GRAVITY FIELD INFORMATION FROM GRAVITY PROBE-B

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ABSTRACT

The GRAVITY PROBE-B Mission will carry the Stanford gyroscope relativity experiment into orbit in the mid 1990's, as well as a GPS receiver whose tracking data will be used to study the earth gravity field. This paper presents estimates of the the likely quality of a gravity field model to be derived from the GPS data, and discusses the significance of this experiment to geodesy and geophysics.

1. INTRODUCTION

By 1995, the GRAVITY PROBE-B (GP-B) spacecraft will carry the Stanford gyroscope relativity experiment, to test the theory of general relativity by measuring two effects predicted by this theory. Both should manifest themselves as very slow precessions of the axis of a spinning body (gyroscope) respect to a frame determined by distant stars, when this body moves relative to a massive object, like the earth. They are illustrated in Figure 1: the geodetic precession occurs because the gyroscope is moving in orbit through curved space-time, around the earth; the motional (or frame-dragging) precession, because the earth is spinning about its own axis. Both effects increase as height decreases, so they are easier to measure from lower orbits. Bright stars like Rigel, to which sensors inside the spacecraft will point, will provide the fixed external reference frame. The gyroscopes themselves (four in number) will be made of fused quartz ground into almost perfect spheres, with a coating of niobium. These will spin while electrostatically suspended inside spherical cavities rigidly attached to a solid block of quartz. The whole assembly will be cooled with liquid helium, so the niobium coating is superconducting. A spinning superconductor developes a magnetic field. For each sphere, this field will point in the direction of the spin axis. Once this field is detected with magnetic sensors, the instantaneous direction of the gyroscope's axis in the frame of the stars can be established. The spacecraft will be nearly axially symmetric, spinning about its axis, to maintain a stable attitude in space and to average out effects due to asymmetries in mass distribution.

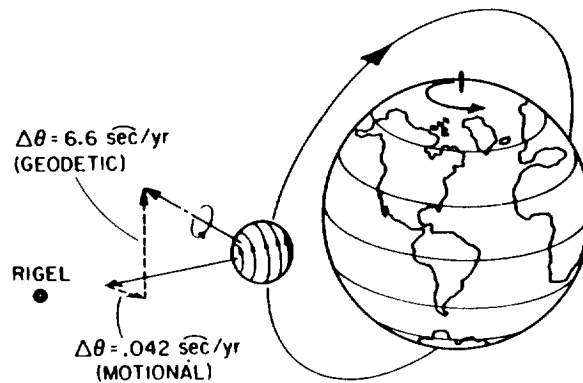## THE TWO GYRO EFFECTS PREDICTED BY SCHIFF

Figure 1. Principle of the Stanford gyroscope experiment. The small spinning sphere is one of the gyroscopes that will orbit the earth on board GP-B.

The spacecraft will also carry a Global Positioning System (GPS) receiver to provide tracking data for studying the earth gravity field and for orbit determination, as well as corner-cube reflectors for geodetic experiments combining GPS and Satellite Laser Ranging (SLR). Also, the spacecraft will have active drag compensation in all three dimensions, to reduce disturbing accelerations on the gyroscopes. To simplify the separation between the relativistic effects, the orbit will be polar. A polar orbit, from the point of view

distribution of the GPS tracking data, provides a complete global coverage, assuming that the full GPS constellation of up to 24 Block II spacecraft is operating. A low, near circular orbit increases the gravitational signal at shorter spatial wavelengths, thus allowing higher resolution in the resulting field model. The drag compensation (which also compensates all the other surface forces, such as solar radiation pressure) will give an orbit shaped purely by gravitation, and thus much "cleaner" from the point of view of geodesy, than those of ordinary, uncompensated, low-orbit satellites.

In coming years, worldwide terrestrial nets of GPS stations are likely to be established for the purpose of providing a framework for geodynamics studies (crustal motion, tides, earth rotation), and for computing very precise GPS ephemerides and satellite clock corrections using tracking data from around the world. The tracking from those stations, or the ephemerides and clock corrections obtained from that tracking, will be needed to analyse the GPS data from the receiver on board GP-B. Conversely, data from a receiver in a relatively low spacecraft like GP-B, obtained simultaneously with that from the ground stations, can be used to improve the geometry of the network by adding to it a moving station high above the ground (Yunk et al., 1985). Therefore, the GPS experiment on GP-B and the operations of the terrestrial network will benefit from each other. Finally, the possibility of tracking GP-B from SLR stations will help to connect and unify the reference frames of the laser and the GPS geodetic networks. At the same time, GP-B will be carrying out its main experiment in relativity. In this way, a single space mission may advance three separate scientific undertakings: one in fundamental physics, and two in geodynamics.

The GP-B mission will follow that of the oceanographic satellite TOPEX, wich will carry a radar altimeter as its main experiment, and will have also a GPS receiver. This receiver will be the first of its kind to operate in space for a prolongued period of time (up to two years). For some studies that can be carried out from the higher orbit of TOPEX as well as from the lower one of GP-B (connection of reference frames, long-wavelength mapping of the gravity field, tidal studies, etc.), the two missions will provide continuity over a period of several years (GP-B is scheduled to begin at about the time when TOPEX is likely to be reaching the end of its useful life).

2. CONCEPT OF A GP-B/GPS GEOPOTENTIAL MAPPING EXPERIMENT

For the purposes of this study, the following assumptions were made:

--- The GP-B spacecraft is drag-free; its orbit is polar, circular, 600 km in altitude, and repeats monthly.

--- The mission lasts for two years, during which an average of 7 GPS satellites from the full constellation of 24 are tracked simultaneously, at all times, from GP-B. The sampling rate is once per second, and there are four simultaneous measurements per GPS satellite: a pseudorange and a carrier phase, in the L1 and also in the L2 bands. The measurements are stored in the spacecraft computer memory and dumped via the TDRSS relay satellites to ground stations, together with telemetry and the relativity experiment data.

--- Mean values of the differences between simultaneous pseudorange and carrier phase measurements, averaged over some tens of minutes (up to a maximum of half a revolution of GP-B, or 40 minutes) are used to estimate the biases in the L1 and L2 carrier phases. The biases are fixed to the integer numbers of wavelengths closest to those averages. If the uncertainty for a given average is much less than one wavelength, the corresponding bias is likely to be resolved. If the uncertainty is large, it is probable that a residual bias remains, which is constant over one averaging interval, but changes value randomly from one interval to the next.

--- The estimated biases are added to the corresponding carrier phase ranges, resulting in full L1 and L2 ranges, which are then combined to correct the effect of ionospheric refraction. The corrected ranges have larger uncertainties than the original L1 and L2 measurements. Next, these corrected ranges are substractd from ranges to the same GPS satellites measured simultaneously from ground receivers. This differencing eliminates the GPS spacecraft clock errors. The resulting single differences are substracted from each other to form double differences, to suppress both the clock errors of the ground receivers and of the receiver on GP-B. At the same time, this double-differencing increases the noise further. Finally, double differences involving four or more GPS satellites altogether, could be combined to estimate geometrically the instantaneous position of GP-B. The uncertainty of the ranges, propagated into those of the x, y, z coordinates (here aligned with the across, along, and radial orbital coordinates), is approximately the uncertainty of the corrected ranges multiplied by the Geometric Dispersion of Precision (GDOP) per coordinate. This quantity depends on the GPS/GP-B geometry at measurement time. The set of measurements can then be compressed, to save work, by estimating from the noisy coordinates of several consecutive points those of a middle (or "normal") point. These normal points then may be used (with their formal accuracies) as

Actual data for the orbits/gravity field parameters estimation. Alternatively, the double differences themselves can be used as data (after being compressed into "normal" data points). In either case, errors in the final data (double differences or coordinates) will be affected by the original measurement errors (unresolved biases, noise), and also by the errors in the ephemerides of GPS.

--- The compressed data set is used to estimate corrections to the orbits of both GP-B and of all GPS satellites involved, simultaneously with the potential coefficients of the gravity field up to a high degree and order. This is done by a least squares technique, using a mathematical description of the compressed data linearized about the a priori values of the orbit and gravity field parameters.

3. MISSION ERROR ANALYSIS

The expected accuracies of estimates of potential coefficients obtained from instantaneous x, y, z data, derived from GPS ranges as indicated in the previous section, have been calculated by setting up and inverting the normal matrix of the adjustment of those parameters. An approximate analytical theory described in Colombo (1986, Ch. 1), based on the linearized dynamics of a circular orbit, has been used to derive a mathematical model for the gravitational perturbations of the coordinates of GP-B. The theory is based on the trigonometric expansions of the gravitational potential and accelerations, when these are given as functions of time. These expansions become true Fourier series when the orbit repeats precisely, their fundamental frequency being that of this repeat, and the orbital perturbations in x, y, and z are also Fourier series with the same frequencies. As explained in the extended abstract on the analysis of satellite gradiometry (Colombo, this issue), use of this model leads to a normal matrix that is very sparse. Moreover, the non-zero elements can be computed analytically, and with a proper arrangement of the unknowns, the matrix becomes block-diagonal. All this permits the very efficient calculation of the inverse, and thus of the variances and covariances of the estimated coefficients. In practice, the more conventional techniques current today in space geodesy are likely to be used to analyze the GPS data, because of their greater flexibility and accuracy. However, those methods would have to be implemented in a supercomputer, given the unusually large size of the adjustment, and work to this end is underway at GSFC. The approach adopted here is good enough for guessing the uncertainties in the potential coefficients obtained by those methods, and requires little effort and modest computer resources.

Measurement errors have been treated as consisting of 1 cm rms white noise in the carrier phase, after correcting for the ionosphere; residual biases have been treated as 10 cm process noise (before double differencing and making the ionospheric correction), with a triangular covariance function and a 15 minutes' correlation length. This error has been propagated into those of the instantaneous x, y, z coordinates, considering the effect of the ionospheric correction, double differencing, and the geometric dispersion of precision. The GDOP per coordinate was assumed to be, on average, 1.7, based on a study by Martin and McCarthy (1987) for a GPS/space shuttle mission. The GPS antenna on GP-B is supposed to be "looking up" (i.e., not tracking any satellites below the local horizontal plane of the receiver).

A worst and a best case have been studied, too pesimistic and too optimistic, respectively, providing upper and lower bounds for the errors in the coefficients. The worst case assumes the loss of all information with the same frequency content as that of the orbit errors (multiples of once per revolution of GP-B plus/minus multiples of one cycle per twelve hours, which is the orbital frequency of GPS, spread out further over plus/minus one cycle per day, supposing that one-day orbits are estimated). It also assumes residual 10 cm standard deviation for the 15 minutes' residual range biases. The best case assumes that all biases have been resolved and there are no GPS orbit errors. The main reason for obtaining an upper and a lower bound was the difficulty of dealing with the orbit errors of GPS while using the approximate, analytical method adopted here. Figure 2 shows the mean of the expected errors in the normalized potential coefficients, per degree ([degree variance/(2n+1)]$^{1/2}$), as a function of the spherical harmonic degree n. The curves for the best and worst cases are very close to each other, narrowly bracketing the actual error, except at very low degrees. Clearly, unresolved biases and orbit errors (both low frequency effects) are the main reason for the low-degree departures. There is a gradual raise followed by a quick exponential one (appearing in the log-linear plot as a straight line), mostly due to attenuation of the signal with height. Where this line intercepts the spectrum of the full gravitational signal (top), the error is 100% of the signal, and the correponding degree n indicates the maximum resolution possible with the data. In this case this degre is close to n = 65, so the size of the smallest detail of the gravity field that can be resolved on the earth's surface is about 300 km, when using GPS data from GP-B. Figure 2 also shows the accuracy spectra of the gravity model GEM-T1, and of those models likely to result from the combination of conventional tracking with altimetry in the early 1990's (dotted line). GP-B/GPS models are likely to be more than two orders of magnitude better through degree and order 20, and substantially better beyond that. The improvement at low degrees looks particularly impressive.
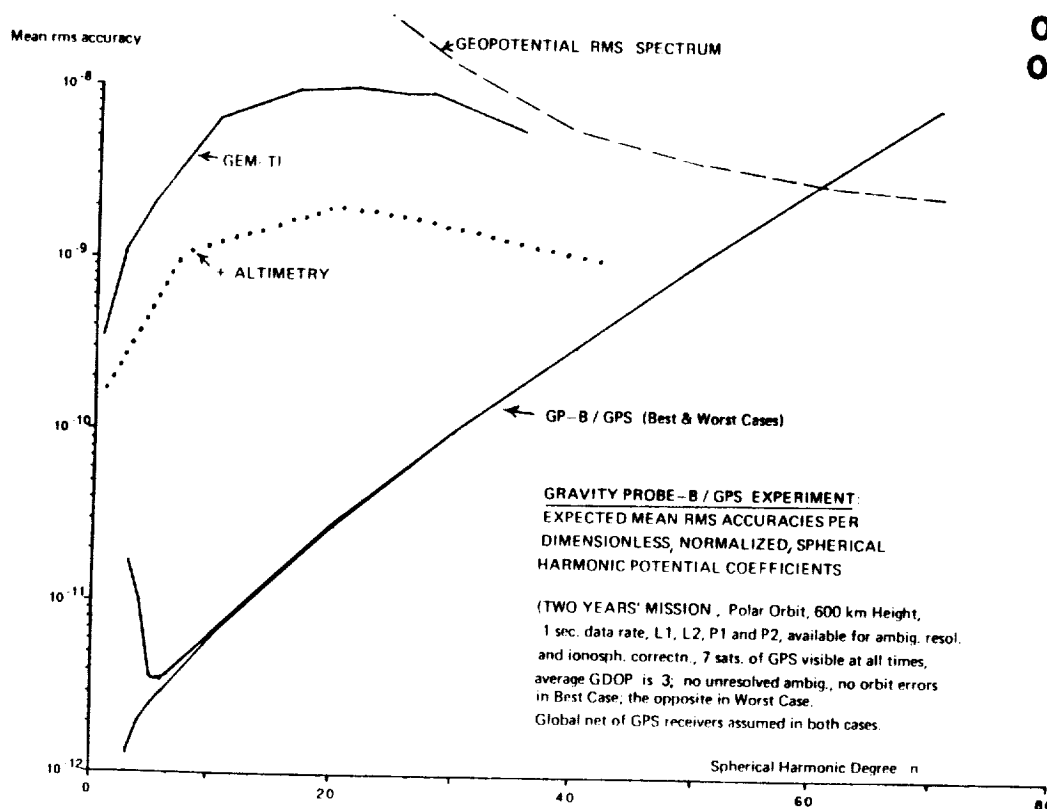
Figure 2. Mean accuracies per degree of normalized potential coefficients estimated with GP-B/GPS data, best and worst cases (as explained in text).

The coefficient errors, translated into rms cumulative geoid errors, are shown in Figure 3. The geoid error grows by about one order of magnitude every ten degrees, reaching 10 cm at about degree 40.
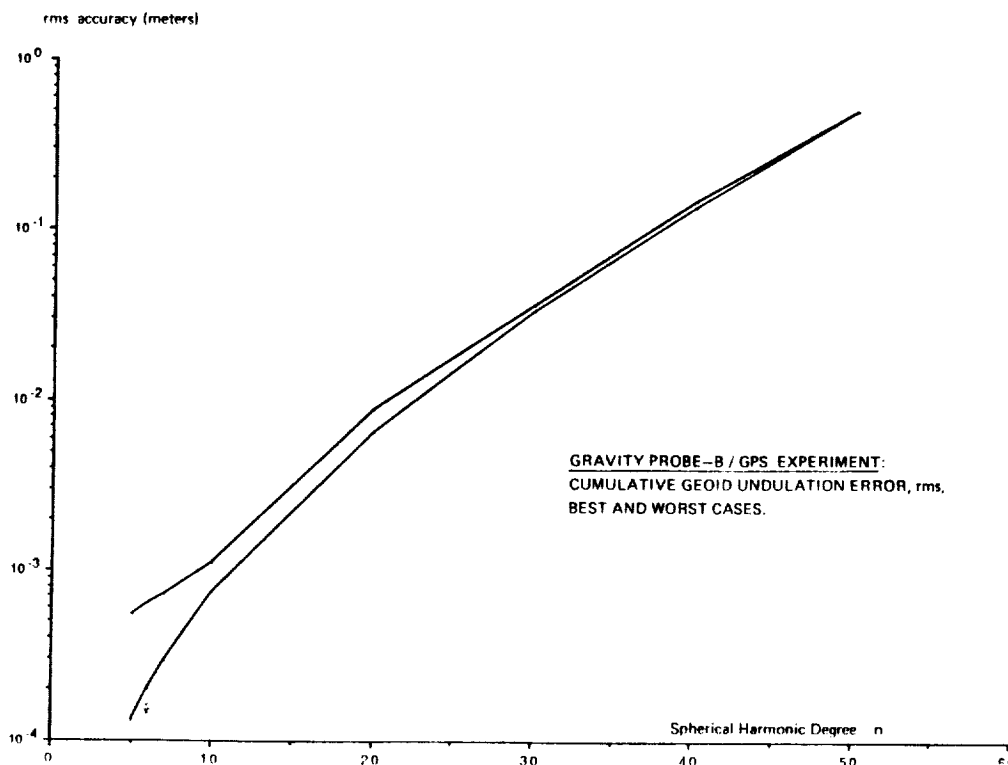

Figure 3. Cumulative geoidal undulation errors per degree, corresponding to the potential coefficient accuracies shown in Figure 1.

The accuracies of the spectral powers of various signals of geophysical interest are compared to the accuracies of the potential coefficients in Figure 4. At the top is the spectrum of the mean sea surface topography, associated with the average global circulation (obtained from the charts of Levitus). The spectrum of the gravitational effects of the M2 tide (according to Schwiderski's model), and of two years of post-glacial rebound are shown as well. In all three cases, the accuracy of the geopotential coefficients is considerably smaller than the variance of the signal, suggesting that a good separation of these geophysical signals is possible. The results are, if anything, pesimistic for the tides; even from the much less powerful conventional tracking data used for GEM-T1, a good deal of low-degree information on M2 and other major ocean tidal components has been extracted, mostly from spacecraft in much higher orbits than GP-B. This is possible because there are orbital resonances associated with the tides, and not considered here, that can produce large perturbations in the motions of spacecraft.
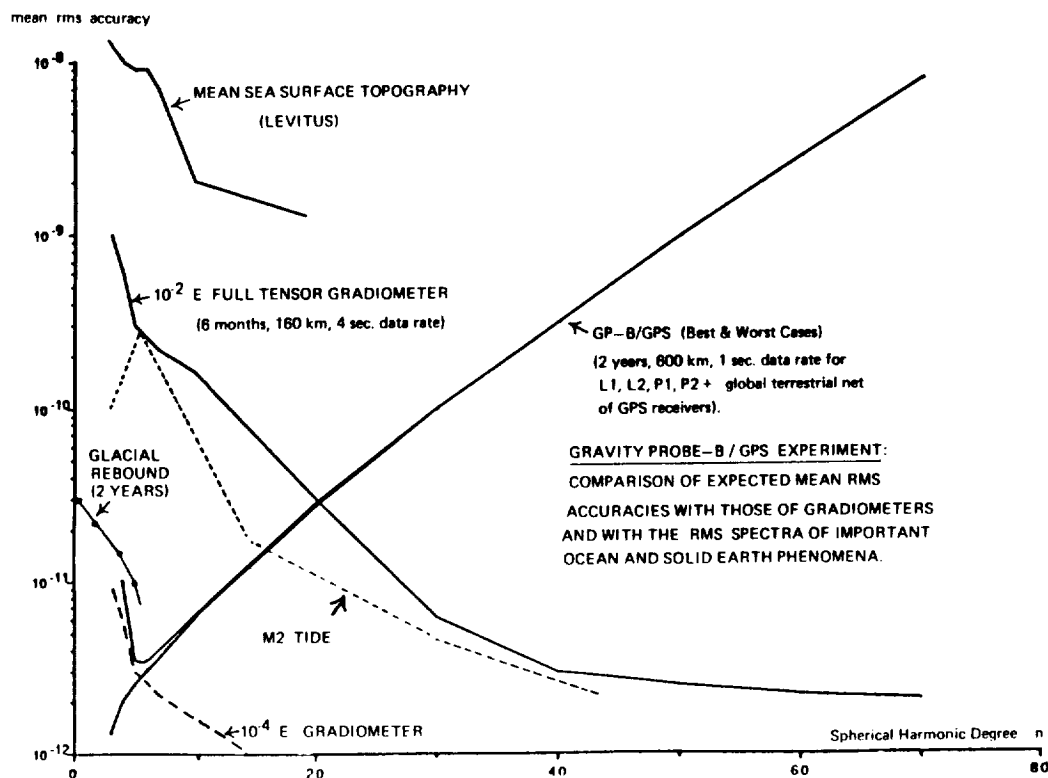

Figure 4. Accuracies of potential coefficients from GP-B/GPS data compared to those of conventional models and to the spectra of various geophysical phenomena.

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