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Geographically Correlated Orbit Error

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ABSTRACT

The dominant error source in estimating the orbital position of a satellite from ground based tracking data is the modeling of the Earth's gravity field. The resulting orbit error due to gravity field model errors are predominantly long wavelength in nature. This results in an orbit error signature that is strongly correlated over distances on the size of ocean basins. *Anderle and Hoskin* [1977] have shown that the orbit error along a given ground track also is correlated to some degree with the orbit error along adjacent ground tracks. This cross track correlation is verified here and is found to be significant out to nearly 1000 kilometers in the case of TOPEX/POSEIDON when using the GEM-T1 gravity model. Finally, it is has been determined that even the orbit error at points where ascending and descending ground traces cross is somewhat correlated. The implication of these various correlations is that the orbit error due to gravity error is geographically correlated. Such correlations have direct implications when using altimetry to recover oceanographic signals.

1. INTRODUCTION

In order to fully exploit the scientific value of satellite altimetry measurements it is necessary to have accurate independent knowledge of the orbital position of the satellite. To date, it has not been possible to determine the position of altimetric satellites to a level of accuracy commensurate with the altimeter measurement accuracy. Thus the determinations of sea surface height above some reference surface are primarily corrupted by the errors in the estimate of the satellite altitude. In turn, these errors in the satellite altitude are primarily caused by errors in the modeling of the Earth's gravity field. Thus, the error spectrum of the sea surface heights is driven by the error spectrum of the satellite orbit altitude errors. This spectrum can be evaluated under some limiting assumptions and if the error spectrum of the Earth's gravity model is known. Evaluation of the spectrum will aid in determining to what extent the sea surface height data can be used in the detection of oceanographic signals.

This paper will principally address the geographic characteristics of the orbit error spectrum. That is, how large is the error in any given geographic region and how much is the orbit error in any one location correlated with the orbit error at another location. This question was originally addressed by *Anderle and Hoskin* [1977] who found a large correlation of the error along the direction of satellite motion and a much smaller though significant correlation of the error for adjacent ground traces. Their results were obtained through a simulation of the SEASAT orbit error based on the difference between two contemporary gravity models. The approach used here is analytic in nature and will be applied to the proposed TOPEX/POSEIDON orbit. Additionally, the gravity model error will be specified by the gravity error covariance of the recent GEM-T1 model [*Marsh, et. al.*, 1988].

2. ANALYTICAL DEVELOPMENT

The objective is to obtain the variance and covariance of sea surface height errors (derived from satellite altimetry) as a function of geographic location. In this development it will be assumed that these errors are solely a result of satellite altitude error due to gravity model error. Thus, the variance and covariance of the sea surface height errors will be equivalently represented by the variance and covariance of the orbit altitude errors. Also, it is assumed that the satellite geodetic altitude error can be accurately approximated by the satellite radial orbit error.

To obtain the variance and covariance of the radial orbit error due to gravity model error requires a relationship between the radial error and the error in the gravity model parameters. Such a relationship can be obtained through application of Kaula's solution [*Kaula*, 1966] which gives the orbit element perturbations based on a spherical harmonic development of the gravity field. The general result for the corresponding radial orbit perturbation is given in *Rosborough and Tapley* [1987]. This result, which gives the radial orbit error as a function of time, also can be expressed as a function of the satellites geographic location if the orbit is assumed to be nearly circular (as in the case of altimetry satellites) [*Rosborough*, 1986; *Engelis*, 1987]. The functional form of this geographic representation of the radial orbit error, δr, due to gravity model error is,

$$\delta r = \sum_{l=2}^{\infty} \sum_{m=0}^{l} [\Phi_{lm}(\phi)(\delta C_{lm} \cos m\lambda + \delta S_{lm} \sin m\lambda) \pm \Phi^*_{lm}(\phi)(\delta C_{lm} \sin m\lambda + \delta S_{lm} \cos m\lambda)]$$

where δC_{lm} and δS_{lm} are the errors in the gravity coefficients of degree l and order m, and Φ_{lm} and Φ^*_{lm} are functions of the satellites body-fixed latitude, ϕ, and mean orbital elements, and λ is the satellites body-fixed longitude. The choice in signs is determined by the satellites directional motion. If the satellite is on an ascending track then the sign is positive, if the satellite is on a descending track then the sign is negative.

This relationship shows the radial orbit error to be stationary along the ascending and descending tracks. That is, the orbit error repeats exactly along any given ground track. This is not the case for actual orbits which have been determined from ground based tracking. To emulate the real case requires this equation to be augmented with additional terms that account for the effect of errors in the initial conditions [*Colombo*, 1984]. These errors will differ for each determination of the initial conditions depending on the distribution of the tracking data. More importantly the resulting radial error due to initial condition error will in general not be stationary in the geographic frame. This effect will certainly augment the results of the next section which are based on the above equation alone. Future studies will attempt to introduce the initial condition error effect.

Given the above linear relationship between the gravity coefficient errors and the radial orbit error it is possible to determine the radial error variance and covariance if the gravity error covariance is known. In this context, the variance of the radial orbit error is the uncertainty in the radial component at any given geographic location, and the covariance is the correlation between the radial error at two different locations. Such variances and covariances have been evaluated using the GEM-T1 gravity error covariance and the mean orbital elements of TOPEX/POSEIDON. These results are given in the following section.

3. RESULTS

The standard deviation of the radial orbit error for TOPEX/POSEIDON due solely to the errors in the GEM-T1 gravity model are given in Figures 1 and 2. These figures show the standard deviation of the error along the ascending and descending tracks respectively. Again it should be emphasized that these results only account for the gravity error. In an actual orbit determination problem the resulting error will differ depending on the accuracy and distribution of tracking data and how much of the gravity induced orbit error can be absorbed through an adjustment of the initial conditions. Nonetheless, these figures illustrate how gravity error, in an ideal case (no initial condition error), is mapped into the radial component of the orbit. There is an obvious strong along track correlation of the error (which will be directly evaluated) and an obvious geographic correlation of the error.

The geographic correlation appears to be a consequence of the non-global distribution of tracking data that was used to construct the GEM-T1 model. Since GEM-T1 was based solely on satellite tracking data this non-global distribution is inevitable. Models which include surface

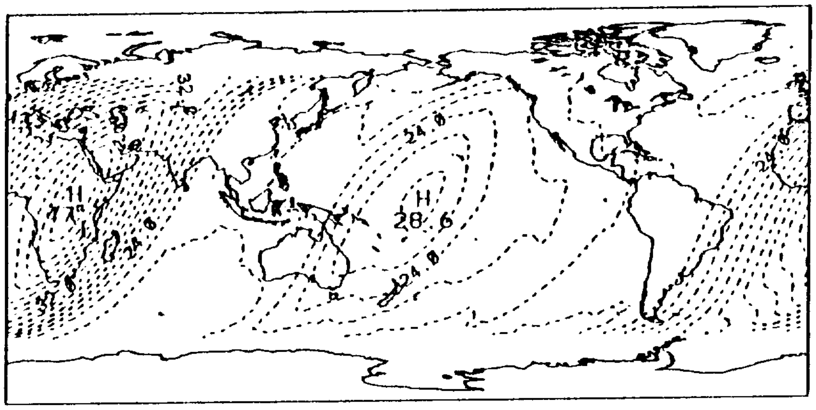

Figure 1: Standard deviation of the TOPEX/POSEIDON radial orbit error along the ascending ground tracks, using the GEM-T1 gravity error covariance. The contour interval is 2 cm.

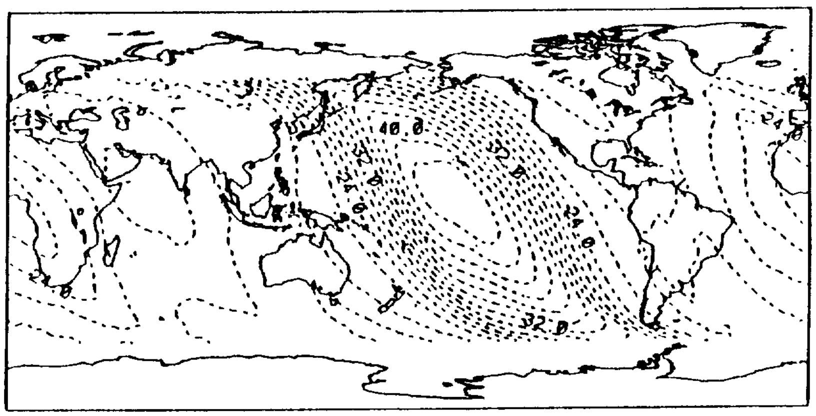

Figure 2: Standard deviation of the TOPEX/POSEIDON radial orbit error along the descending ground tracks, using the GEM-T1 gravity error covariance. The contour interval is 2 cm.

gravity data and altimeter data can be expected to have much better global characteristics and the resulting orbit error would not be expected to be as non-uniform as the case for this satellite only model.

To demonstrate the spatial correlation of the radial orbit error an example is presented that shows how the error at one geographic location is correlated with the errors in the neighboring geographic region. The reference point is located at 200° East longitude on the equator and and corresponds to the radial error along an ascending track passing over that point. The correlation of this error with the errors on all other ascending tracks through the region is given in Figure 3. The correlation value of 1.0 locates the reference orbit error with which all the other errors are correlated against. This plot clearly shows the strong along track correlation and it also shows the significant cross track correlation. That is, the errors on adjacent ascending tracks have significant correlation even when separated by distances of up to a 1000 km. The correlation distances evident in Figure 3 compare with those found by *Anderle and Hoskin* [1977] although the cross track correlation appears stronger here due to the higher altitude of TOPEX/POSEIDON as compared to SEASAT (thus, TOPEX/POSEIDON is not as sensitive to the higher degree coefficient errors which would decorrelate the orbit error at shorter scales).

The spatial correlation of the radial errors at crossover points has also been investigated. In this case, at a given crossover location (where an ascending track crosses a descending track) the correlation between the error on each track is computed. Overall the correlations were found to vary between ±.3 for the case of TOPEX/POSEIDON and using the GEM-T1 gravity error covariance. This is very significant for those applications that attempt to use crossover data to remove the remaining orbit error in an altimetric satellite ephemerides. Due to the correlations of the error along the two tracks (even though widely separated in time) the crossovers do not

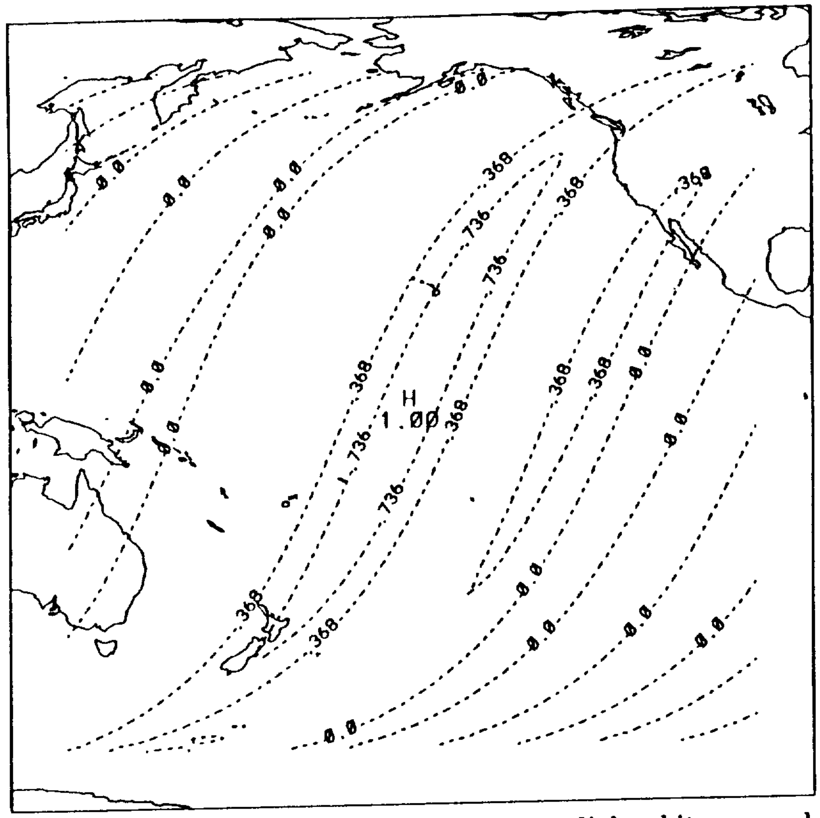

Figure 3: Correlation between the TOPEX/POSEIDON radial orbit error along ascending tracks with the ascending error located at 200° East longitude on the equator. The contour interval is 0.368.

provide an absolute measure of the radial orbit error. In general there is some component of the orbit error (usually referred to as the geographically correlated component) that is unobservable in the crossovers.

4. Conclusions

Orbit error due to gravity model error is very systematic when examined geographically. This strong geographic dependence is a hindrance in the utilization of satellite altimeter data to measure oceanographic signals. The results presented here are based on some simplifying assumptions (near circular orbit) and do not account for other possible model error sources or the effect of fitting the orbit to a set of tracking data. However, it should be expected that the effects presented here will be manifested into the actual orbital ephemerides at some level. Elimination of these correlations can only be accomplished through the elimination of the gravity model error. Thus, it can be expected that ephemerides for altimetric satellites have errors which are strongly correlated and which can in turn be aliased into the oceanographic signals trying to be recovered from the altimetric data.

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