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EFFECTIVE TECHNIQUES FOR THE IDENTIFICATION AND ACCOMMODATION OF DISTURBANCES

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1. Introduction

The successful control of dynamic systems such as space stations, launch vehicles, etc. requires a controller design methodology that acknowledges and addresses the disruptive effects caused by external and internal disturbances that inevitably act on such systems. These disturbances, technically defined as "uncontrollable inputs," typically vary with time in an uncertain manner and usually cannot be directly measured in real time.

Traditionally, control designers have employed two basic techniques for coping with uncertain disturbances. If the disturbance essentially behaves as an unknown constant, the well–known technique of integral–control is quite effective. In those cases where the disturbance behaves like random, erratic noise (radio static, sensor noise, etc.) the technique known as stochastic (statistical) control is often used.

However, in many realistic cases of practical interest today, the significant disturbances are not as simple as "unknown constants" and not as erratic and capricious as "random noise". Moreover several such disturbances, with perhaps dissimilar characteristics, may enter the system at different locations, thus creating a situation of multi–input disturbances. The accurate positioning control of a space station in the face of uncertain crew motions, equipment movements, gravity gradient torques, structural deflections, etc. is an example of the kind of problem we have in mind.

In this paper we will first describe a relatively new non–statistical technique for modeling, and (on–line) identification, of those complex uncertain disturbances that are not as erratic and capricious as random noise. This technique applies to multi–input cases and to many of the practical disturbances associated with the control of space stations, launch vehicles, etc. Then, we describe a collection of new "smart controller" design techniques that allow controlled dynamic systems, with possible multi–input controls, to accommodate (cope with) such disturbances with extraordinary effectiveness. These new "smart controllers" are designed by non–statistical techniques and typically turn out to be unconventional forms of dynamic linear controllers (compensators) with constant coefficients. The simplicity and reliability of linear, constant coefficient controllers is well–known in the aerospace field.

This paper is written in a tutorial style. Derivations and other technical details of the material outlined here are contained in (refs. 1–34) listed at the end of the paper. To help the reader quickly access specific details of interest, an unusually large number of topical citations to those references are given throughout the text.

2. A Critique of Stochastic Control

The uncertain, time–varying nature of typical disturbances encountered by dynamic systems has led many control designers to conclude that such disturbances are best modeled as random processes and should be characterized by the statistics of their long–term average behavior such as: mean–value, variance, power spectral density, higher–order moments, etc. Using this approach, realistic disturbances are often treated as classical random "noise" (white or colored noise with known statistical properties) and designers then employ the mathematical theories of stochastic control to design controllers that yield good long–term "average" performance in the face of such imagined "noisy disturbances". If the actual disturbances really do behave like radio static, wide–band sensor noise, etc., such a controller is usually effective.

The potential trouble with this stochastic approach to disturbance modeling and control is twofold. First, the actual disturbances encountered by the system might not behave like erratic random noise. Second, knowledge of the long–term statistical averages of disturbance behavior, as expressed by mean, variance, etc., may have little, if any, relevance to the problem of making real–time control decisions for fast–acting, high–performance dynamic systems. Thus, a stochastic controller that is "optimal" in the long–term average sense might yield unsatisfactory performance in the face of realistic disturbances and dynamic systems with time–stressed performance requirements, eg. tight set–point or servo–tracking requirements with specified short settling–times. This latter point is rather subtle and warrants further elaboration.

In order to obtain meaningful numerical values for the statistical mean, variance, etc., of an uncertain disturbance w it is necessary to observe and analyze the disturbance time–behavior w(t) over a sufficiently long window of time $t_o \le t \le T_s$ as shown in Figure 1. Otherwise, the computed "mean" $\bar{w}$ (for instance) will vary unpredictably with the length of the observation window and perhaps with the particular sample function w(t) being considered, thereby contradicting the concept of statistical mean. On the other hand, the performance time–window $t_o \le t \le T_p$, during which a controller must grapple with the disturbance and accomplish the specified control task, might be significantly shorter than the minimal window $t_o \le t \le T_s$ needed to evaluate the disturbance's statistics. In the latter event, knowledge of the disturbance's long–term average mean, variance, etc. would offer little, if any, help in making real–time control decisions; see Figure 1. To make matters even worse, some of the most elementary forms of practical disturbances w(t) (eg. random constant disturbances) do not satisfy the ergodic hypothesis; ie. the hypothesis that ensemble–averages equal time–averages for "almost all" sample functions. This hypothesis forms the foundation upon which most stochastic control principals are based.

3. Essential Disturbance Information for Real–Time Control Decisions

According to the preceding arguments, information about long–term statistical averages of disturbance behavior is of little help in making real–time control decisions over short performance windows. Thus, it is natural to ask: what disturbance information is essential for making "good" control decisions in such cases? The answer is best stated in terms of two subcases.

3.1 The Idealistic Case

It can be shown that in the idealistic case, where the unknown disturbance w(t) is nevertheless a theoretically "completely determined function" over the performance window $t_o \le t \le T_p$, the optimum real–time control decision at each $t_o \le t \le T_p$ requires complete knowledge of the future disturbance behavior (function) $w(t_f)$ over the remaining performance window $t \le t_f \le T_p$. This kind of information is not available in most practical applications. However, the result has certain theoretical importance. A special case of the result was established by Kalman (ref. 14) for linear–quadratic optimal control problems and was extended to a general class of plants and performance criteria in (ref. 5).

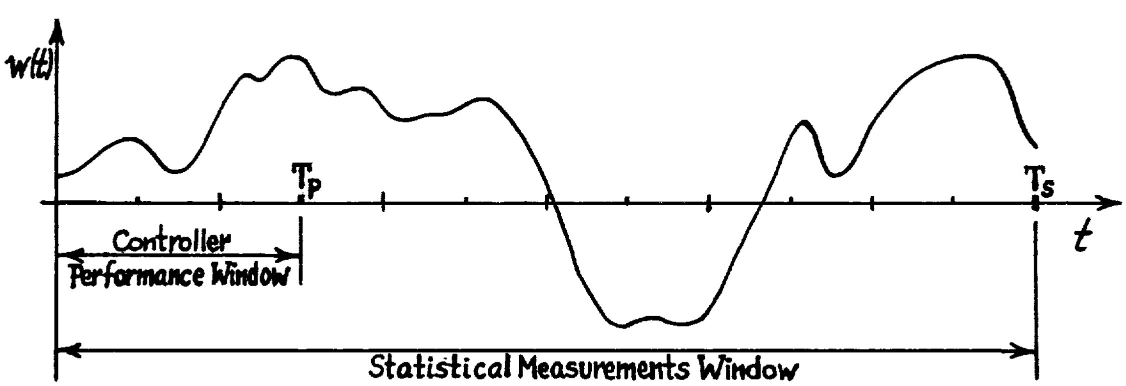

Fig. 1 – Comparison of Time Windows for Statistical Measurements and Controller Performance.

3.2 The Practical Case

In most practical cases the unknown disturbance w(t) is not a theoretically "completely determined function" over $t_o \le t \le T_p$ but rather is "determined" only over sequential subsets (cells) $t_i \le t \le t_{i+1}$ which partition the performance interval $t_o \le t \le T_p$ as shown in Figure 2. At the boundaries t_i, t_{i+1} of each cell abrupt indeterminable (a priori) random–like jumps occur in the value of w(t) and/or one or more of its time derivatives. Since these indeterminable jumps occur only at the cell boundaries t_i, t_{i+1}, the unknown function w(t) is said to be theoretically "determined" within the interior $t_i < t < t_{i+1}$ of each cell. In such cases the conditionally optimum real–time control decision for each cell–interior time $t_i < t_{real} < t_{i+1}$ requires complete knowledge of future $w(t_f)$ behavior over the remaining cell length $t_{real} \le t_f < t_{i+1}$; see Figure 2. Here, the term "conditionally optimum" reflects the fact that behavior of the disturbance function w(t) is mathematically indeterminable (a priori) beyond the current (real–time) cell $t_i < t < t_{i+1}$, and therefore in making real–time control decisions within a cell it is desirable but theoretically impossible to account rationally for future disturbance behavior beyond that current cell. This fundamental theoretical handicap to the optimal control decision process is inescapable and cannot be mitigated by any rational procedure. On the other hand, if within a current cell one is willing to gamble on the likely behavior of the indeterminable function w(t) over future cells, it is possible that "luck of the draw" or a "fortuitous guess" can sometimes result in a control decision, for a particular moment of time, that turns out (in retrospect) to be "better" than the rational, conditionally optimum decision described above. This gambling in function spaces is exciting entertainment but is not recommended as a means for improving conditionally optimum control decisions for space stations, launch vehicles, etc.

It would appear that "conditionally optimum" control decisions are themselves physically unrealizable, in general, since they require knowledge of "local" future behavior $w(t_f)$, $t_{real} \le t_f < t_{i+1}$ within each cell, as shown in Figure 2. In the remaining sections of this paper we will describe a new approach to disturbance modeling and estimation that makes it possible to (easily) implement "conditionally optimum" control decisions in practical applications.

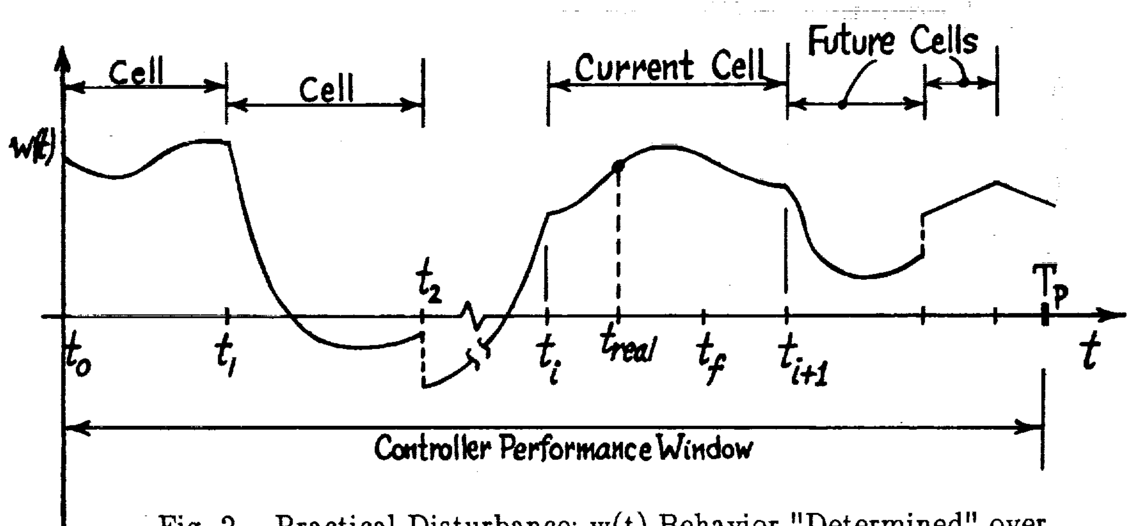

Fig. 2 – Practical Disturbance; w(t) Behavior "Determined" over Sequential Time–Cells, $t_i<t<t_{i+1}$.

3.3 The Limiting Case of Random Noise

In the limiting case where the cell–lengths $\ell_i = t_{i+1} - t_i$ all approach zero, the disturbance function w(t) becomes indeterminable for all $t_o \le t \le T_p$ and thus w(t) reduces to a classical "random noise" process. In that event it becomes theoretically impossible to account for the local, real–time behavior of the disturbance w(t) in making real–time control decisions. Consequently for such cases the only determinable information about w(t) is that embodied in the long–term average statistics (mean, variance, etc.) of w(t), as measured a priori. The conventional theory of stochastic control uses those long–term statistical averages of w(t) to arrive at (long–term) optimum control decisions for such limiting cases.

4. The Idea of Waveform Structure and State Models for Uncertain Disturbances

In the remainder of this paper, we will focus attention on the "practical case" of disturbances as described in Section 3.2. In that case we will say the unknown disturbance w(t) has (linear) waveform structure if, over each cell, the function w(t), $t_i < t < t_{i+1}$, can be mathematically modeled by an expression of the form

$$w(t) = c_1 f_1(t) + c_2 f_2(t) + \cdots + c_M f_M(t), \qquad t_i < t < t_{i+1} \tag{1}$$

where the weighting coefficients $\{c_1, c_2, \cdots, c_M\}$ are unknown "constants" that may jump in value at the cell boundaries, and the functions $\{f_1(t), f_2(t), \cdots, f_M(t)\}$ are completely known a priori. The representation (1) is a generalized spline–function model, hereafter called "waveform model," and the $f_k(t)$ are referred to as the basis functions for that spline model.

In practice, one selects the $f_k(t)$ in (1) to closely match the natural waveform modes (waveform patterns) actually observed in representative samples of w(t). For example, if w(t) is observed to be periodic in nature, one would choose the $f_k(t)$ to be the natural harmonic components ($\sin\omega_k t$, $\cos\omega_k t$), $k = 1,2,\cdots$, of w(t), just as in a Fourier series. In other cases, the natural choices for the $f_k(t)$ might be one or more elements from the set $\{1, t, t^2, \cdots, t^N e^{\alpha t}, te^{\alpha t}, e^{\alpha t}\sin\omega t, \text{etc.}\}$. In some cases the natural basis functions for w(t) are not clearly defined by the available data. For such cases it is usually effective to use a "polynomial spline" waveform–model of the form

$$w(t) = c_1 + c_2 t + c_3 t^2 + \cdots + c_M t^{M-1}, \quad M = 1,2,\cdots. \tag{2}$$

Practical experience with (2) has shown that M–values in the range $1 \le M \le 4$ are adequate for most disturbances encountered in applications. In the case of multi–variable disturbances a separate descriptor (1), (2) is used for each independent $w_i(t)$.

The idea of modeling unknown disturbances w(t) by spline–type waveform–models (1),(2) was developed in a series of papers published in the period 1968–71, (refs. 1,2,3,4,5). That idea now forms the foundation for a new branch of control theory called Disturbance–Accommodating Control (DAC), (refs. 6,7,8,9,10), which we will discuss in the next section. It should be emphasized that in DAC theory the values of the arbitrary weighting coefficients c_i in (1),(2) are assumed piecewise–constant, with "once–in–a–while" jumps, but otherwise completely unknown. No statistical properties or probabilistic structures are assumed about the time behavior of the c_i. Thus, for instance, the traditional statistical properties of uncertainty such as mean, covariance and power–spectral density of w(t) are assumed completely unknown in (1),(2) and, in fact, are of no concern in DAC theory. This means that assumptions about the disturbance's ergodic behavior, stationary statistics, etc. are not required in DAC designs using the disturbance models (1),(2).

When an uncertain disturbance has waveform structure, the use of waveform models (1),(2) and DAC design techniques allows the controller to make more effective real–time control decisions than are possible using long–term mean, covariance, etc. statistical properties of w(t).

4.1 State Models of Disturbances with Waveform Structure

The waveform model (1) is the key idea behind our approach to disturbance modeling. However, the "information" reflected in the model (1) must be encoded into an alternative format before it can be used effectively in identification and control design recipes. That alternative format is called a "disturbance state–model" in DAC theory and consists of a differential equation for which (1), with the c_i viewed as constants, is the general solution. In other words, one must solve the following inverse–problem in differential equations: Given the general solution (1), with arbitrary constants c_i, find the (a) differential equation. There are many interesting ramifications to this latter problem (ref. 6; pg. 402,417). However, in practical applications of DAC theory, the basis functions $f_i(t)$ are almost always such that this step leads to a linear differential equation. Consequently, one obtains a state–model of (1) in the form (a.e. means "almost everywhere");

$$\frac{d^\rho w}{dt^\rho} + \beta_\rho(t)\frac{d^{\rho-1} w}{dt^{\rho-1}} + \cdots + \beta_2(t)\frac{dw}{dt} + \beta_1(t)\, w = 0, \qquad \text{a.e.} \tag{3}$$

where the coefficients $\beta_1(t),\cdots,\beta_\rho(t)$ in (3) are completely determined by the (known) basis functions $f_1(t),\cdots,f_M(t)$ in (1). That is, the $\beta_i(t)$ are not functions of the (unknown) weighting coefficients c_i in (1). In the case of a multi–variable (vector) disturbance $w(t) = (w_1(t),\cdots,w_p(t))$, a differential equation similar to (3) would be obtained for each independent disturbance component $w_i(t)$. In the latter case, the differential equation for $w_i(t)$ may contain coupling–terms involving the other $w_j(t)$ etc.

The final step in constructing a state–model for (1) is to write the differential equation(s) (3) in the form of a set of simultaneous first–order differential equations, (ref. 6, p. 405,406). The end result, in the general case of a vector disturbance $w= (w_1,\cdots,w_p)$, has the form:

$$w = H(t)z \quad , \quad z = (z_1,\cdots,z_\rho) \tag{4–a}$$

$$\dot{z} = D(t)z + \sigma(t) \tag{4–b}$$

where H(t), D(t) are completely known matrices and z(t) is a ρ–vector called the "state" of the disturbance w. The elements z_i of z embody the disturbance components $w_1,\cdots,w_p$ and certain of their higher–derivatives. The term $\sigma(t)= (\sigma_1(t),\cdots,\sigma_\rho(t))$ is a symbolic representation of a vector sequence of impulses with completely unknown "once–in–a–while" arrival times and completely unknown random–like intensities. Thus, the basis–functions $f_i(t)$ in (1) appear in (4) as the principle mode solutions of the homogeneous disturbance state equation $\dot{z} = D(t)z$ in (4–b). As shown in (refs. 11,6,8,12,17) the disturbance state–model (4) can be generalized to include terms involving the plant state x, the plant control u and conventional noise inputs.

The completely unknown impulses of $\sigma(t)$ in (4) represent the source of the uncertain, once–in–a–while jumps in the values of the piecewise–constant weighting coefficients c_i in (1). In DAC theory it is assumed that adjacent impulses in $\sigma(t)$ are separated by a finite time–spacing (cell–length in Figure 2) not less than μ, where μ is the controller's closed–loop settling–time; i.e.

$\sigma(t)$ consists of a sparsely populated sequence of unknown impulses. If the impulses of $\sigma(t)$ arrive "too fast", the c_i in (1) will jump in value too often and the DAC controller will then be unable to respond properly. In that case we say the disturbance w(t) looses its waveform structure and becomes "noise"; see Section 3.3. In particular, if the impulses arrive arbitrarily close (and are totally uncorrelated) the $\sigma(t)$ sequence then behaves like a vector "while–noise" process (Bode–Shannon realization) and our disturbance state–model (4) then appears similar to the white–noise coloring filters traditionally used in stochastic control. However, note the subtle differences. Namely, in DAC theory the matrices H(t), D(t) in (4) are determined by the disturbance's natural waveform patterns rather than by long–term statistical means, variances, etc. Moreover, the homogeneous part of the DAC disturbance model (4) is not required to satisfy stability conditions. In fact, even though w(t) itself is usually bounded and well–behaved, it is common in DAC applications to find that many of the natural principle mode solutions $f_i(t)$ of the disturbance equation $\dot{z} = D(t)z$ are unstable, (i.e. grow with time in an unbounded fashion). For example, a uniformly bounded, well–behaved disturbance w(t) can have the natural waveform–model $w(t) = c_1e^t + c_2te^{-10t} + c_3t^2$, where the "constants" c_i jump in a strategically correlated manner determined by the physical process that produces w(t). Such behavior of (4) is not permitted of the coloring filters in conventional stochastic control theories. This constitutes a unique and practically important feature of our disturbance modeling technique (1),(3),(4).

5. Real–Time Identification of the State z(t) of a Waveform–Structured Disturbance

As we stated in the Introduction, disturbances w(t) associated with dynamic system control problems usually cannot be directly measured in real–time. It should be mentioned in passing, however, that some noteworthy exceptions to this truism are found in the field of chemical process control. In any event, control engineers have traditionally argued that if the uncertain disturbances w(t) could somehow be directly measured in real–time the system performance could be easily managed by feeding–forward the disturbance measurements to strategic points in the system. This concept seems plausible, but turns–out to be flawed when examined from the scientific viewpoint. Namely, if the uncertain disturbance w(t) has at least some waveform structure (as virtually all realistic disturbances do) then it is not just the real–time value of w(t), but rather the real–time value of the state z(t) of w(t), that is important to the real–time control decision process. This important fact is established in (ref. 5) and can be summarized as the following principle.

The Principle of Optimal Disturbance Accommodation

Suppose a controlled dynamical system is acted upon by uncertain disturbances w(t) that are known to have waveform structure in the sense of (1). Then, for a broad class of performance criteria the corresponding optimal real–time choice for the control $u^o(\cdot)$, at time t, can be expressed in terms of the current plant state x(t) and the current disturbance state z(t); i.e. $u^o(t)$ has the generic "control law" format

$$u^o(t) = \phi(x(t), z(t), t) \quad . \tag{5}$$

Remarks

This principle implies that at each t the current disturbance state z(t) embodies enough information about w(t) to allow a rational scientific choice for the real–time control u(t) --- even though the actual future behavior of w(t) is uncertain (in fact, indeterminable!) beyond the "current" cell in Figure 2.. This result enables the control designer to derive a DAC deterministic control policy (5) for accommodating the presence of uncertain disturbances. Because that control policy is based on the real–time dynamic behavior of z(t) in (4), [not the forecasted long–term mean, variance, etc. of w(t)] it can achieve the conditionally optimum control decisions described in Section 3.2. In particular the DAC controller automatically adapts control actions to the actual real–time waveform patterns of the disturbance function w(t) as those patterns evolve with time over each cell. In control problems involving set–point or servo "commands," the generic controller (5) will also be a function of the current "state" c(t) of the command signal; see eq. (29) in the next section and also (refs. 4,22).

If, in fact, the disturbance w(t) can be directly measured, this principle shows that one should not "feedforward" only w(t) but rather should feedforward the "state" z(t) of w(t), as obtained from a real–time z(t) estimator described in the next section; see also (ref. 6, p. 431 and 434).

5.1 The Use of Composite Observers to Generate Real–Time Estimates $\hat{x}(t)$, $\hat{z}(t)$

Generally speaking, it is not possible to directly measure x(t) and/or z(t) in practical applications. Therefore, the practical implementation of DAC control laws of the form (5) is accomplished by using a special form of on–line, real–time observer (or Kalman filter) to generate real–time estimates of both the plant state x(t) and disturbance state z(t). That observer, called a composite–state observer, processes the control input measurements u(t) and plant output measurements y(t) to simultaneously generate reliable estimates $\hat{x}(t)$, $\hat{z}(t)$ of the current plant and disturbance states. Those estimates are then used in place of x, z, in (5). The theory and explicit design recipes for DAC composite observers is covered in refs. (6,11,12) for the continuous–time (analog) control case and in (refs. 7,8,13) for the discrete–time (digital) control case; see also (refs. 4,10). These DAC composite observers are typically linear in structure and enjoy all the features one usually associates with conventional state observers and Kalman filters; a DAC composite observer based on Kalman filter ideas is used when some disturbances are truly "noisy" in nature, see (refs. 12,13).

5.2 Example of a Full–Order Composite State Observer for Linear Dynamical Systems

In this section, we will illustrate the DAC technique for real–time identification of the disturbance state z(t) for a waveform–structured disturbance w(t). In particular, we will consider the class of linear dynamical systems (plants)

$$\dot{x} = A(t)x + B(t)u + F(t)w \quad ; \qquad \begin{array}{l} x = n\text{–vector plant state} \\ u = r\text{–vector control} \end{array} \qquad (6\text{–a})$$

$$y = C(t)x \qquad \begin{array}{l} w = p\text{–vector disturbance} \\ y = m\text{–vector plant output} \end{array} \qquad (6\text{–b})$$

where each element of the vector uncertain disturbance $w = (w_1, w_2, \cdots w_p)$ is assumed to be independent and have waveform structure in the sense of (1). It is further assumed that the set of natural basis functions $\{f_i(t)\}$ in (1) is such that the associated disturbance state–model has the linear form (4). In that case the composite, time–varying dynamic model of the plant and disturbance states is found by consolidating (4), (6) to obtain

$$\left[\frac{\dot{x}}{\dot{z}}\right] = \left[\begin{array}{c|c} A & FH \\ \hline O & D \end{array}\right] \left[\frac{x}{z}\right] + \left[\frac{B}{O}\right]^{u} + \left[\frac{O}{\sigma}\right] \qquad (7\text{–a})$$

$$y = [C \,|\, O] \left[\frac{x}{z}\right] \qquad (7\text{–b})$$

All matrices shown in (7) are allowed to vary with time in a known manner; see (ref. 10) for the case of uncertain matrices.

Setting $\bar{x} = \left[\frac{x}{z}\right]$ we can write (7) in the compact form

$$\dot{\bar{x}} = \bar{A}(t)\bar{x} + \bar{B}(t)u + \bar{\sigma} \tag{8-a}$$

$$y = \bar{C}(t)\bar{x} \tag{8-b}$$

where the meanings of $(\bar{A},\bar{B},\bar{C},\bar{\sigma})$ are evident from (7). The DAC composite state $\bar{x}$ in (8) is sometimes called the system "metastate".

The composite system (7) is clearly uncontrollable, since $\dot{z}$ does not depend on either x or u. However, this conclusion is not necessarily true for the generalizations of (4) considered in (ref. 5; ref. 6, p. 416; ref. 11, p. 826; ref. 17, eq. (14)). On the other hand, it is common to find that the composite system (7) is completely observable. That is, the pair $(\bar{A},\bar{C})$ in (7), (8) satisfies the Kalman criterion for complete observability (ref. 14). In the time–invariant case $(\bar{A},\bar{C})$ = constant, this implies that, in principle, one can always generate reliable, real–time estimates $\hat{x}(t)$, $\hat{z}(t)$, between arrivals of the "sparse" impulses of $\sigma(t)$, using a conventional full–order state observer for (7), (8). If $(\bar{A},\bar{C})$ are not constant this latter feat is still possible (between $\sigma(t)$ impulses) provided the pair $(\bar{A},\bar{C})$ satisfies a stronger observability condition known as "uniform complete observability," on every positive sub–interval of time between impulses of $\sigma(t)$; see (refs. 14; 4, p. 223). Thus, assuming the appropriate observability condition is satisfied one can generate the real–time estimates $\hat{x}(t)$, $\hat{z}(t)$ by employing a conventional full–order observer for (7), (8). That observer, called a "composite–state" or "metastate" observer in DAC theory, is given by (ref. 5, p. 222)

$$\left[\begin{array}{c}\dot{\hat{x}}\\ \hline \dot{\hat{z}}\end{array}\right] = \left[\begin{array}{c|c} A & FH \\ \hline O & D\end{array}\right]\left[\begin{array}{c}\hat{x}\\ \hline \hat{z}\end{array}\right] + \left[\begin{array}{c} B\\ \hline O\end{array}\right]u - \left[\begin{array}{c} K_{01}\\ \hline K_{02}\end{array}\right](y-C\hat{x}) \tag{9}$$

where (K_{01},K_{02}) are observer gain matrices to be designed. The dynamics of the estimation error $\varepsilon = \left[\begin{array}{c}x\\ \hline z\end{array}\right] - \left[\begin{array}{c}\hat{x}\\ \hline \hat{z}\end{array}\right]$ associated with (7), (8), (9) is easily shown to be (between impulses of $\sigma(t)$)

$$\dot{\varepsilon} = [\bar{A}(t) + \bar{K}_o(t)\bar{C}(t)]\varepsilon \quad ; \quad \bar{K}_o = \left[\begin{array}{c} K_{01}\\ \hline K_{02}\end{array}\right] \tag{10}$$

and thus $\bar{K}_o(t)$ should be designed to make $\varepsilon(t) \rightarrow o$ rapidly, between impulses of $\sigma(t)$. Effective recipes for designing such $\bar{K}_o(t)$ are described in (refs. 4,5,6), provided one corrects a recurring sign error therein, as explained in (ref. 15).

In summary, under the assumptions stated the linear dynamical data–processing algorithm (observer) (9) will process the real–time measurements of $\{u(t),y(t)\}$ to generate reliable, real–time estimates $\hat{x}(t)$, $\hat{z}(t)$ of the plant and disturbance states (between impulses of $\sigma(t)$. Those state estimates can then be used in an appropriate DAC "control law" (5) to achieve optimal accommodation of real–time uncertain disturbances w(t). A diagram of the disturbed plant (6), (4) with generic DAC controller (5), (9) installed is shown in Fig. 3. Note that the DAC controller contains an "internal copy" of the external disturbance process (4). This feature is characteristic of all DAC controllers and was first discussed in a 1970 paper (ref. 2; Fig. 2 and pp. 225, 226); see also (ref. 5; Fig. 2 and p. 229), (ref. 22; Fig. 5). As pointed out in (ref. 5, p. 222), the estimator (9) is valid for arbitrary control inputs u(t). Thus, (9) accurately estimates x(t), z(t) even if $u(t) \equiv 0$, and even if (5) is an arbitrary form of nonlinear control law.

A discrete–time (difference–equation) version of (9) is described in (refs. 7,8,13) for use in digital computer implementations. Also, a reduced–order version of (9) is described in (refs.

4,6,11). The modification of (9) to account for state–dependent disturbances and/or the direct measurement of some of the disturbance components w_i is presented in (ref. 6; p. 431 and 434); see also (ref. 8) for the discrete–time counterparts.

5.3 Real–Time Identification of "Plant Parameter Perturbation" Disturbances Using the DAC Method: A New Approach to Adaptive Control

The waveform–model idea (1), (4) can also be applied to the problem of identifying (and compensating for) internal disturbances in the form of uncertain perturbations in plant parameters. In practice these uncertain parameter perturbations can result from: parameter modeling errors, effects of neglected non–linear terms, reduced–order models, and actual real–time changes in plant parameters caused by operating environment and aging effects. For instance, in the case of a linear dynamical system (6) it may happen that one or more of the coefficients a_{ij} of the matrix A(t) are subject to uncertain perturbations δa_{ij} away from their known nominal values a_{iN}. Thus, A(t) in (6) can be written as

$$A(t) = A_N(t) + [\delta A(t)] \quad ; \quad A_N = \text{known nominal value.} \tag{11}$$

In this case, (6–a) can be written

$$\dot{x} = A_N(t)x + B(t)u + [\delta A(t)]x + F(t)w \tag{12}$$

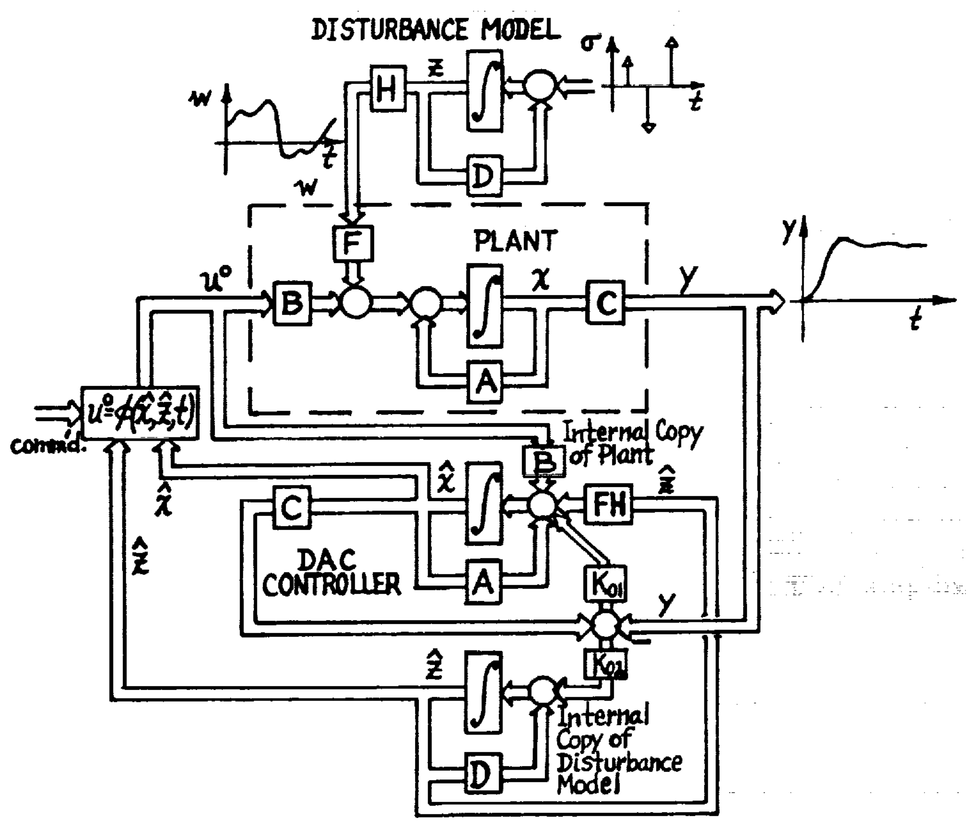

Fig. 3 – General Block-Diagram of Plant and Disturbance (6),(4) with DAC Controller (5),(9) Installed.

It is now clear from (12) that the perturbation term

$$w_a(t) = [\delta A(t)]x(t) \tag{13}$$

acts on the plant as an uncertain "disturbance" just like the conventional disturbance term w(t). Moreover, the nature of the practical time–variations in $w_a(t)$ in (13) suggests that $w_a(t)$ should possess "waveform–structure" in the sense of (1).

Thus, it is plausible that the same DAC techniques (1),(4),(5),(9) used for w(t) can also be used to: (i) identify the "state" $z_a(t)$ of $w_a(t)$ in real–time, and, (ii) design a control law (5) to optimally compensate for $w_a(t)$ in real–time. This concept differs radically from conventional approaches to identification and compensation of parameter perturbations. In particular, all conventional approaches first estimate the n^2 elements of $[\delta A]$, using elaborate nonlinear estimation schemes, and then <u>compose</u> the estimate $\hat{w}_a(t)$ of (13) by setting

$$\hat{w}_a(t) = \widehat{[\delta A]}\hat{x}\,. \tag{14}$$

Our unorthodox approach recognizes that the real "disturbance" in (12) is <u>not</u> $[\delta A]$ but rather the n–vector (n elements) $w_a = [\delta A]x$. Thus, instead of generating the conventional "product of estimates" (14) we generate the "estimate of the product"

$$\hat{w}_a(t) = \widehat{[\delta A]x} \tag{15}$$

using a DAC state–model for w_a and a special composite observer to generate $\hat{z}_a(t)$. The virtue of (15) is that estimation of the product (15) is much easier and quicker than generating the product of estimates (14). In particular, (15) can be generated by an <u>all–linear</u>, constant coefficient estimation algorithm (observer).

In (ref. 6; pp. 413–415) the DAC approach (11)–(13), (15) to plant parameter "disturbances" was advocated using a polynomial–spline waveform model (2) for each element w_{ai} of the uncertain "disturbance" $w_a(t)$. The model (2) leads to a state–model (3) of the Mth–order integrator type

$$\frac{d^M w_{ai}}{dt^M} = 0 \quad , \quad \text{a.e.} \tag{16}$$

which results in a particularly simple model (4) and disturbance state observer (9); see (ref. 6; eq. (32)). The effectiveness of (16) in identifying $\hat{w}_a(t)$ is quite good --- provided $w_a(t)$ changes slowly. This limitation has now been largely removed by the recent discovery of a more efficient "natural" set of basis functions $\{f_i(t)\}$ for $w_a(t)$ in (13). In particular it has been shown in (refs. 10,16,17,18,19,20) that if x in (12) denotes the "error–state" (refs. 4, art. VI; 6, p. 450; 10, p. 31; 19, p. 2453), and $[\delta A] \approx$ constant, then during "ideal–model" response $x(t) \to 0$ each independent element w_{ai} of the n–vector $w_a(t)$ in (13) is closely modeled by a "natural" state–model (3) having the special form [compare with (16)]

$$\frac{d^n w_{ai}}{dt^n} + \beta_n^* \frac{d^{n-1} w_{ai}}{dt^{n-1}} + \cdots + \beta_2^* \frac{dw_{ai}}{dt} + \beta_1^* w_{ai} = 0, \ \text{a.e.} \tag{17}$$

where $\{\beta_1^*, \beta_2^*, \cdots, \beta_n^*\}$ are constant, known coefficients defined by

$$\det[\lambda I - A_M] = \lambda^n + \beta_n^* \lambda^{n-1} + \cdots + \beta_2^* \lambda + \beta_1^* = 0 \tag{18}$$

and A_M is the (presumed given, constant) matrix that <u>specifies</u> the desired (ideal–model) error–state response $x(t) \to 0$ via the expression

$$\dot{x}_{ideal} = A_M x_{ideal} \quad , \quad x = \text{"error–state"}. \tag{19}$$

In other words,the "natural" eigenvalues of (17) correspond to the desired closed–loop poles specified for the plant "error–dynamics" (19). This discovery allows the DAC approach (11)–(13), (15) to be successfully applied to cases in which $w_a(t)$ in (13) changes rapidly. Moreover, even though $[\delta A]$ itself has been assumed ≈ constant in the theoretical development of (17), it turns–out that significant time–variations in $[\delta A(t)]$ can be accommodated provided that a polynomial–spline model (2) with M = 2,3 is <u>added to</u> the natural state–model (17), see (refs. 19, p. 2458 and Fig. 7; 10, Fig. 5).

6. <u>A New Family of "Smart" Controllers for Real–Time Accommodation of Disturbances</u>

The fundamental advantage of our waveform–modeling technique (1),(3),(4), compared to conventional long–term averaged statistical models of uncertain disturbances, is that the waveform–model allows one to estimate the <u>actual real–time dynamic behavior</u> (state) <u>of each individual disturbance function</u> w(t) as it evolves in real–time. In other words, to use a term from stochastic control, our waveform–modeling technique allows the DAC controller to recognize and deal–with the unique behavior of each individual disturbance "sample–function" (ref. 4, footnote 8). The conventional stochastic controller has no means of recognizing this actual real–time disturbance behavior and must instead rely on the disturbance's long–term statistical <u>ensemble averages</u> as measured by some earlier experiment. Since at any given moment the current disturbance behavior can differ greatly from the long–term statistical averages measured earlier, it follows that real–time controller decisions based on current disturbance behavior will tend to be smarter and more effective than those based on long–term statistical averages. This advantage can be rather significant in those cases where the controller's performance window $t_o \le t \le T_p$ is relatively short compared to the window $t_o \le t \le T_s$ used to measure statistical averages of the disturbance. This consideration prompted the original idea for DAC theory (ref. 1, p. 417) and my own experience suggests that, in practice, those cases occur more frequently than (most) control designers and theoreticians realize.

The systematic design of "smart" (DAC) controllers in Figure 3 that can take advantage of real–time disturbance "state" information z(t) is a rather lengthy topic that is covered, in detail, in (refs. 4,5,6,7,8,10,12,13,21). Here,we will only be able to outline the main ideas and final results. For this purpose, it is convenient to sub–divide the discussion into three parts, corresponding to the three fundamental strategies for "accommodating" disturbances.

6.1 <u>Modes of Disturbance–Accommodation; Design Options Unique to DAC Theory</u>

One of the most attractive features of DAC theory is the unique flexibility it offers the control designer in selecting strategies for coping with multi–variable uncertain disturbances. In fact, prior to the introduction of DAC theory control designers used essentially only one strategy (= cancellation) in regard to accommodating disturbances. In DAC theory there are basically three strategies one can choose from, each having several possible variations. Those basic strategies of accommodation can best be illustrated in terms of the well–known multi–variable linear plant model (6) which is repeated here for convenience

$$\dot{x} = A(t)x + B(t)u + F(t)w(t) \; ; \qquad u = (u_1, \cdots, u_r) \tag{20–a}$$
$$w = (w_1, \cdots, w_p)$$

$$y = C(t)x \qquad y = (y_1, \cdots, y_m) \tag{20–b}$$

For simplicity, we will assume A, B, F, C are all constant; see (ref. 6) for a treatment of the time–varying case and further generalizations of (20).

The Disturbance Cancellation Mode of Accommodation

The strategy of disturbance–cancellation, sometimes called disturbance–absorption or rejection, consists of designing the control u(t) to completely cancel–out the effects of the disturbance w(t) on the plant behavior. This strategy is prompted by the common attitude that disturbances cause only unwanted disruptions or perturbations in the plant behavior. In terms of the specific plant (20), and disturbance model (4) the disturbance–cancellation design procedure goes like this. First, one agrees to split (allocate) the total control action u(t) into two parts

$$u = u_p + u_d \tag{21}$$

where u_d is responsible for the disturbance–cancellation task and u_p is responsible for accomplishing the primary control task such as stabilization, set–point regulation, servo–tracking, etc. Substituting (21) into (20) yields

$$\dot{x} = Ax + Bu_p + Bu_d + Fw(t); \quad y = Cx \tag{22}$$

In terms of (22) and (4), the task of u_d is to achieve and maintain the condition of complete cancellation:

$$Bu_d(t) = -Fw(t) = -FHz(t), \quad z \in E^\rho, \; t_o \le t \le T\,. \tag{23}$$

The n.a.s.c. for satisfaction of (23), by some u_d, is

$$\mathrm{rank}[B\,|\,FH] = \mathrm{rank}\,[B] \tag{24}$$

which is called the "complete cancellation" condition of DAC theory. Condition (24) implies $FH = B\Gamma$ for some (possibly non–unique) matrix Γ, in which case the control u_d in (23) can be ideally chosen as

$$u_d(t) = -\Gamma z(t) \tag{25}$$

where for implementation purposes one would use an observer–produced estimate $\hat{z}(t)$ in place of z(t) in (25). Substitution of (25) into (22) yields

$$\dot{x} = Ax + Bu_p \tag{26}$$

so that one can now proceed to design u_p by conventional methods. It is remarked that the technique of splitting (allocating) the total control effort u into task–oriented parts, as illustrated in (21), is a simple but notably effective design idea that appears to be unique to DAC theory, as far as modern state–variable control theories are concerned.

Suppose the designer is concerned about cancelling only that subset of disturbance effects that appear in the plant output y(t) in (20). This is called "output disturbance–cancellation" and is achieved as follows. Let $u_p = Kx + \tilde{u}_p$ and $u_d = \Lambda z$, where K, Λ are to be designed and $\tilde{u}_p$ denotes terms of u_p which do not involve x (such as set–points, etc.). Then, the n.a.s.c. for complete cancellation of disturbances in the output y(t) is:

$$C[\hat{B}\,|\,\bar{A}\hat{B}\,|\,\bar{A}^2\hat{B}\,|\cdots|\,\bar{A}^{(n-1)}\hat{B}] = 0; \qquad \begin{aligned}\hat{B} &= B\Lambda + FH\\ \bar{A} &= A + BK\end{aligned} \tag{27}$$

Thus, one first designs K to satisfy the primary control task and then chooses Λ to satisfy the output cancellation condition (27). Several such iterations on the design of K, Λ may be required since the solution Λ of (27) depends on K, while the effect of residual disturbances on the primary control task (and therefore on the choice of K) may depend on Λ. Further details are given in (refs. 6,22,23). The condition (27) implies that Λ should be chosen so that the "controllable subspace" of $(\bar{A},\hat{B})$ becomes totally unobservable. In DAC theory the latter subspace is called the "disturbable subspace" [24].

In addition to the "observer–based" disturbance cancellation theory just outlined, there are two other DAC theories for designing disturbance cancellation controllers. Those two alternative theories, known as the "Optimal Control Method" and the "Algebraic/Stabilization Method", are based on different concepts and employ different mathematical procedures. The details are outlined in (ref. 9) where the original references are also given.

The Disturbance–Minimization Mode of Accommodation

Suppose the complete cancellation condition rank $[B|FH]$ = rank $[B]$ fails to be satisfied. Then, there does not exist a control $u_d(t)$ that can satisfy (23). In that event, the designer can invoke the alternative strategy of disturbance–minimization control (DMC) in which the objective is to choose $u_d(t)$ so as to "minimize" the disturbance effects in (22) in some specified sense. There are literally hundreds of variations on this problem, depending on which disturbance effect(s) one chooses to minimize in (22). For example, one natural approximation to (23) is to choose u_d to minimize $\|Bu_d + FHz\|$. The minimum–norm control that solves this latter problem is $u_d^o = B^\dagger FHz$ where $(\cdot)^\dagger$ denotes the Moore–Penrose generalized inverse. Alternatively, one can choose u_d to cancel the effects of certain selected components of $w = (w_1,\cdots,w_p)$ or can choose u_d to cancel the total disturbance effect(s) as they appear on certain selected components of $x = (x_1,\cdots,x_n)$. The latter is called disturbance cancellation for "critical" state–variables, and represents a generalization of the "output cancellation" idea (27) where $y = Cx$ plays the role of a vector of critical variables. Finally, there is the innovative technique called "indirect disturbance cancellation" which can be explored as an option under the disturbance–minimization mode. In that option the control u_d itself doesn't directly counteract the disturbance but rather u_d maneuvers certain noncritical state–variables $x_j(t)$ into such a position that the $x_j(t)$ themselves perform the disturbance cancellation; see [6; pp. 465–468]. Further details of the disturbance–minimization mode may be found in (refs. 4,6,8).

The Disturbance–Utilization Mode of Accommodation

The most intriguing mode of disturbance–accommodation in DAC theory is called disturbance–utilization control (DUC) and is based on the recognition that disturbance effects might not be altogether detrimental to the primary control task(s). In other words, it is conceivable that if the disturbances are pushing in the right direction, at just the right time,they might usefully "assist" the control u in achieving the primary control task(s). If that were the case, it would be foolish and wasteful to apply a control strategy of cancelling or minimizing those "useful" disturbance effects. What one should do in that case is manipulate the control u(t) so as to exploit and take maximal advantage of all useful energy and other beneficial effects in the disturbances. Needless–to–say this latter feat requires extraordinary finesse in making real–time control decisions.

The systematic design of optimum disturbance–utilizing controllers is relatively straightforward using DAC theory. First, one constructs a state–model (4) of the disturbance

$w = (w_1, \cdots, w_p)$. Then, a performance index (functional) J must be chosen with the property that minimization of J with respect to u simultaneously achieves two things: 1) it achieves the primary control task(s), and 2) it makes maximum utilization of the disturbances w(t) to assist in achieving the primary control task(s). One possible candidate for J, which also happens to be computationally attractive, is the familiar error–quadratic performance index

$$J = \varepsilon^T(T_p) S\, \varepsilon(T_p) + \int_{t_o}^{T_p} [\varepsilon^T(t)Q(t)\varepsilon(t) + u^T(t)R(t)u(t)]dt \qquad (28)$$

where $\varepsilon(t) = x^*(t) - x(t)$, [or $\varepsilon(t) = y^*(t) - y(t)$], *–denotes desired value, and where (typically) S,Q,R are symmetric, positive–definite weighting matrices chosen by the designer. It is remarked that a rationale and theory for admitting a range of <u>indefinite</u> and <u>negative–definite</u> Q in (28) has recently appeared in (refs. 25,26). The control penalty term u^TRu in (28) automatically encourages u(t) to "let w(t) do the driving" whenever that policy is cost–effective. If the disturbance <u>can</u> provide useful assistance to the control u(t) we say the disturbance has <u>positive</u> "utility". On the other hand if w(t) is only a hindrance to achieving the primary control task(s) we then say the disturbance has <u>negative</u> utility. It is possible to derive a "utility function" $\mathcal{U} = \mathcal{U}(x,z,t,.)$ which actually measures the sign and degree of optimum disturbance utility, (refs. 8,27). During the performance–interval, $t_o \le t \le T_p$, the disturbance utility function $\mathcal{U}$ may change sign back and forth, which is further evidence of the cunningness required in real–time control decisions in order to actually utilize disturbances to maximum advantage.

The DAC method of deriving the optimal disturbance–utilizing control $u^o(t)$ consists of appending (4) to (20), together with a dynamic model of the "commanded" (desired) behavior $x^*(t)$ or $y^*(t)$ of the form [compare with (4)]

$$y^* = y_c = Gc \quad ; \quad G,E = \text{known} \qquad (29\text{–a})$$

$$\dot{c} = Ec + \bar{\mu}(t) \;, \qquad (29\text{–b})$$

where $\bar{\mu}(t)$ = a sparse sequence of unknown impulses and c is the "state" of the command input $x^*(t)$ or $y^*(t)$. The idea of using a state model of the type (29) to represent uncertain set–points and servo–commands was proposed in (ref. 4, eqs. 40,41); see also (refs. 6,22). Next, one introduces the composite state vector $\tilde{x} = (x|c|z)$ so that $\varepsilon(t)$ in (28) can be expressed as $\varepsilon = [-C|G|0]\tilde{x}$. Then, (28) can be minimized subject to the composite dynamics of $\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$ by applying standard methods of Linear–Quadratic Optimal Control Theory (ref. 28). The details of this procedure are given in (refs. 4,5,6,8,27). The final form of the optimal disturbance–utilizing control u^o is

$$u^o = -R^{-1}B^T[K_1(t)x + K_2(t)c + K_3(t)z] \qquad (30)$$

where the gain matrices $K_i(t)$ are independent of x, c, z, and are determined by off–line solution of an auxiliary system of unilaterally–coupled matrix differential equations with known boundary conditions at $t = T_p$; see (ref. 4; pg. 641; 6; pg. 470).

It is important to note that in designing the disturbance utilizing control u^o in (30) we <u>do not</u> split (allocate) the total control u into parts as was done in (21) for the cancellation and minimization modes of accommodation. Moreover, the disturbance–utilizing control policy (30) continues to yield optimum control decisions <u>even if the disturbances w(t) have no useful effect</u>

(i.e. have only negative utility). In the latter case, the control (30) automatically minimizes the inevitable performance losses (= increase in J) due to "non–useful" disturbances. Thus, the control law (30) is a universally attractive substitute for the traditional Linear–Quadratic Optimal Control Law

$$u_{LQ} = -R^{-1}B^{T}K(t)x, \tag{31}$$

as presented in control textbooks and currently widely used in industry. Such a substitution is easy to implement (graceful upgrading) because the gain matrix K(t) in (31) coincides exactly with the matrix $K_1(t)$ in (30); see (Refs. 4,6). Moreover, the more general "disturbance–utilizing control law" (30) automatically reduces to the traditional Linear–Quadratic control law (31) whenever the disturbance w(t) disappears, [i.e., whenever z(t) becomes zero, assuming c = 0 also]. Note that the term K_2c in (30) represents DAC " feedforward" control of the command state c in (29). The importance of feeding–forward the command state c(t), rather than just the servo–command $y^*(t)$, is underscored throughout DAC theory; see (ref. 22, Figs. 3,4,5) and also (refs. 4,6,29).

The use of a disturbance–utilizing control law can result in significant savings in the consumption of u(t) control energy, without jeopardizing the performance quality of the primary control task. In fact, performance quality may also be significantly improved (refs. 30,31). This capability represents an exciting new domain of control design options and is unique with DAC theory.

Multi–Mode Accommodation of Disturbances

The three primary modes of accommodation just outlined can be blended in various ways to obtain a multi–mode disturbance controller which, for instance, performs disturbance–utilization during the initial phase of the control period and performs disturbance cancellation during the final (terminal) phase of control. This further widens the range of controller design options that DAC provides for coping with disturbances.

7. Summary

The disturbance modeling, identification and accommodation techniques outlined in this paper, collectively known as DAC theory, have attractive performance and design features which make them viable candidates for consideration in stabilization, set–point regulation, servo–tracking and model–reference adaptive control design problems in which uncertain external and internal disturbances play an important role. As such, DAC theory represents an effective alternative to existing stochastic control and adaptive control theories for dynamical systems with uncertain parameters and persistently acting, uncertain external disturbances.

Of course, in real–life control engineering problems, the day–to–day disturbances that act on a controlled system always deviate, to some extent, from the idealistic disturbance model originally used in the controller design. Thus any approach to controller design for disturbances will, of necessity, be less–than–optimal with respect to the actual disturbances. Therefore, in view of this inherent uncertainty in developing disturbance models, it is our opinion that designers should not choose a priori between DAC, stochastic, or other design methods, but rather in each application they should design an assortment of candidate controllers using all reasonable design methods. Then, by exercising each of the candidate controllers against the same family of representative real–life disturbances and parameter perturbations (or simulations thereof) one can decide which candidate controller is "best" for that particular application. In this respect, DAC theory simply provides an additional candidate in the competition for "best".

8. Epilogue

The DAC waveform modeling technique (1)–(4), and disturbance control law design methodologies in Section 6, originated in a small NASA–funded study during the period 1966–67 (refs. 32,33) and has evolved over the past 21 years into an effective general theory for the control of systems with complex, uncertain multivariable disturbances. Some representative applications of that theory, and many additional references, are described in (ref. 9); see also (refs. 21,34). The DAC theory outlined here is now beginning to appear as a standard topic in control engineering texts and university courses. In a few cases, the nomenclature and lineage presented therein differs from that presented here and reflected in the original literature.

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