Refinement of Earth's Gravity Field with Topex GPS Measurements

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1. INTRODUCTION

The NASA Ocean Topography Experiment satellite Topex will carry a microwave altimeter accurate to a few centimeters for the measurement of ocean height. The capability can be fully exploited only if Topex altitude can be independently determined to 15 cm or better [*Born, et al.* 1985]. This in turn requires an accurate gravity model. The gravity will be tuned with selected nine 10-day arcs of laser ranging, which will be the baseline tracking data type, collected in the first six months of Topex flight. Topex will also carry onboard an experimental GPS flight receiver capable of simultaneously observing six GPS satellites above its horizon to demonstrate the capability of GPS carrier phase and P-code pseudorange for precise determination of the Topex orbit. It has been found that sub-decimeter orbit accuracy can be achieved with a mere two-hour arc of GPS tracking data, provided that simultaneous measurements are also made at six or more ground tracking sites [*Yunck, et al.* 1986]. The precision GPS data from Topex are also valuable for refining the gravity model. This paper presents an efficient technique for gravity tuning using GPS measurements. Unlike conventional *global* gravity tuning, this technique solves for far fewer gravity parameters in each filter run. These gravity parameters yield *local* gravity anomalies which can later be combined with the solutions over other parts of the earth to generate a *global* gravity map. No supercomputing power will be needed for such combining. The following describes the approaches used in this study and presents preliminary results of a covariance analysis.

2. GRAVITY ANOMALY INFORMATION CONTENT IN TOPEX GPS MEASUREMENTS

In this section a theoretical formulation is derived for the information content of GPS measurements, in particular carrier phase, from Topex for the recovery of gravity anomalies. The acceleration (force) on Topex in a rotating local $\mathbf{h}-\mathbf{c}-\boldsymbol{\ell}$ coordinate (Fig. 1), where $\mathbf{h}$ is the radial, $\mathbf{c}$ the cross-track and $\boldsymbol{\ell}$ the down-track directions, can be written as

$$\ddot{\mathbf{h}} = \frac{\Delta\dot{\mathbf{h}}}{\Delta t} - \frac{\dot{\boldsymbol{\ell}}^2}{(r_E+\mathbf{h})} ,$$

$$\ddot{\mathbf{c}} = \frac{\Delta\dot{\mathbf{c}}}{\Delta t} ,$$

and

$$\ddot{\boldsymbol{\ell}} = \frac{\Delta\dot{\boldsymbol{\ell}}}{\Delta t} + \frac{\dot{\mathbf{h}}\dot{\boldsymbol{\ell}}}{(r_E+\mathbf{h})}$$

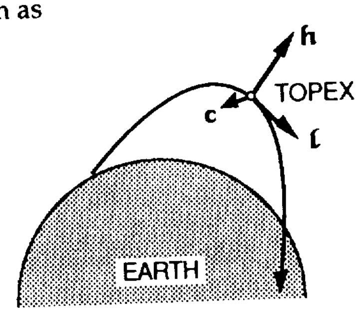

Fig. 1. Rotating $\mathbf{h}-\mathbf{c}-\boldsymbol{\ell}$ coordinate

where single and double dots denote first and second time derivatives, respectively, and r_E is earth's mean radius. For given uncertainties in Topex velocity and its change over a time increment Δt, the corresponding uncertainty in the determination of the force is

$$\sigma_{\ddot{\mathbf{h}}} < \frac{\sigma_{\Delta\dot{\mathbf{h}}}}{\Delta t} + \frac{2\dot{\boldsymbol{\ell}}\sigma_{\dot{\boldsymbol{\ell}}}}{(r_E+\mathbf{h})} , \qquad \sigma_{\ddot{\mathbf{c}}} = \frac{\sigma_{\Delta\dot{\mathbf{c}}}}{\Delta t} \qquad \text{and} \qquad \sigma_{\ddot{\boldsymbol{\ell}}} < \frac{\sigma_{\Delta\dot{\boldsymbol{\ell}}}}{\Delta t} + \frac{\dot{\boldsymbol{\ell}}\sigma_{\dot{\mathbf{h}}}}{(r_E+\mathbf{h})}$$

Previous analyses have indicated that Topex orbit determination with GPS measurements yields

$$\sigma_{\dot{h}},\ \sigma_{\dot{\ell}},\ \sigma_{\Delta\dot{h}},\ \sigma_{\Delta\dot{c}},\ \sigma_{\Delta\dot{\ell}} < 0.5\,\text{mm/sec}$$

Hence, with an altitude h = 1,334 km and nominal velocity $\dot{\ell}$ = 7.2 km/sec for Topex, and for a sampling time of Δt = 2 minutes, local gravity anomaly along a Topex flight path can be determined to an accuracy of better than 0.6 mgal in the radial component and better than 0.5 mgal in the other two components.

3. SIMULATION ANALYSIS

Next, a simulation analysis was performed to numerically estimate the accuracy with which a given, but assumed *unknown*, gravity anomaly can be determined by solving for the coefficients of a number of spherical harmonics. In this analysis, a network of six globally distributed tracking sites is used. These include the three NASA Deep Space Tracking (DSN) Sites at Goldstone, California; Madrid, Spain and Canberra, Australia; and three other sites at Japan, Brazil and South Africa. The "truth" model for the gravity anomaly is assumed to consist of a sum of eleven zonal harmonics $J_{15}, J_{16}, \ldots, J_{25}$, each with a normalized magnitude of 10^{-6}. In the filtering process different subsets of these terms are estimated and the lumped effects of the selected terms are computed and compared with the truth model to assess the accuracy with which the local gravity anomaly can be recovered. Other assumptions used in the analysis are included in Table 1.

Data Type:	0.5-cm GPS Carrier Phase
Data Span:	2 hrs
Data Intervals:	2 minutes
Gravity Anomaly:	$J_{15}, J_{16}, \cdots, J_{25}$ (10^{-6} each, normalized)
Station Coordinates:	DSN Sites: 3 cm each component (fixed) other: 10 cm each component (adjusted)
Clock Bias:	3 μsec (adjusted as white noise)
Carrier Phase Bias:	10 km (adjusted)
GPS Epoch States:	3 m; 0.3 mm/sec (adjusted)
Topex Epoch States:	10 m; 1 cm/sec (adjusted)
Zenith Troposphere:	2 cm

Table 1. Simulation Analysis Model

Fig. 2 shows the results when four different subsets of gravity terms are estimated. Note that good agreement with the truth can be achieved when proper model is used in the estimation (Fig. 2a and 2b). On the other hand, a larger discrepancy occurs if the model is not flexible enough to represent the truth (Fig. 2c and 2d). This leads us to the use of a piecewise constant model for its high flexibility, provided that the step size is fine enough to follow the variation in the truth model. The solution using a piecewise constant model with a 2-minute step size is shown in Fig. 3. Note that the RMS discrepancy between the solution and the truth model (0.54 mgal) is consistent with the theoretical prediction.

4. THE GRAVITY BIN TECHNIQUE

Following the above encouraging findings, a more general and efficient technique was investigated. This technique solves for "gravity bins", which are 3-D positional deviations on Topex, with three parameters at each measurement time point over a period of a few hours. The epoch state of Topex and other pertinent parameters are also simultaneously adjusted. These are related to Topex current state by the following linearized equation.

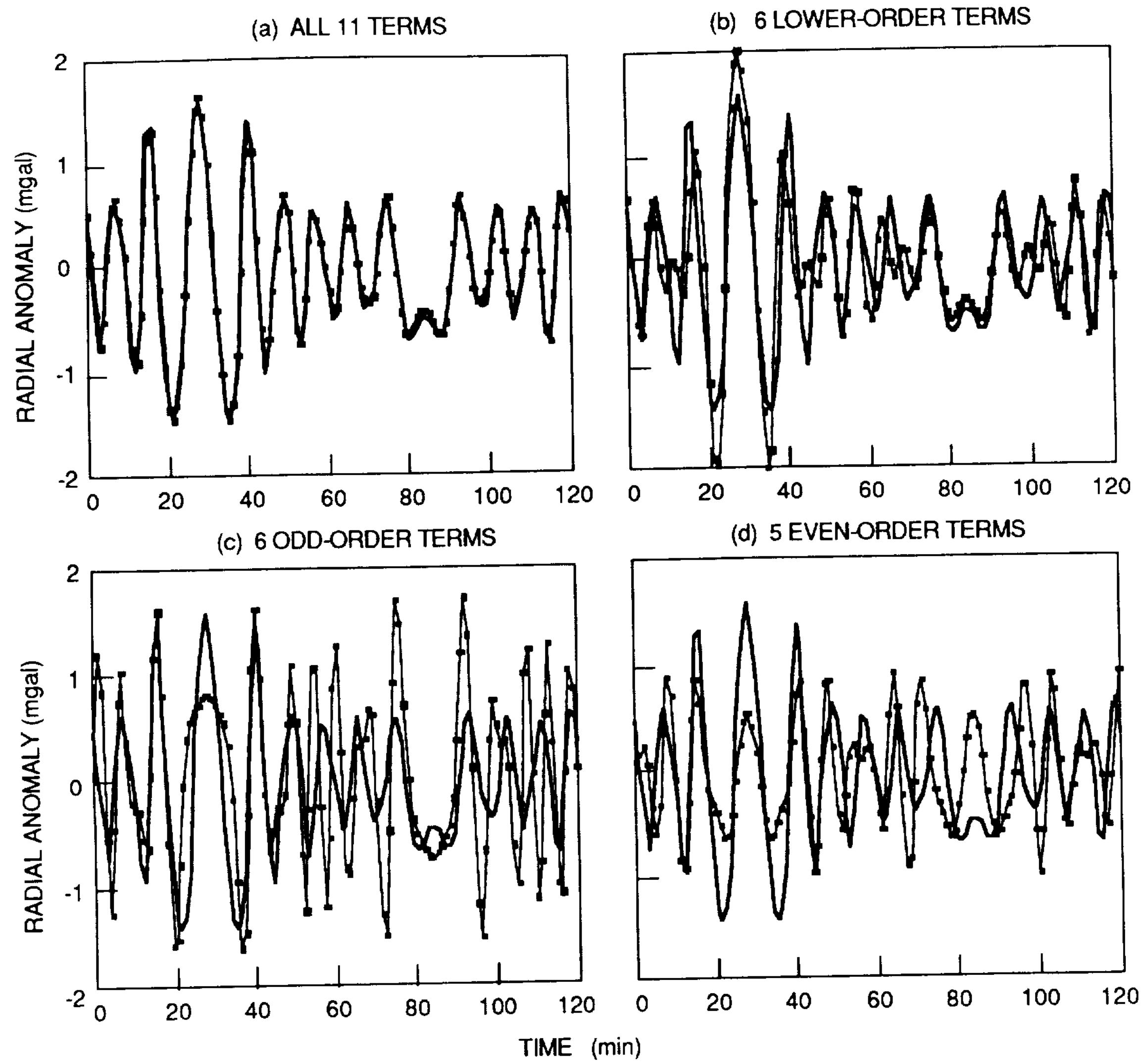

Fig. 2. Sample results of simulation analysis (— truth, -•- solution)

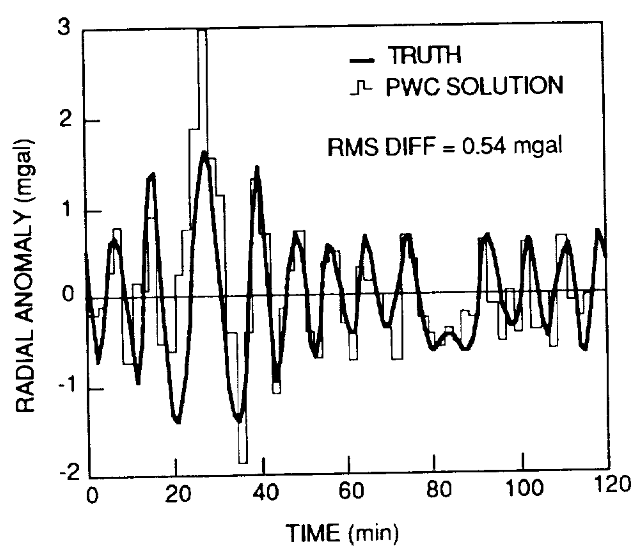

Fig. 3. Simulation result with piecewise constant model

$$r(t) = \frac{\partial r}{\partial r_0} r_0 + \frac{\partial r}{\partial v_0} v_0 + \delta(t)$$

where r(t) is the current position of Topex, r_0 and v_0 are position and velocity at epoch, and δ(t) is the gravity bin parameter at time t. Since δ remains the same for orbits over repeat ground tracks, which will occur every 10 days for Topex, information from repeat orbits can be combined to increase the estimation accuracy. Gravity bins δ for orbits over different ground tracks are independent from one another and can be determined separately. Hence only moderate number of parameters are estimated in each filter process. Local gravity anomaly Δg can be computed from the solutions of gravity bin parameters δ by

$$\Delta g(t) = \ddot{\delta}(t) - \frac{\partial g}{\partial r} \delta(t)$$

where g is the nominal gravity field. The *global* gravity anomaly expressed in terms of spherical harmonics can be constructed from the *local* gravity anomaly by a process similar to the Fourier transformation.

Fig. 4 shows the result of a covariance analysis using a 2.5-minute bin size over a 2-hour data span which is nearly the Topex orbit period. Note that local gravity anomaly can be determined to an accuracy of 0.2 mgal under most circumstances with GPS measurements over one Topex orbit. This accuracy will improve monotonically by combining multiple data arcs over repeat Topex ground tracks [*Wu and Yunck*, 1988]. It is anticipated that gravity anomalies of medium wavelengths (1,500 to 3,000 km) can be greatly refined with GPS measurements onboard Topex.

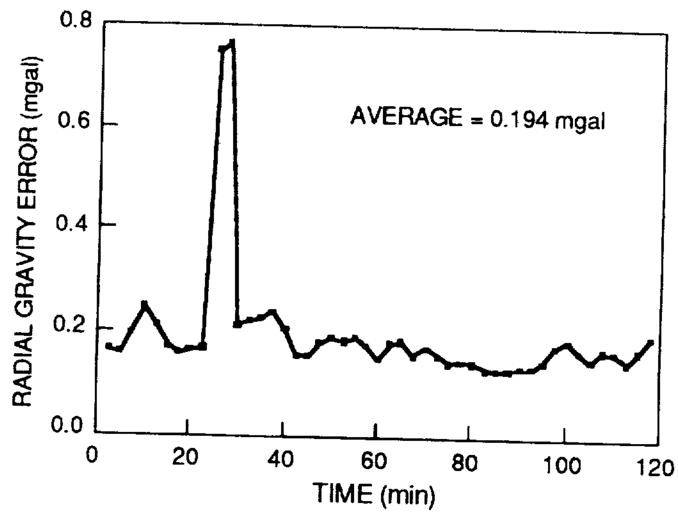

Fig. 4. Performance of gravity bin technique

The computation involved in the transformation of global gravity anomaly from a collection of gravity bins over the entire globe is well known and can be done without the need for a supercomputer. Hence, the gravity bin technique is efficient for global gravity recovery. Note that gravity bins solution can be directly applied as a calibration for the effects of gravity mismodeling on Topex orbit determination. Currently under investigation is the application of the gravity bin technique to Topex with the baseline (laser ranging) tracking data type, where large data gaps exist.

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