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Performance of FFT methods in local gravity field modelling

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Abstract and introduction

Fast Fourier transform methods provide a fast and efficient means of processing large amounts of gravity or geoid data in local gravity field modelling. The FFT methods, however, has a number of theoretical and practical limitations, especially the use of flat-earth approximation, and the requirements for gridded data. In spite of this the method often yields excellent results in practice when compared to other more rigorous (and computationally expensive) methods, such as least-squares collocation.

The good performance of the FFT methods illustrate that the theoretical approximations are offset by the capability of taking into account more data in larger areas, especially important for geoid predictions. For best results good data gridding algorithms are essential. In practice "truncated" collocation approaches may be used. For large areas at high latitudes the gridding must be done using suitable map projections such as UTM, to avoid trivial errors caused by the meridian convergence.

In the paper the FFT methods are compared to "ground thruth" data in New Mexico (ξ,η from Δg), Scandinavia (N from Δg, the geoid fits to 15 cm over 2000 km), and areas of the Atlantic (Δg from satellite altimetry using Wiener filtering). In all cases the FFT methods yields results comparable or superior to other methods.

Gravity field modelling by FFT methods

The Fourier transformation methods have become practical tools in geodesy after the advent of high-degree (180x180) spherical harmonic expansions of the geopotential. Using such a geopotential reference model, the residual field may with good approximation be treated within the flat-earth approximation of the Fourier methods.

For the (residual) anomalous potential T, two-dimensional Fourier transformation

$$\tilde{T}(k_x,k_y) = \iint_{-\infty}^{\infty} T(x,y)\; e^{-i(k_x x + k_y y)}\,dxdy \qquad (1)$$

yields the familiar simple frequency domain relationships

$$\begin{aligned} &\text{- upward continuation:} && \tilde{T}(k_x,k_y,z) = \tilde{T}(k_x,k_y)\; e^{-kz} \\ &\text{- geoid prediction:} && \tilde{N} = \frac{1}{\gamma}\frac{1}{k}\,\widetilde{\Delta g} \qquad (2) \\ &\text{- deflections:} && \left.\begin{matrix}\tilde{\xi}\\ \tilde{\eta}\end{matrix}\right\} = -\frac{i}{\gamma}\frac{1}{k}\left\{\begin{matrix}k_x\\ k_y\end{matrix}\right\}\widetilde{\Delta g} \end{aligned}$$

where γ is normal gravity and $k = (k_x^2 + k_y^2)^{1/2}$ the radial wavenumber. The above equations may be evaluated by the Fast Fourier Transform algoritm, and the final results obtained by an inverse transform. The use of FFT requires gridded data, and introduces errors due to periodicity assumptions and aliasing. For details see, e.g., Kearsley et al. (1985).

The "inverse Stokes" transformation N to Δg is a high pass filtering operation, often seriously affected by noise when data is from satellite altimetry. In this case Wiener filtering may be used. Assuming Kaula's rule to be valid, the PSD of the geoid heights ϕ_{NN} will decay like k^{-4} (Forsberg, 1984). In the presence of white noise w in data, the optimum Δg-estimate will be

$$\tilde{\Delta g} = k \frac{\phi_{NN}}{\phi_{NN} + \phi_{ww}} \tilde{\gamma N} = k \frac{1}{1 + ck^4} \tilde{\gamma N} = k\alpha(k)\tilde{\gamma N} \tag{4}$$

The constant c depends on data noise and local variability of the gravity field. It may be specified indirectly through the wanted resolution of the solution, the resolution here defined as the wavelength corresponding to the wavenumber k_R with $\alpha(k_R) = 0.5$. Resolutions around 20 km give good results.

The FFT prediction examples in the sequel have been done using the GI programme modules GEOGRID (fast gridding using truncated collocation with a second-order Markov model), GEOFOUR (FFT manipulation), and TCIP (interpolation from result grids with local splines).

Examples of gravity field modelling by FFT

1. Δg to (ξ,η) - White Sands area, New Mexico.

Gravity data was gridded in a 4^o x 3^o area on a 2' x 2' grid using terrain reduced data. The FFT prediction results were compared to 384 deflection pairs from astrogeodetic observations, yielding

	ξ"	η"
R.m.s. observed deflections	2.69	6.16
Difference FFT minus observed	0.73	0.85

For a comparison r.m.s. prediction accuracies in the range 0.9-1.0" for ξ and 1.1-1.8" for η were reported in Kearsley et al (1985), using the same data with four different prediction methods (ring integration, collocation, collocation/integration, and FFT on 5' mean data)

2. Δg to N for a large area - Scandinavia.

The geoid was computed for the region 54^o-71^oN, 4^o-32^oE by two FFT solutions gridding the available gravity data on either a 6' x 12' geographical grid, or a 10 km x 10 km UTM grid (zone 33). The FFT results are compared to an earlier comprehensive blocked collocation prediction consisting of 26 3^ox 6^o solution blocks (Tscherning and Forsberg, 1986), and all solutions are compared to 41 GPS derived geoid undulations (fig. 1) observed by the Institut fur Erdmessung, Hannover (Denker, pers.comm.).

The FFT solutions were both based on GPM2 and a thinned out gravity data set (6' x 12' pixels), as for the collocation solution, with some new data added in northern Germany and off northern Norway. A total of some 20000 gravity stations were gridded onto a 210x180 (geographic) or 256x256 point grid (UTM). The comparison of the predicted geoids to the GPS results are shown below and in fig. 2.

Geoid comparison at 41 GPS stations (m):	mean	std.dev.
Difference FFT (geographic) minus GPS	0.07	0.17
- FFT (UTM zone 33) - GPS	-0.09	0.15
- collocation - GPS	-1.31	0.54
Difference FFT (geographic) - FFT (UTM)	0.16	0.04

The FFT solutions are clearly superior to the collocation solution, illustrating the benefit of being able to take more data into account. The UTM

FFT solution provided best results as expected (GRS80 bias uncertain), with the difference between the two FFT solutions being quite large: On 1044 $0.5^{o}x1^{o}$ grid points the difference had a mean value of 15 cm, std.dev. of 12 cm, and a maximal discrepancy of 45 cm, unacceptable for a precision geoid.

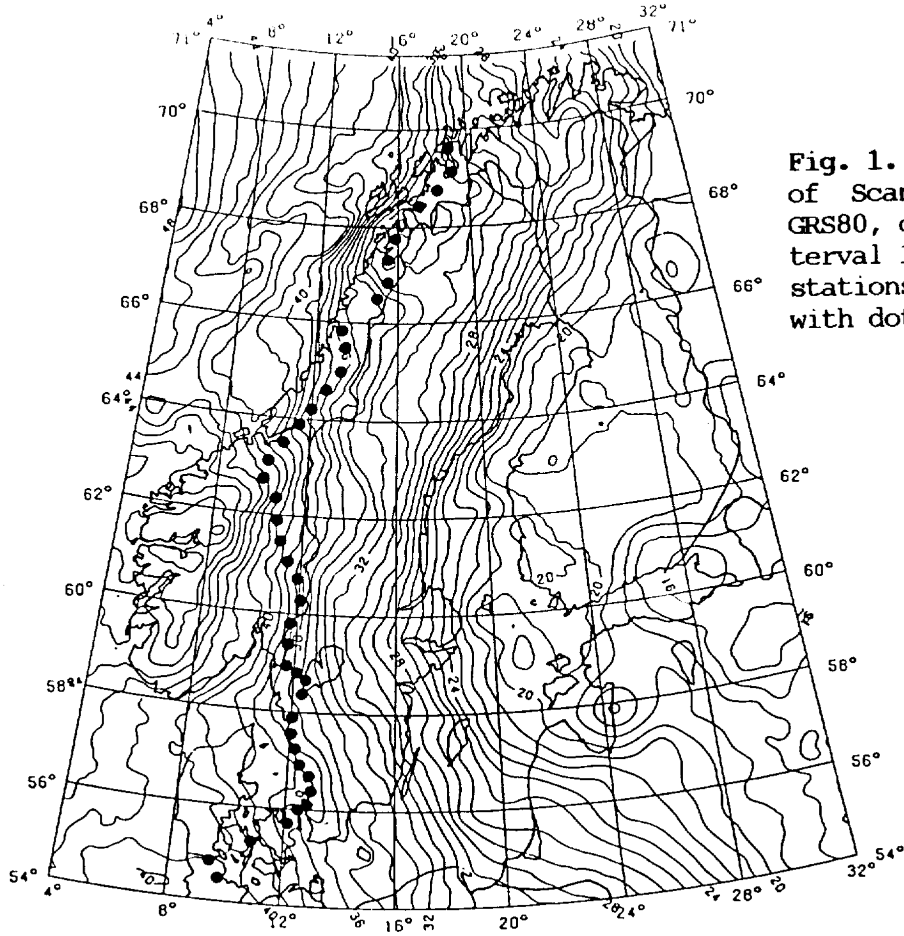

Fig. 1. FFT geoid of Scandinavia in GRS80, contour interval 1.0 m. GPS stations shown with dots.

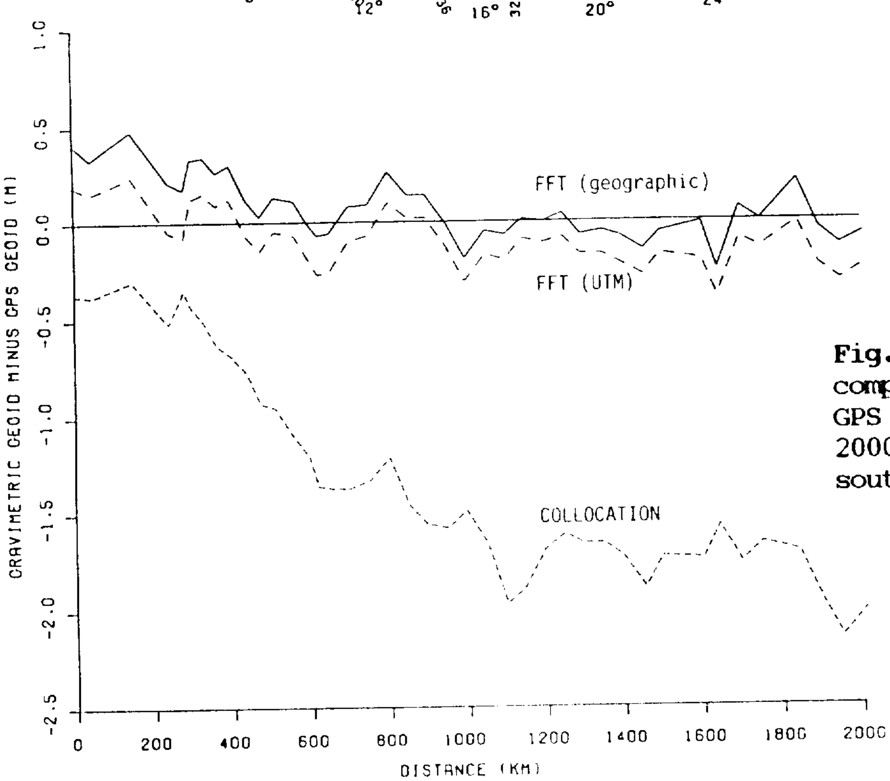

Fig. 2. Comparison of computed geoids versus GPS results along the 2000 km profile from south to north.

3. N to Δg - satellite altimetry off Norway and Puerto Rico trench.

In this example Wiener filtering with resolution k_R^{-1} at 20-30 km is used. With the simple FFT approach useful Δg values were not obtained. In both cases the satellite altimetry data was submitted to a local bias cross-over adjustment prior to gridding and FFT.

The test area S of Lofoten, Norway, is a shelf area. Available SEASAT data within a 5°x 8° were gridded at 3'x 6' using GPM2 as reference field. The adjusted orbits had r.m.s. cross-overs at 8 cm. The computed gravity anomalies are compared to recent SK gravimetry in a 2.5°x 4° area (fig. 3). For comparison a collocation solution (GEOCOL) was also done.

In the Puerto Rico area combined SEASAT/GEOS3 data were compared to all ship gravimetry in a 3°x 3° area over the trench. The r.m.s. crossovers after adjustment was 31 cm. The FFT solution was done by a 6'x 6' grid in a 8°x 8° area (without reference field). Results are shown below, together with OSU collocation results (Kadir, Knudsen and Rapp, pers.comm).

Area	Δg observed		FFT difference		Collocation diff.	
	mean	std.dev.	mean	std.dev.	mean	stddev.
Norway	12.2	33.8	-1.0	10.9	1.2	12.8
Puerto Rico	-125.8	130.5	-12.7	15.3	-10.5	15.9

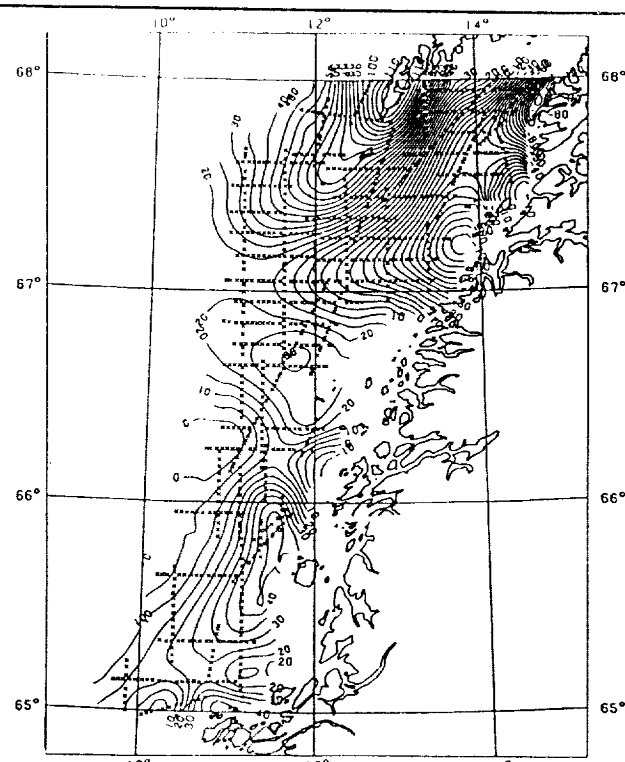

Fig. 3. Gravity anomalies computed by FFT from SEASAT altimetry south of Lofoten, Norway. 5 mgal contour interval. Comparison gravimetry tracks shown with crosses.

Conclusions

The FFT methods work, theoretical approximations are offset by the ability to handle more data. For large blocks proper map projections must be used.

References

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