surface for an area being investigated, i.e. a regional, relative, geoid. By this we mean that height differences are precisely known, but that all the values may be affected by a common bias. (Clearly, it would be better, if we could compute an absolute geoid, but this will require that e.g. a global gradiometric satellite mission is carried through).

At the Geodetic Institute there has been developed a software package for gravity field modelling "GRAVSOFT", which may be, and have been, used for geoid determination (Tscherning and Forsberg, 1986). The package includes programs for gravity modelling using collocation, (GEOCOL), and FFT (GEOFOUR), as well as programs for the estimation of statistical parameters for the gravity field (EMPCOV, COVFIT).

It is our intension to use GRAVSOFT for geoid determination as a part of our participation in the ESA ERS-1 project. Therefore we wanted to test the programs in a kind of worst-case situation, namely where the geoid variation was large. On the other hand, the distribution of the gravity data should be good, and nearly no oceanographic phenomena should influence the satellite altimeter data, which we wanted to use in our evaluation. Such a situation is found in the central Mediterranean Sea, see Fig. 1.

In the following we will describe the data and the result of the evaluation.

2. Available Data

Gravity data were made available to us by D. Arabelos, University of Thessaloniki, in the form of data in a 0.125^{0} grid digitized from the maps published by Morelli et al.(1975). Data in the sea area shown in Fig. 1 was used.

Since the use of the FFT requires data to be available in a regular grid all the missing values were predicted using a fast collocation procedure implemented in the program module GEOGRID. On the other hand does collocation not permit the use of all the 4194 values, since a full system of equations with this dimension must be solved. Therefore, when using collocation for geoid computation, only the 0.25^{0} grid points were used outside the 4^{0} $x4^{0}$ inner area, where the geoid was computed.

Cross-over adjusted SEASAT-data (Cruz and Rapp, 1982) were made available to us by R.H. Rapp. A local cross-over adjustment, using the data in the $4^{0}x4^{0}$ area, made the standard deviation of the cross-over values of the six used tracks decrease from 0.05 cm to 0.02 cm. Raw GEOSAT data were also adjusted, and cross errors (mainly due to data over land, see Fig. 1) were removed. The data covered a 1/2 year period, and contained therefore up to 10 repeat tracks. Originally the dataset consisted of 3096 points, which before the removal of gross errors had a standard deviation of 5.53, and with 97 values removed had a standard deviation of 2.87 m. The result of a cross-over adjustment with only bias gave a standard deviation of the cross-over differences of 0.05 m compared to 5.29 m before the adjustment.

3. Gravimetric Geoid Computations

First the contribution of the spherical harmonic expansion GPM2 was subtracted. Using these "reduced" values, empirical auto- and cross -covariance functions were estimated by EMPCOV, using the gravity and the GEOSAT data, regarded as geoid heights. An analytic expression for the covariance function was then determined using COVFIT (Knudsen,1987),

$$C(\psi) = a \cdot \sum_{i=o}^{N} e_i \, P_i(\cos\psi) + \sum_{i=N+1}^{\infty} \frac{A(i-1)}{(i-2)(i+4)} \left(\frac{R_B}{R}\right)^{2i+4} P_i(\cos\psi).$$

Here ψ is the spherical distance between two gravity anomaly values (at the sea surface), e_i the error degree-variances of GPM2, a, A scale factors and R_B the radius of the so-called Bjerhammar sphere. R is the mean radius of the Earth, and P_i are the Legendre polynomials. Values of N=120, a=0.88, A=444 and R-R_B=3.75 km was found to give a nearly perfect agreement between the analytic expression and the empirical auto- and cross covariances.

The gravity data were then used to compute geoid heights for the $4^{0}x4^{0}$ area. The use of collocation look more than 10 times as long time as the use of FFT. A comparison with the altimeter measurements were then made, and the results are given in Table 1. In Fig. 2 are shown the FFT, collocation and GEOSAT heights along the longest track in the open sea.

Table 1. Comparison of FFT and collocation gravimetric geoids with SEASAT and GEOSAT adjusted altimeter heights.

	Mean m	Standard Dev. m
GEOSAT-data with GPM2 subtracted	-1.24	0.62
Difference GEOSAT-FFT geoid	-2.18	0.15
Difference GEOSAT-Collocation geoid	-1.37	0.15
Difference SEASAT-FFT geoid	-0.54	0.20
Difference FFT-Collocation geoid	-0.82	0.20

The difference between the FFT and collocation geoid heights are shown in Fig. 3. The large mean difference and standard deviation may be caused by the way the two methods accounts for the long-wavelength information. Also the standard deviation of the differences is surprisingly large, considering that both methods agree so well with the GEOSAT data.

A detailed analysis of the differences between the GEOSAT heights and the gravimetric geoid heights along the individual tracks, see Fig. 4, showed that altimeter data close to the coast (<50 km distance) have a larger variation than points at the open sea. This indicates a possible coastal current, the existence of which must be verified.

4. Conclusion

The result of the investigation shows (as expected) that the GEOSAT data in this area are slightly superior to the SEASAT data. Also, considering the error in the altimeter data, we have demonstrated that it is possible to compute a regional, relative geoid, at the decimeter level, using the GRAVSOFT programs. It is obvious, that FFT should be used if the data configuration and quality permits it. Otherwise collocation should be used, since it puts few requirements on the data configuration, and also makes it possible to include the adjusted altimeter data as observations. The quite large differences between FFT and collocation must be further studied.

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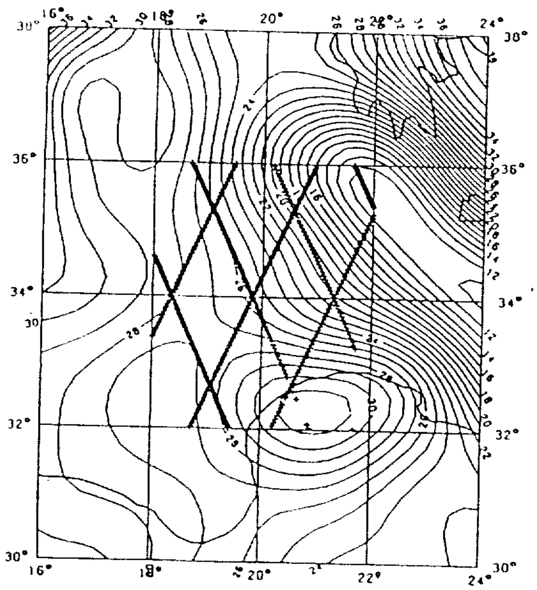

Fig. 1. Geoid heights (contour interval 1 m) derived from GPM2 with the used GEOSAT tracks.

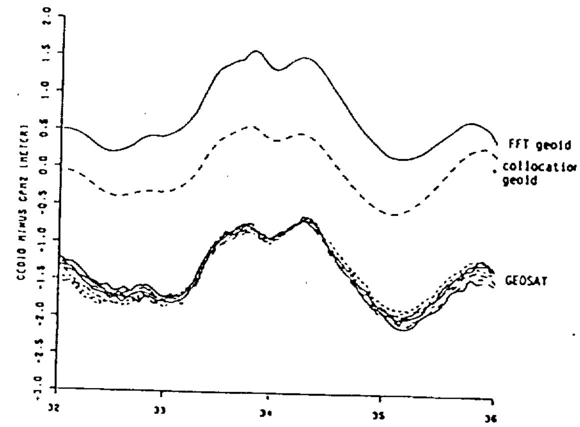

Fig. 2. Adjusted GEOSAT heights and corresponding FFT and collocation geoid heights along the longest track. The contribution from GPM2 is subtracted.

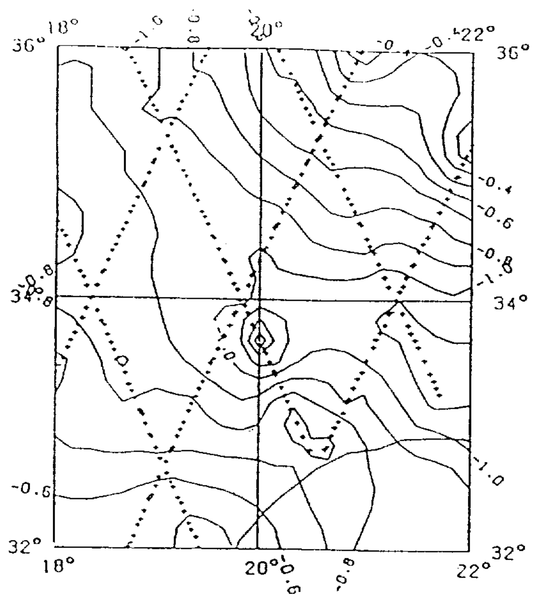

Fig. 3. Difference between geoid heights computed by FFT and collocation. Contour interval 0.1 m.

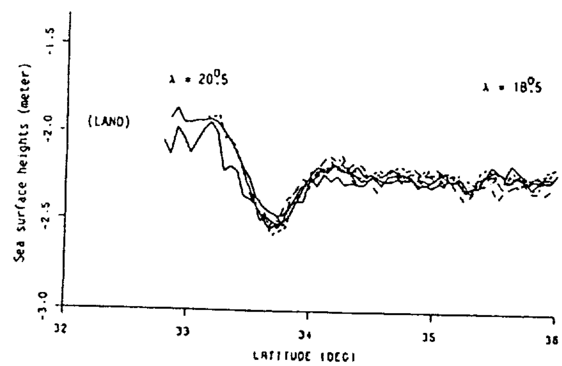

Fig. 4. Sea-surface heights computed as the difference between the GEOSAT heights and the gravimetric geoid heights computed by FFT.

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