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A Trajectory Generation and System Characterization Model for Cislunar Low-Thrust Spacecraft

Volume II—Technical Manual

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TABLE OF CONTENTS

I. INTRODUCTION

With the formation of the NASA Office of Exploration and the July 20, 1989 speech by President George Bush initiating a space exploration planning study, more attention has been focused on the planning and technologies required to establish a base on the lunar surface. Many different technologies have been considered to support such an endeavor with a great deal of attention being given to transportation systems. Many past studies of various lunar base strategies have led to the conclusion that a major cost driver of a base on the moon would be the transportation of the infrastructure from Earth to the lunar surface. One element of the transportation system is the transfer of crew and cargo between low-Earth orbit and lunar orbit and the subsequent return of such payloads. Many technologies have been applied to the space transportation in order to reduce the program costs. One such technology that may be applied to orbital transfer is that of constant, low-thrust propulsion systems. These systems, generally more efficient than conventional chemical propulsion systems tend to reduce the fuel requirement by several orders of magnitude and thus reduce the Earth to orbit launch requirements which results in a cost savings to a lunar base program. However, it must be noted that low-thrust propulsion systems are not applicable to crew transfer as the longer trip times would expose the crew to the hazards of the Van Allen radiation belts and long-term degradation of human physiological systems. Therefore, most studies addressing the low-thrust transfer of payloads from low-Earth orbit to lunar orbit have been primarily in unmanned cargo vehicles. This is the primary focus of this study.

The analysis contained herein focuses on both the definition of the vehicle and its supporting systems in conjunction with the guidance and control algorithms required to fly the spacecraft from its departure orbit to its destination orbit. The vehicle definition and systems characterization portion of this study concentrated on collecting state-of-the-art information from such institutions as

NASA Lewis Research Center, Los Alamos National Laboratory, the Auburn Space Power Institute, United States Air Force Space Division, and the Strategic Defense Initiative Organization in order to develop sizing algorithms to characterize the most state-of-the-art spacecraft for assessment of its performance. The trajectory analysis part of this study is the primary state-of-the-art advancement of this study as it is a three dimensional analysis formulated in the restricted three-body problem with perturbations.

In the past there has been a number of studies conducted on the low-thrust spacecraft, but most of these studies have either been dedicated to system characterization or trajectory analysis with very few being a combination of the two. Most of the trajectory analyses performed have been done in in-plane without out-of-plane thrusting required to change inclination of line of ascending node. This leads to a good initial characterization of a vehicle, but hardly lends itself to what is required for a complete, robust conceptual design and mission plan. Other past studies have also concentrated on a centric trajectory analysis. That is, a problem formulation in a single gravitational field with a spacecraft in orbit and spiraling outward (or inward) until escape (or capture) and then changing to a different centric coordinate systems for continued analysis. It should be noted that the Earth-Moon systems does not lend itself to this type of analysis as the Moon is in close proximity to the Earth's sphere of influence. Therefore, the problem formulation was developed in the restricted three-body problem to facilitate a more accurate simulation of the spacecraft's operation.

Since the trajectory analysis technique chosen here is based on a numerical integration, the vehicle must be initially sized before the trajectory analysis can begin. Therefore, an initial estimation model for the spacecraft size was developed which then transfers data to the trajectory analysis model. After the trajectory analysis is completed, the spacecraft is re-sized to fit the computed trajectory. This iteration scheme can be repeated until a suitable level of fidelity is reached.

II. PROBLEM FORMULATION

To facilitate the development of the low-thrust cislunar spacecraft trajectory determination and sizing model it was important to segment the total problem into discrete problems that could be more easily addressed. From a top-level perspective, the problem was divided into two functional areas: system characterization, and trajectory generation and guidance. These two functional areas were further divided to better enable a solution to determined.

In the case of the system characterization portion of the model it was necessary to determine how multiple technologies for various components were to be integrated into a complete system. Also of concern was the determination of the amount of propellant required by the vehicle. The propellant requirement is needed to enable the determination of an appropriate initial mass of the spacecraft in the departure orbit. The value of the spacecraft's initial mass is used to determine the initial acceleration of the spacecraft and the subsequent acceleration throughout the trajectory generation.

For the trajectory generation and guidance segment of the model, two reference systems were chosen to allow the greatest ease of numerical integration, conceptual understanding, and control for various portions of the spacecraft's flight. The first reference system is a non-dimensionalized geocentric frame in which six orbital elements describe the shape and nature of the spacecraft's orbit. This is the equinoctial coordinate system with modifications. The second system is a rotating, non-dimensionalized, right handed coordinate system with the origin at the Earth-Moon system center of mass. This coordinate system is known as the restricted three-body system. Both reference systems are explained in later sections.

2.1 Vehicle System Characterization

The characterization of a low-thrust orbital transfer vehicle (OTV) for Earth-Moon transfers was performed by dividing the spacecraft into its major systems. These are power, propulsion, thermal control, propellant tankage, and structural mass. Each of these systems is defined in terms of interactions between the other systems. For example, the propulsion systems needs power, thermal control, propellant, and structure, the power system needs thermal control, and structure, the thermal control system needs power, and structure, etc....

The payload mass is the primary driver when characterizing the OTV. The user can input any desired size for the payload and the system sizing model will characterize a spacecraft based on the payload mass. The propulsion system is chosen by the user to define the performance of the system. The propulsion system performance characteristics, such as specific impulse, and power per engine, along with the technology choice of the propulsion system are required user inputs. These inputs allow the model to adequately define the propulsion system of the OTV. The propulsion system then has certain requirements that the other systems must provide in order to have a functioning spacecraft. The technology choice of the power system is another user input that completes the necessary user characterization of the OTV. The power system obtains the information regarding its electrical requirements primarily from the propulsion system. The thermal control system obtains the required heat rejection data from the propulsion and power systems and the structural requirements are developed from the mass and volume requirements of the propulsion, power, and thermal systems.

Sizing the subsystems required for an OTV is a formidable task due to the large amount of data needed to cover all of the possible system choices. In order to implement a computer-based vehicle systems sizing model, it was determined that the use of parametric equations would be more effective than using a large design database to generate system sizing parameters. The use of parametric equations results in a robust design model. For example, the parametric equations used in the power system sizing subroutine were derived by determining mathematical relationships between such system parameters as output power and system mass. Using these relationships, the power system mass can be calculated for any desired output power within the range of the power system technology. All of the system sizing subroutines use parametric equations, derived from data in current technical publications, to generate the system mass estimates.

2.2 Trajectory Generation

In previous studies on the development of trajectories for low-thrust cislunar OTVs, little attention has been directed toward the robust algorithms required for three-dimensional guidance and control of the spacecraft. The guidance and control of the spacecraft and the determination of the appropriate trajectory are highly coupled. The guidance, control, and trajectory determination are closely related problems which by necessity must be treated with equal importance[1]. A major problem in the design of low-thrust OTVs and their associated trajectories is the lack of an end-to-end computer simulation for the spacecraft trajectory between Earth and lunar orbit. The physical capability of a low-thrust spacecraft to travel between the Earth and the Moon is known[2]. The current question is how the vehicle will behave at the proposed thrust level and how it will be guided on its trajectory. To adequately model the dynamic forces on the spacecraft in its travel in the Earth-Moon system, two problem formulations were used to generate the trajectory. The equinoctial formulation of the equations of motion was used for Earth escape, and the restricted three-body formulation was used for the for the midcourse and capture phases.

The standard formulation of a spacecraft's orbit is through the use of classical elements. These correspond to the semi-major axis of the orbit (a), eccentricity (e), inclination (i), mean anomaly (M), longitude of periapsis (ω), and longitude of the ascending node (Ω). These classical elements are usually defined in terms of a geocentric equatorial coordinate system with the positive X-axis parallel to a vector from the sun to the Earth at vernal equinox, and the Z-axis through the north pole of the Earth (see Figure 1). These six elements are traditionally used to describe the orbit of a spacecraft.

In the classical formulation of the two-body problem, the angles of mean anomaly (M), eccentric anomaly (E), and true anomaly (v), are well known and much used. Low-thrust spacecraft trajectories will always have portions of their trajectories where the orbit is circular or the eccentricity is very small (<<1). Also, it is possible to have small to zero inclination orbits when transferring to geosynchronous orbits or passing through the equatorial plane. The three angles, M, E, v, are not well suited for small eccentricity or circular orbits because they are measured from the perigee or periapsis point of the orbit. When a orbit is circular the periapsis point of the orbit is not defined. Additionally, ω and Ω are ill-defined for zero inclination orbits. This causes singularities during numerical integration of orbits with classical orbital elements. For these reasons, integration of low-thrust trajectories in classical orbital elements is not feasible or desirable.

2.2.1 Equinoctial Formulation

An alternate set of orbital elements can eliminate the singularities experienced by classical orbital elements. These elements are known an equinoctial elements and are defined in terms of classical elements,

$$
\begin{aligned}
a &= a, \\
h &= e \sin(\omega + \Omega), \\
k &= e \cos(\omega + \Omega), \\
\lambda &= M + \omega + \Omega, \\
p &= \tan i/2 \sin \Omega, \text{ and} \\
q &= \tan i/2 \cos \Omega.
\end{aligned}
$$

The equinoctial element formulation, $p_\alpha = (a, h, k, \lambda, p, q)$, has no singularities at e=0 or i=0. The semi-major axis, a, remains the same while h, k, p, and q are just convenient mathematical formulations, and λ is the sum of M, E, and v and is referred to as the mean longitude.

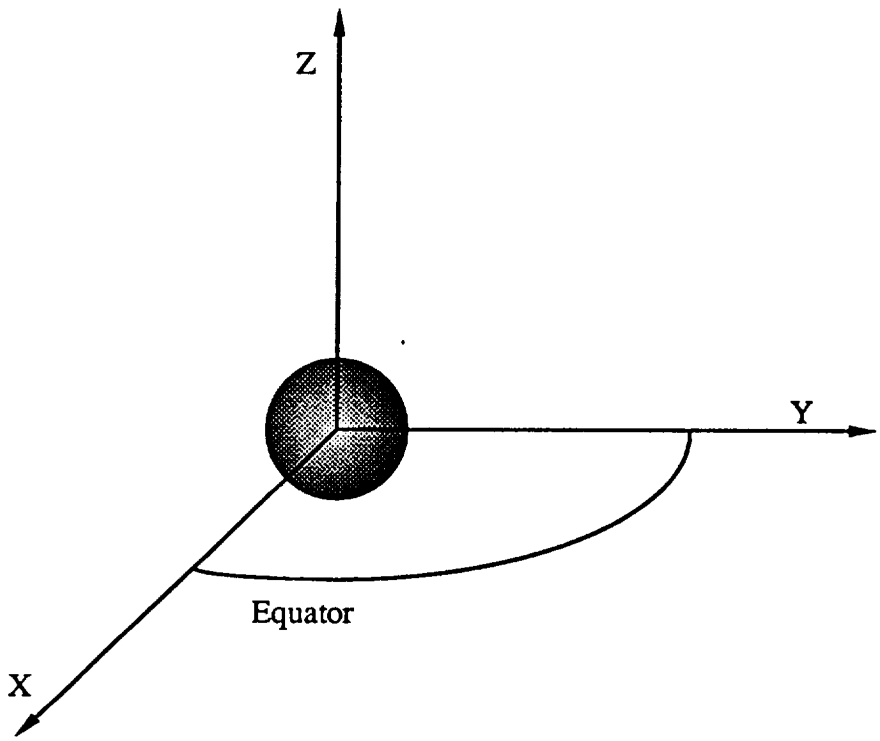

Figure 1 - Geocentric Equatorial Coordinate System

The equinoctial coordinate frame is defined by unit vectors $\hat{f}$, $\hat{g}$, and $\hat{w}$ illustrated in Figure 2 and defined as follows.

$$\hat{f} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 1 - p^2 + q^2 \\ 2pq \\ -2p \end{bmatrix},$$

$$\hat{g} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 2pq \\ 1 + p^2 - q^2 \\ 2q \end{bmatrix},$$

$$\hat{w} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 2p \\ -2q \\ 1 - p^2 - q^2 \end{bmatrix}.$$

Where each column vector in brackets contains the X, Y, and Z component of the unit vectors $\hat{f}$, $\hat{g}$, and $\hat{w}$.

The trajectory of a low-thrust OTV can be integrated in this formulation through the use of a method known as variation of parameters. A complete derivation of this method is included in Appendix A. The equinoctial elements are integrated using the variation of parameters equation written in equinoctial form:

$$\dot{P}_\alpha = \frac{\partial p_\alpha}{\partial \dot{\vec{x}}} \cdot \vec{F}, \qquad \{1\}$$

where the right hand side of {1} is the dot product of the partials with respect to velocity with the rectangular components of the perturbing acceleration $\vec{F}$. For the low-thrust spacecraft, the perturbing acceleration vector, $\vec{F}$, consists of the acceleration due to the spacecraft's propulsion system, the perturbing acceleration of the Moon, the gravitational pull due to the oblateness of the Earth, and the acceleration on the spacecraft due to atmospheric drag.

The equinoctial elements change slowly during integration and are smoothly varying functions. This allows the numerical integrator to take large step sizes, thereby speeding calculation of the

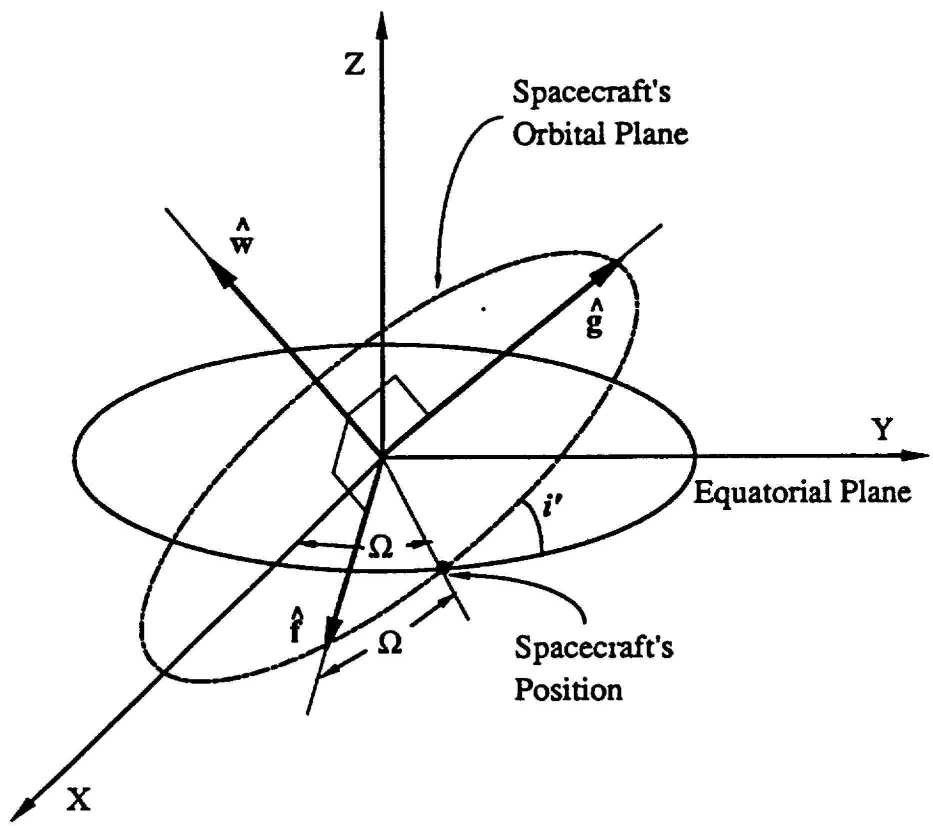

Figure 2 - Equinoctial Coordinate Frame

trajectory. For both the increase in integration speed and the elimination of singularities, the equinoctial formulation is used to generate the trajectory during Earth escape.

2.2.2 Restricted Three-Body Formulation

To adequately understand the dynamics of motion of the low-thrust spacecraft, the gravitational effects of the Earth and the Moon on the spacecraft must be included for the full duration of the trajectory. The thrusting acceleration of low-thrust OTVs in high Earth orbit is the same order of magnitude as the perturbing force due to the Moon. To model the Earth-Moon system with the necessary accuracy and achieve computational efficiency, the restricted three-body formulation of the dynamical equations is utilized as the governing equations of motion.

The problem of three bodies was first formulated in 1772 by Lagrange. Further studies by Poincare, Laplace, Hill and Szebehely have resulted in a detailed treatment of the problem and a general understanding of the interactions between the two primary gravitational fields. Various formulations are available to represent the problem of three bodies. Discussions with Victor Szebehely at The University of Texas at Austin led the authors to use the non-dimensional formulation of the restricted problem of three-bodies for the cislunar transfer and capture portion of the trajectory generation[3]. This formulation has several advantages over the general three-body problem. The order of equations to be integrated are reduced from 18th order for the general problem of three bodies to 6th order for the restricted problem of three-bodies. This reduction in order dramatically decreases integration time. Additionally, the equations of motion are non-dimensionalized by the Earth-Moon distance and the angular rate of the Earth-Moon system about the systems center of mass (barycenter). The Earth-Moon system in restricted three-body formulation is shown in Figure 3.

Many realistic orbital cases of interest permit treatment as restricted three-body situations. A case of particular interest is that

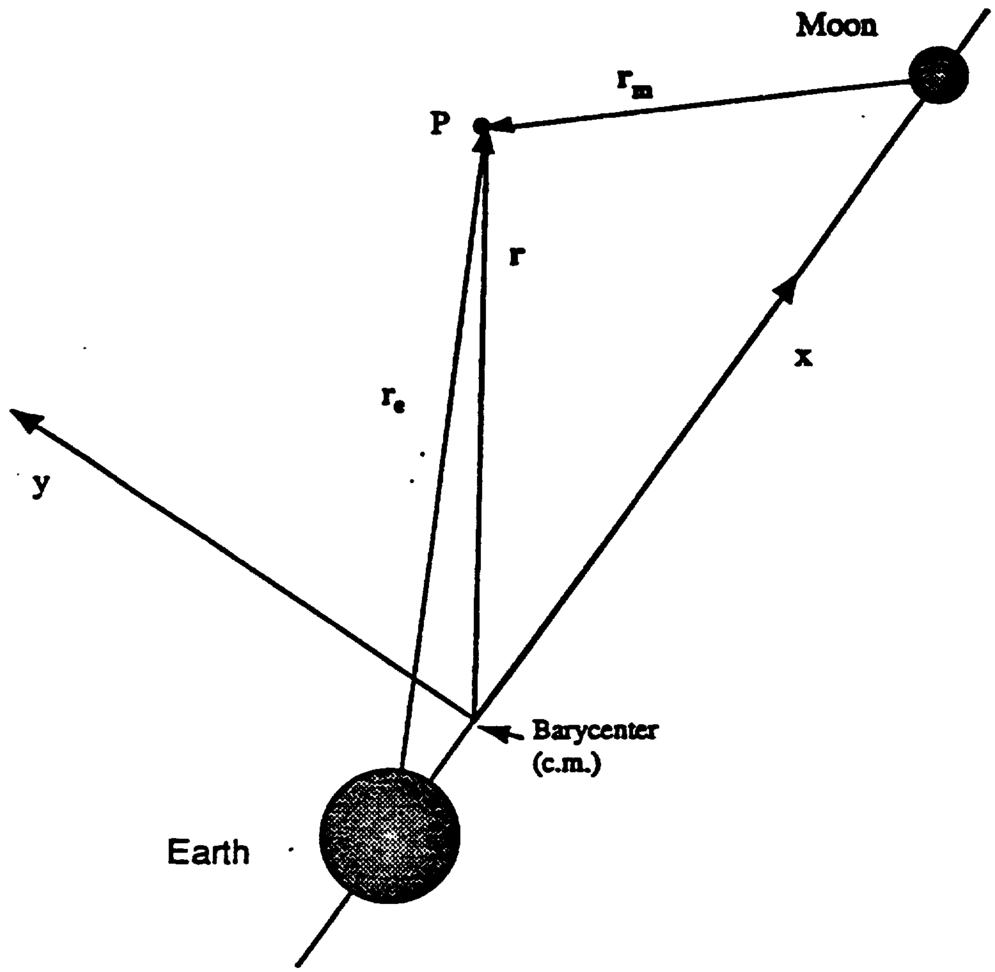

Figure 3 - Restricted Three-Body Problem Formulation

of a spacecraft moving in the Earth-Moon system. Certain assumptions can be made about the nature of the Earth-Moon system that permit a simplified formulation at a slight loss of accuracy. It is assumed that the motion of both the Earth and Moon is circular and coplanar about their barycenter. The Earth-Moon line is the x-axis of a rotating coordinate system and the z-axis is parallel to the angular momentum vector of the Earth-Moon plane. The y-axis is perpendicular to both the x-axis and the z-axis. The position of the Earth and the Moon are constant in the restricted three-body formulation. The spacecraft, at point P, is assumed to have negligible mass and to have no impact on the motion of the Earth or the Moon. The motion of the spacecraft is governed by the relative gravitational attraction of the Earth and the Moon rotating about the barycenter. The equations of motion for the spacecraft in the restricted three-body non-dimensionalized rotating coordinate system are:

$$\ddot{x} - 2\dot{y} = \Omega_x, \qquad \{2\}$$

$$\ddot{y} + 2\dot{x} = \Omega_y, \text{ and} \qquad \{3\}$$

$$\ddot{z} = \Omega_z, \qquad \{4\}$$

where

$$\Omega = \frac{1}{2}\left[(1-\mu) r_1^2 + \mu r_2^2 \right] + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}, \qquad \{5\}$$

$$r_1 = \left[(x-\mu)^2 + y^2 + z^2\right]^{1/2}, \text{ and}$$

$$r_2 = \left[(x+1-\mu)^2 + y^2 + z^2\right]^{1/2}.$$

A derivation of the equations of motion for the restricted three-body problem can be found in Victor Szebehely's book, *Theory of Orbits*[4].

In the restricted three-body formulation of the equations of motion the Keplerian (potential and kinetic) energy of the spacecraft is not conserved. This is due to the assumptions regarding the motion of the Earth and Moon and the spacecraft's effect upon their motion. However, the sum of the angular momentum, velocity in the rotating coordinate system, and potential energy of the spacecraft is conserved. This result is determined through the derivation of the equation known as the Jacobian integral. The integral and the constant, C, it produces are named after mathematician Karl Gustav Jacobi who first formulated this integral in 1836. This integral can be derived from equations {2}, {3}, {4}, and {5}. Initially, equations {2}, {3}, and {4} are multiplied by $\dot{x}$, $\dot{y}$, and $\dot{z}$ respectively. This yields,

$$\ddot{x}\,\dot{x} - 2\dot{y}\,\dot{x} = \Omega_x\,\dot{x}\ ,$$
$$\ddot{y}\,\dot{y} + 2\dot{x}\,\dot{y} = \Omega_y\,\dot{y}\ , \text{ and}$$
$$\ddot{z}\,\dot{z} = \Omega_z\,\dot{z}\ .$$

These three equations are added together to form,

$$\ddot{x}\,\dot{x} + \ddot{y}\,\dot{y} + \ddot{z}\,\dot{z} = \Omega_x\,\dot{x} + \Omega_y\,\dot{y} + \Omega_z\,\dot{z}\ ,$$

which can be integrated and rearranged to find the integral,

$$C = 2\Omega - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2). \qquad \{6\}$$

This constant, C, can be determined for any set of position and velocity conditions in the restricted three-body problem. If there is no force acting on the spacecraft other than the Earth and Moon the value of C, the Jacobian constant, will remain constant. So for a nonperturbed orbit about the Earth the value of C will remain the same. For various arbitrary combinations of position and velocity differing values of C will be found. Conversely, if C is initially determined to be a particular value from equation {6}, say the value that corresponds to the position and velocity of a spacecraft in orbit about the Earth, there will be many combinations of position and velocity the will give the same value of C. The potential, Ω, shown in equation {5} is dependent solely upon the position of the spacecraft in the restricted three-body system. If $\dot{x}$, $\dot{y}$, and $\dot{z}$ were each set equal to zero in equation {6} and a value of C was previously determined from a spacecraft's position and velocity, then equation {5} will give the x, y, and z coordinates where the spacecraft would have zero velocity. These coordinates form a surface in the three-dimensional coordinate system of the restricted three-body formulation. This surface bounds the locus of points where the spacecraft can have motion given the value of its Jacobian constant, C, in the Earth-Moon system. On this surface a spacecraft with a given C will have zero velocity. Only on the inside of the curve will the spacecraft have any velocity, thus restricting the motion of the vehicle to the inside. Figure 4 shows a series of cross sections of zero velocity surfaces as curves in the Earth-Moon system's x-y plane[5]. The value of the spacecraft's Jacobian constant is used during the midcourse targeting phase of the trajectory generation. It is used as an indicator of the spacecraft's ability to achieve the desired cislunar transfer.

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[1] Battin, R. H., Miller, J. S., "Trajectories and Guidance Theory for a Continuous Low-Thrust Lunar Reconnaissance Vehicle," 6th Symposium on Ballistic Missile and Aerospace Technology, 1961.

[2] Hill, P.G., and Peterson, C.R., Mechanics and Thermodynamics of Propulsion, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1965, Chapter 10.

[3] Szebehely, Victor, Personal Communication, October 1988.

[4] Szebehely, Victor, Theory of Orbits, Academic Press, Inc., New York, 1967

[5] Kaplan, Marshall H., Modern Spacecraft Dynamics and Control, John Wiley and Sons, New York, 1976.

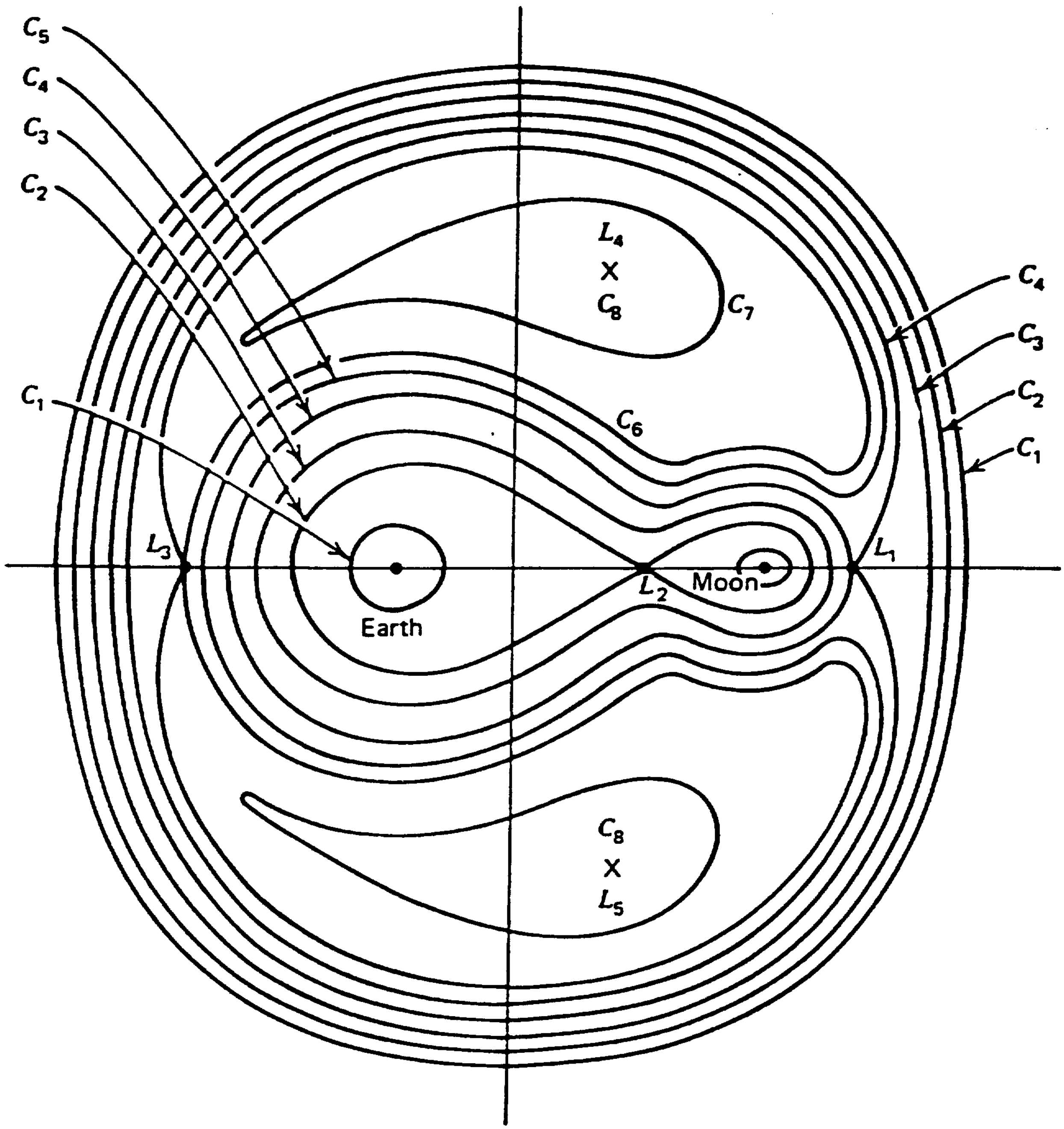


Figure 4 - Zero Velocity Curves in the Earth-Moon System

III. PROGRAM METHODOLOGY

3.1 Vehicle Systems Sizing

The vehicle system sizing model contains parametric models of the major vehicle subsystems: power generation and distribution, propulsion, support and propellant tankage structure, and thermal control. In some cases multiple technologies are parameterized for each system with the technology choice left to the model user. The interrelationships between these systems are mapped out as a set of iterating functions that are scaled according to the users system technology and payload size choice. Figure 5 shows some of the interrelationships between the various systems. It is assumed that the OTV does not refuel at the Moon and that no propellant tanks are discarded. The total dry mass of the OTV is calculated by summing the system component masses and desired payload mass.

A propellant estimation model has been developed to determine the appropriate amount of propellant necessary for a cislunar transfer. A functional relation between the spacecraft's thrust to weight ratio and the propellant fraction of the vehicle has been developed using a two-dimensional constant thrust lunar trajectory program developed by the Large Scale Programs Institute under a grant from the NASA Johnson Space Center. Using this functional relationship the propellant mass can be determined which leads to an initial estimate of the total system mass. The initial total system mass of the spacecraft, along with the propulsion parameters (mass flow rate and specific impulse (Isp)), is used as model drivers by the trajectory generation model.

A description and overall program flow of the vehicle system sizing model is detailed in the next section. Following the program flow description, the specific formulation of the propellant estimation routine is discussed and assumptions concerning the development of the methodology are detailed.

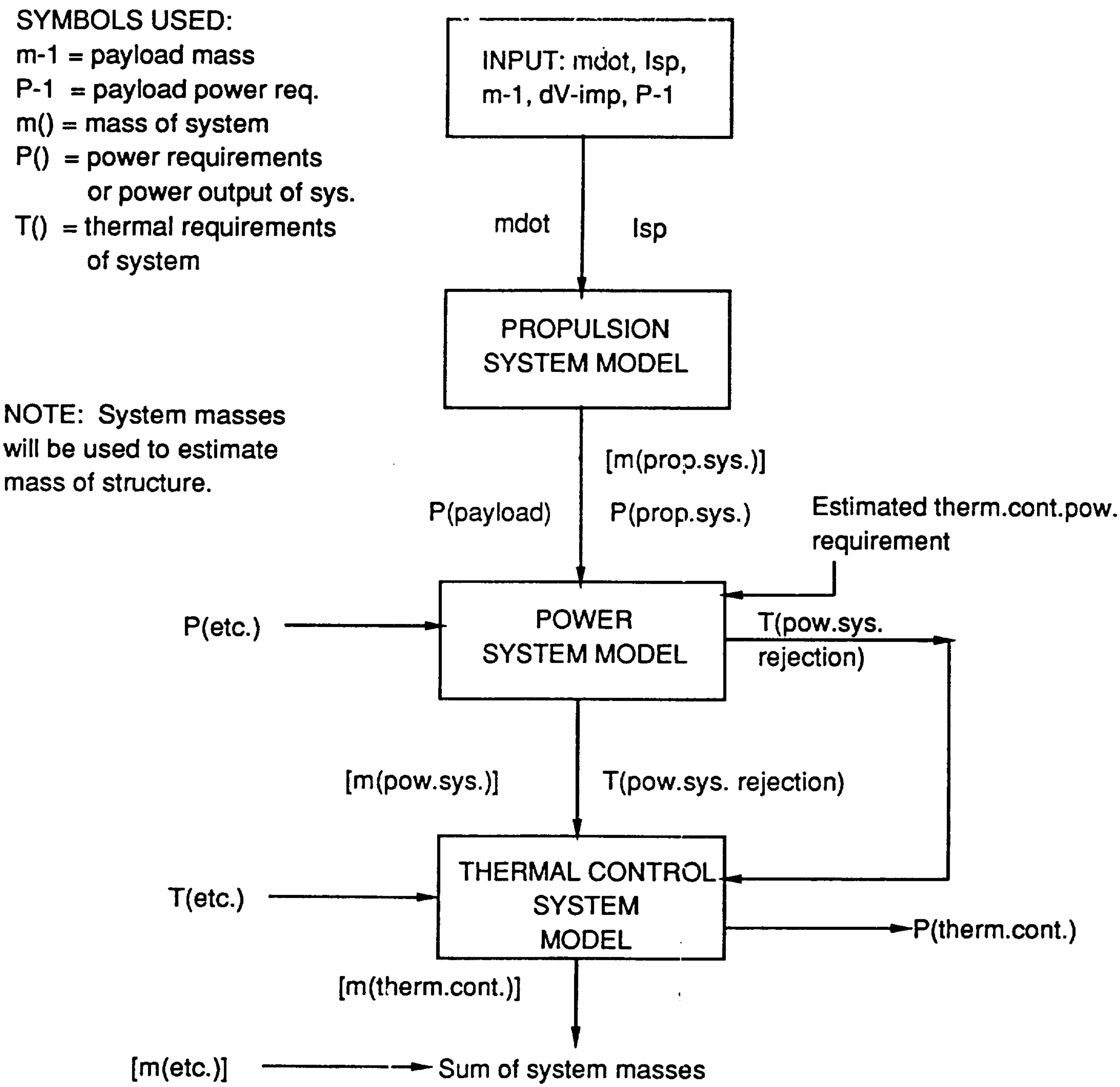


Figure 5 - Basic OTV System Interrelationships

3.1.1 System Model Program Flow

The vehicle system sizing model is segmented into input and systems sizing subroutines. The first subroutine run is the input subroutine, INPUT0, which checks whether the user wishes to run the vehicle system sizing model or specify his own overall vehicle characteristics. If the user wishes to specify the specific vehicle characteristics, the program prompts the user to enter the initial spacecraft mass, the total mass flow rate of the propulsion system, the specific impulse of the propulsion system, and the final mass of the vehicle. These parameters are transferred to the trajectory generation subroutines.

If the user chooses to run the vehicle system sizing model, then the user has the choice of generating the spacecraft's characteristics for a one-way or two-way trip. A one-way trip would consist of a Earth to Moon, or Moon to Earth, transfer. A two-way trip is an Earth-Moon-Earth transfer. At present, even though the system sizing model can generate characteristics for a vehicle supporting two-way transfers, the trajectory generation routine only supports one-way transfers. The characteristics of the spacecraft are output to a file, LOWTHRST.DAT for later evaluation. After these choices have been made, program control is passed to the subroutine, SYSMOD (SYStem MODel), which handles the calls to the various system sizing subroutines. The top level flow of data is shown in figure 6.

SYSMOD begins by calling the input subroutine, INPUT1, which lets the user enter the mass of the payload to be delivered (and returned in the case of the two-way transfer). Next, SYSMOD calls the propulsion system sizing subroutine, PROPMOD (PROPulsion MODel), which allows the user to choose one of four types of electric propulsion systems. The four types of systems available include ion, magnetoplasmadynamic (MPD), arcjet, and a user specified system. There are three options for Ion propulsion: Xenon, Krypton, or Argon propellant. PROPMOD calls one of the four propulsion system models: ION, MPD, ARCJET, or USER1. The subroutines, ION, MPD, and ARCJET,

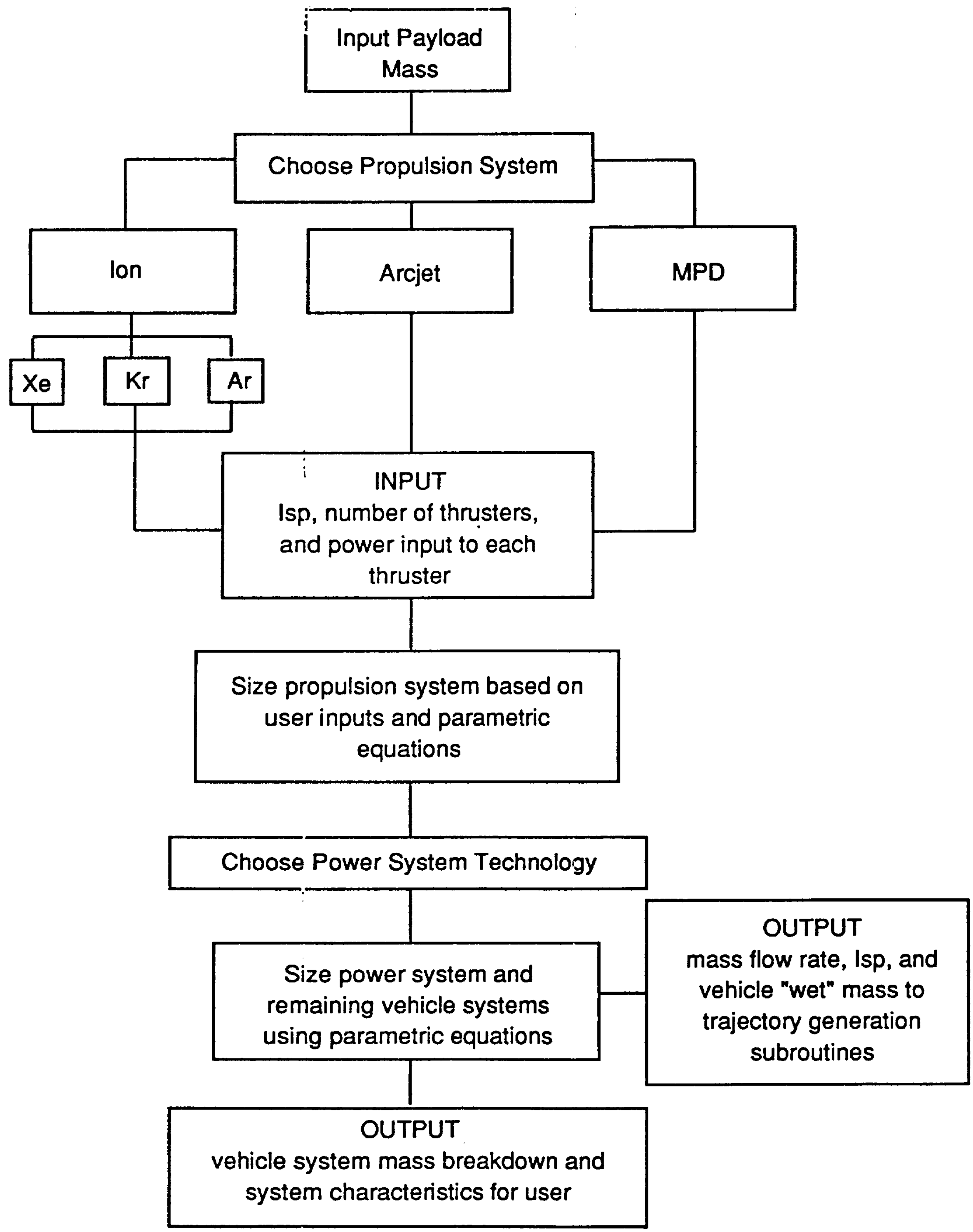


Figure 6 - Top-level Flow of Spacecraft System Sizing Model

allow the user to specify the specific impulse, number of thrusters, and power input to each thruster. These input quantities are used in parametric equations to calculate the efficiency of the thruster system, the dry mass of the propulsion system (i.e. no propellant), and the total power requirements of the propulsion system. The subroutine, USER1, allows the user to specify the specific impulse, mass flow rate per thruster, power input to each thruster, number of thrusters, efficiency, and the dry mass of the propulsion system. Once the propulsion system has been specified, control of the program is returned to SYSMOD which then calls the power system sizing subroutine, PWRMOD.

PWRMOD allows the user to choose one of six power generation and conversion systems[1,2]. The possible choices include:

(1) Liquid Metal Reactor using Rankine Cycle Conversion (1.5 kWe - 50 MWe),
(2) Liquid Metal Reactor- NERVA Derivative using Closed Brayton Cycle Conversion (1.5 kWe - 50 MWe),
(3) Solid Core Reactor using In-Core Thermionic Conversion (10 kWe - 50 MWe),
(4) Liquid Metal Reactor using AMTEC Thermoelectric Conversion (1 kWe - 50 MWe),
(5) NERVA Derivative Reactor using Magnetohydrodynamic Conversion (100 kWe - 100 MWe), and
(6) SP-100 reactor with Thermionic conversion (100 kWe - 500 kWe).

Once the power system is chosen, PWRMOD uses parametric equations to size the reactor, convertor, and control systems based on the power required. In addition, PWRMOD computes the efficiency of the power system and returns program control to SYSMOD to call the thermal control system sizing model, THERM. THERM computes the mass of the thermal control system based on parametric equations concerning the efficiency and power output of the power system.

The final systems sizing subroutine called by SYSMOD is the reaction control system sizing subroutine, RCS. RCS uses parametric equations to estimate the mass of a reaction control system and the propellant required to give the vehicle an initial estimate of a 100 meter per second velocity change. The vehicle mass used for this estimate consists of the system masses already calculated, an estimate of the vehicle's structural mass, and a rough estimate of the propellant required for the trip. Once this system has been sized, the only remaining quantity that must be determined is the amount of propellant required for the trip. The subroutine that contains the propellant estimation algorithm is called PRPEST.

3.1.2 Propellant Estimation

The subroutine PRPEST (PRoPellant ESTimator) computes an estimate of the mass of the propellant required for a one-way trip. The inputs to the subroutine consist of the specific impulse, mass flow rate per thruster, the number of thrusters in the propulsion system, the vehicle initial mass (structural mass plus propulsion, power, thermal control, and RCS system masses), and the payload mass for the one-way flight. The subroutine uses the mass flow rate per thruster and number of thrusters to determine the total mass flow rate for the propulsion system.

PRPEST runs an iterative loop to calculate an estimate of the propellant mass required for the cislunar flight. The vehicle's velocity is initialized to zero, the mass of the spacecraft is the vehicle's initial mass (including the payload mass), and the propellant mass is zero. The total velocity change, based on Apollo 17 data, required to travel between the Earth and Moon is 9000 meters per second[3] . PRPEST calculates an incremental Δv to be given to the vehicle at each time step in the loop. The equation used to calculate this Δv is:

$$\Delta v = mdot*g*Isp*dt/m \qquad \{7\}$$

where,

mdot = mass flow rate (kg/s),

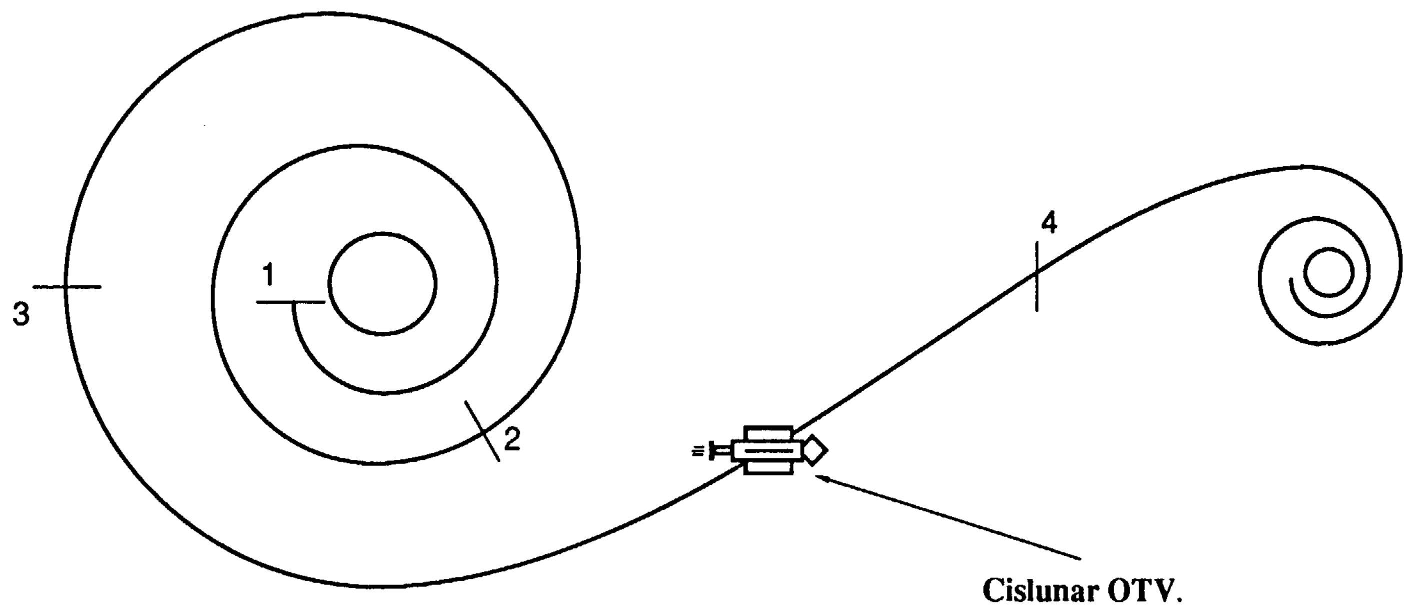
g = constant of gravity (km.sec^2),
Isp = specific impulse of the engines (seconds),
dt = time step (100 seconds), and
m = instantaneous spacecraft mass (kg).

The calculated incremental Δv from equation 7 at each time step is added to the vehicle's current velocity to obtain an updated velocity. The propellant mass is incremented by the total mass flow rate over the 100 second time increment. The propellant tank mass is incremented at each time step using a parametric equation that relates tank mass to propellant mass. The new propellant and tank masses are added to the vehicle's total mass to obtain a new value for the vehicle's instantaneous mass. These equations are repeated until the vehicle's velocity meets or exceeds 9000 m/s.

In order to use PRPEST to estimate the mass of the propellant required for a two-way trip, SYSMOD first generates the total spacecraft mass for the "second leg" of the two-way transfer. To determine the propellant mass for the "first leg" of the transfer, the propellant mass and tankage is assumed to be part of the first leg's payload mass. The propellant mass for the first leg is summed with the propellant mass for the second leg to obtain the total two-way propellant required.

3.2 Trajectory Generation

The trajectory determination and guidance of the cislunar low-thrust OTV is divided into three distinct phases (see Figure 7): Orbital Plane Alignment, Midcourse Targeting, and Capture and Circularization. Orbital Plane Alignment is concerned with the methodology and required guidance and control to drive the spacecraft from its initial orbital plane about the Earth into the plane of motion of the Moon about the Earth. The Midcourse Targeting phase of the trajectory generation deals with achieving the spacecraft position and velocity at Earth or Moon escape to achieve Earth-Moon transfer. The Capture and Circularization phase consists of the required controls to capture the spacecraft about the target



Trajectory Breakdown:
1-2 - Orbit Plane Alignment
2-3 - Transfer to Departure Point
3-4 - Cislunar Transfer
4-5 - Capture
5 - Circularization

Figure 7 - Trajectory Generation Phases

planet and to circularize the capture orbit at the desired final altitude above the surface of the target body. Each of these phases has a different guidance scheme to achieve the overall goal of generating trajectories between the Earth and Moon.

3.2.1 Orbital Plane Alignment

The first phase in any cislunar journey for an OTV is the escape from the initial parking orbit, whether about the Earth or the Moon. For low-thrust spacecraft to achieve escape, a long period of continuous thrusting is necessary. This results in a slowly increasing spiral trajectory from the initial orbit. It can be shown that a near-optimal thrust for a planar orbital transfer should be directed along the velocity vector of the spacecraft for the majority of the trajectory[4,5]. This is referred to as tangential thrust, because the thrusting acceleration will be tangent to the trajectory at all times. This methodology provides the maximum increase in the semi-major axis of the spacecraft's orbit over a specific time interval of thrusting. Tangential thrust is the nominal thrusting approach used in the spiral escape from the departure planet in this program due to its near-optimal propellant usage.

The program allows the user to specify the spacecraft in an orbit about the Earth at any date. To generate a trajectory from the spacecraft's initial orbit to the final desired orbit about the Moon, the plane of the spacecraft's orbit must be aligned with that of the Moon's when the spacecraft is escaping from the Earth. This requires the spacecraft to transfer between its initial plane of motion to the Moon's plane of motion. Current low-thrust trajectory research has not developed adaptive guidance and control algorithms for the transfer of a spacecraft between two planes using low-thrust propulsion. This lacking instigated the development of a new thrusting algorithm for low-thrust transfer between two arbitrary planes about the Earth.

An arbitrary plane of motion in an Earth centered equatorial inertial coordinate system can be defined by two of the classical orbital elements, Ω and i, of an orbit in that plane. The longitude of ascending node (Ω) describes the angle, for a posigrade orbit, from the x-z plane to the line where the orbital plane and x-y plane intersect. The inclination (i) describes the angle between the orbital plane and x-y plane measured at their line of intersection. For any orbit about the central body of the Earth, Ω and i will define the plane of motion of the object. For two arbitrary planes (see Figure 8), the angles Ω_1 and i_1 define the first plane of motion and the angles Ω_2 and i_2 define the second plane of motion. The common angle between the two planes is called the wedge angle, i'. To enable transfers between two arbitrary orbital planes, Plane 1 and Plane 2, the angles Ω_1 and i_1 of the spacecraft's orbit, Plane 1, must be driven to the angles Ω_2 and i_2 of the target orbit, Plane 2. This aligns Plane 1 with Plane 2. In order to change the plane of motion of a spacecraft it is necessary to determine how low-thrust accelerations would effect the the angles Ω and i that describe the plane.

The influence of perturbing forces upon the classical orbital elements is known from Lagrange's Planetary equations[6]. These equations address the rates of change of the classical orbital elements due to perturbing forces on the body in orbit. The equations that determine effect of perturbing forces on the rate of change of i and Ω are:

$$\frac{di}{dt} = F_n \frac{\cos(\nu + \omega)}{V} \tag{8}$$

and

$$\frac{d\Omega}{dt} = F_n \frac{\sin(\nu + \omega)}{V \sin(i)}\ , \tag{9}$$

where F_n is the normal component of the perturbing force, ν is the true anomaly of the orbit, ω is the longitude of periapsis of the orbit and V is the magnitude of the spacecraft's velocity. From equations {8} and {9} it can be seen that changes in i and Ω are only caused by a perturbing force acting normal to the spacecrafts orbital plane.

A novel way to determine the necessary thrusting to change the spacecraft's orbit from one plane to another was developed using

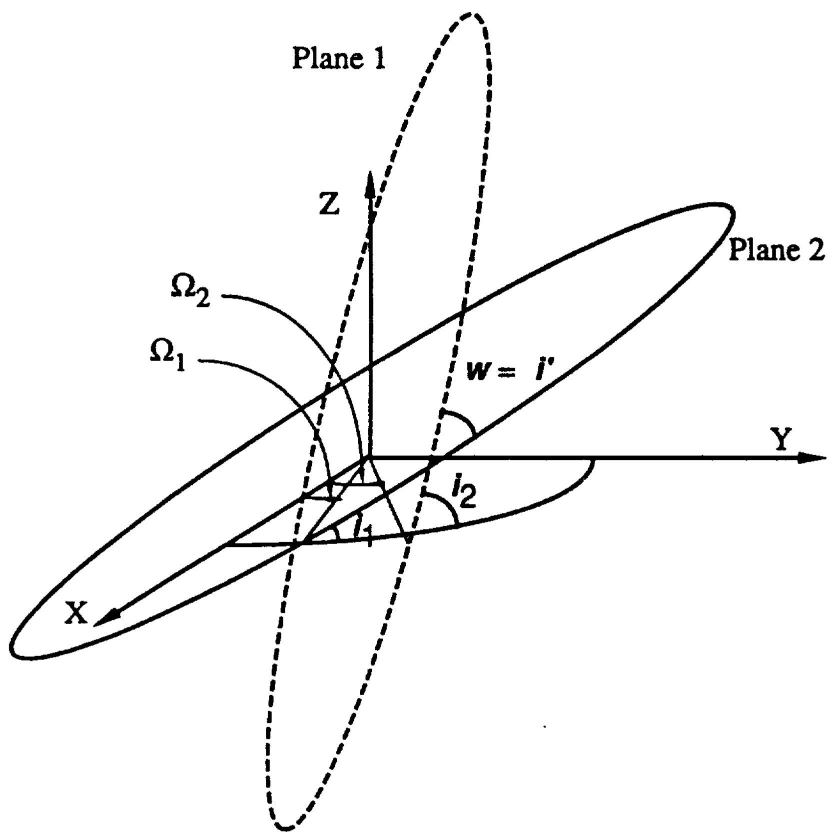


Figure 8 - Two Arbitrary Orbital Planes

Lagrange's Planetary equations as a starting point. First, it is necessary to develop a desired di/dt and dΩ/dt. The desired average rates, di/dt and dΩ/dt, can be determined by calculating the difference (Δi) between i_1 and i_2 and the difference ($\Delta\Omega$) between Ω_1 and Ω_2 and developing a time period (Δt) during which the change should occur. This Δt was determined empirically as a function of the initial acceleration magnitude of the spacecraft. The Δt is the minimum time required to achieve escape velocity for a spacecraft, from starting orbit, under constant tangential thrust. This spiral escape was calculated in a nonperturbed two body formulation of the equations of motion (see Appendix B). Then di/dt and dΩ/dt can be approximated by $\Delta i/\Delta$t and ΔW/Δt. The normal force required to change i and Ω can be derived from {8} and {9} by rearranging the equations to find,

$$F_{ni} = \frac{\Delta i}{\Delta t}\,\frac{V}{\cos(\nu + \omega)} \qquad \{10\}$$

and

$$F_{n\Omega} = \frac{\Delta\Omega}{\Delta t}\,\frac{V\,\sin(i)}{\sin(\nu + \omega)}\ , \qquad \{11\}$$

where F_{ni} is the normal force to change i_1 to i_2, and $F_{n\Omega}$ is the normal force to change Ω_1 to Ω_2.

Since both Ω and i are affected by accelerations normal to the plane of motion, it is not possible for a low-thrust spacecraft to change Ω and i independent of each other. Equations {10} and {11} are coupled, so the normal forces derived may be in opposing directions and effectively cancel during portions of the orbit for a given $\Delta i/\Delta$t and $\Delta\Omega/\Delta$t. Some previous studies have attempted to change only one element, either i or Ω, while ignoring the effect the of the thrust on the other orbital elements[7]. These coupled equations imply that the implementation of the control algorithm that address only one of the orbital elements makes an incorrect simplification. Additionally, determining the appropriate control algorithm from {10} and {11} would be extremely difficult. An essential simplification, however, has been identified to reduce the two control equations from {10} and {11} to one single control.

It should be noted that the assumption that the angle between two arbitrary orbital planes can be closely represented by the difference between the inclinations, i_1 and i_2, is incorrect. The angle between the two planes, commonly called the wedge angle (i'), is a function of both Ω and i. The wedge angle is calculated as follows:

$$i' = \cos^{-1}\left[\cos(\Omega_1 - \Omega_2)\sin(i_1)\sin(i_2) + \cos(i_1)\cos(i_2)\right]. \quad \{12\}$$

The control developed for use in this program is based on driving the wedge angle between the two planes to zero. The concepts behind this control are outlined in the following paragraphs.

The target orbital plane, in the case of an Earth to Moon trajectory, is the Moon's plane of motion about the Earth. This plane is defined as the new x-y plane for the geocentric coordinate system. The x-axis points to the Moon at the time of the spacecraft's escape from the Earth. The z-axis of this coordinate system is coincident with the angular momentum vector of the Earth-Moon system. The classical orbital elements of the spacecraft's orbit are then expressed with respect to this new frame of reference. Since the desired target plane, the Moon's orbital plane, is the x-y plane in the new coordinate system, the spacecraft's inclination in this new system is the actual angle between the planes or wedge angle. If a thrusting control is used to drive the new inclination, or wedge angle, i', to zero, then the Ω of the spacecraft will also change. However, when i' is zero, Ω becomes undefined. This is because for any orbit in the x-y plane the longitude of ascending node, Ω, is undefined. Thus, the only control necessary to bring the two planes together is the control forcing the inclination, or wedge angle, (i'), to zero (Figure 9). This control is identical to equation {10},

$$F_{ni'} = \frac{\Delta i'}{\Delta t}\,\frac{V}{\cos(\nu + \omega)}\,, \quad \{13\}$$

where $F_{ni'}$ is the normal force required to drive the wedge angle to zero, ν is the true anomaly of the orbit in the new reference plane,

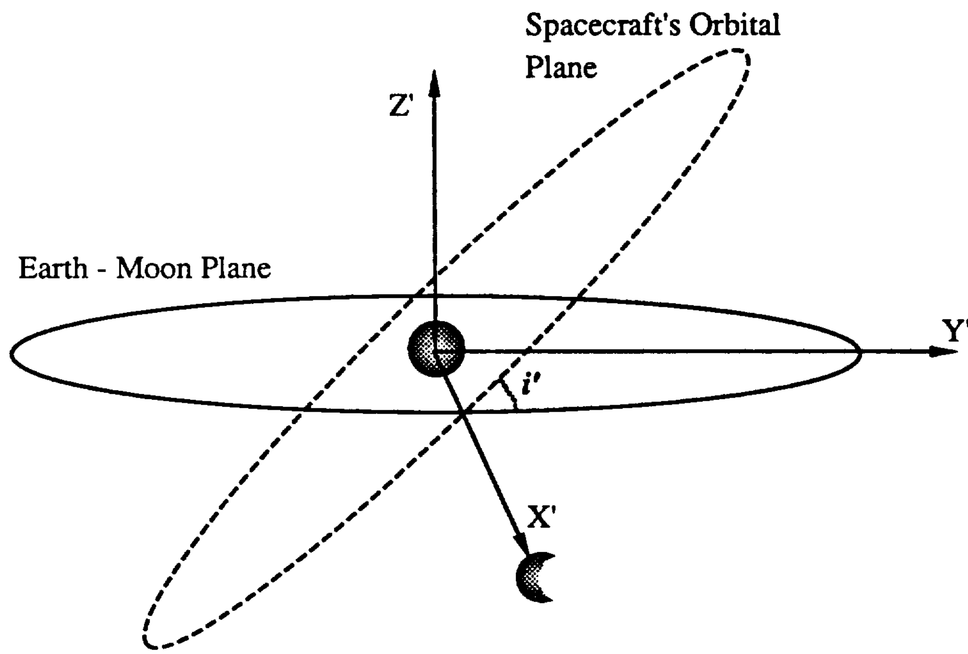


Figure 9 - New Coordinate System for the Simplified Control

and ω is the longitude of periapsis of the orbit in the new reference plane.

This control poses practical problems during the integration of the spacecraft's orbit. If the inclination of the spacecraft's orbit is zero, or close to zero, integration errors can occur. For this reason the integration is performed using equinoctial elements. Atmospheric drag is included in the perturbations to the spacecraft's orbit as is the oblateness effects of the Earth. These perturbations are added to that of the Moon for the entire spiral escape and orbital plane alignment.

When the spacecraft has achieved the required plane change the problem is essentially reduced to a planar problem. This is because the spacecraft's plane of motion is now the Earth-Moon plane and in the absence of out-of-plane perturbations the spacecraft will stay in the Earth-Moon plane. The planar problem of cislunar transfer has been previously address to some degree[8]. The previous work has been extended and modified to account for the three-dimensionality of the current approach and the modification of the coordinate system to non-dimensionalized coordinates. The control of the trajectory generation is passed from the equinoctial integration subroutine, EQUIN, to the R3BGEN (Restricted 3-Body GENeration) subroutine which integrates the trajectory in the restricted three-body problem.

3.2.2 Midcourse Targeting

The midcourse targeting portion of the trajectory generation is concerned with controlling the spacecraft in order to achieve Earth-Moon transfer. The value of the Jacobian constant of a spacecraft is used as an indicator of sufficient energy for the cislunar transfer.

Figure 10 shows a series of zero velocity curves at various values of the Jacobian constant. Szebehely notes that when a spacecraft in the restricted three-body system has a Jacobian constant of approximately 3.3, an equipotential curve like that shown at the $L_{2\ \mathrm{point}}$ in Figure 10 occurs[9]. When the spacecraft has an energy level that indicates an associated Jacobian constant of less than this value, the range of motion of the spacecraft is no longer restricted only to orbits about the Earth or Moon, but can include transfer orbits between the neighborhoods of the Earth and Moon.

The midcourse control is based on the desire for the spacecraft to have slightly under 3.3 for the value of its Jacobian constant when the spacecraft is in the vicinity of the opening in the zero velocity curve about the Earth and Moon. This opening occurs at the L_2 point shown in figure 10. The spacecraft needs to have the appropriate velocity vector in order to pass through the opening between the Earth and Moon. It is possible, even likely, that the spacecraft will have enough energy for the opening between the Earth and Moon to occur, but be in the incorrect position in its orbit to achieve cislunar transfer. A control is required that will enable the spacecraft to be travelling in the correct direction to achieve Earth-Moon transfer when the zero velocity curve opens.

The Jacobian constant of the spacecraft is calculated at each integration step as the spacecraft nears escape from the initial orbit. As the value of the spacecraft's Jacobian constant decreases, the spacecraft's energy in the three-body system (potential, kinetic, and angular) increases. Figure 11 shows the midcourse control concept. It is desired that the spacecraft have enough energy at position 1 so that when the spacecraft reaches position 2 the zero velocity curve will be open. Different spacecraft acceleration levels correspond to different rates of change of the spacecraft's Jacobian constant. This means the spacecraft must be very close to escape energy at position 1 in order to achieve the Jacobian constant of less than 3.3 when the spacecraft reaches position 2. The midcourse control is based on determining the spacecraft's energy level or Jacobian constant at position 1 in order to achieve position 2 using tangential thrust.

Before the midcourse targeting portion of the trajectory is run, the program presents the user with the option of using a parametric default value for the Jacobian constant at position 1 or entering a different number. The default values of the Jacobian constant are

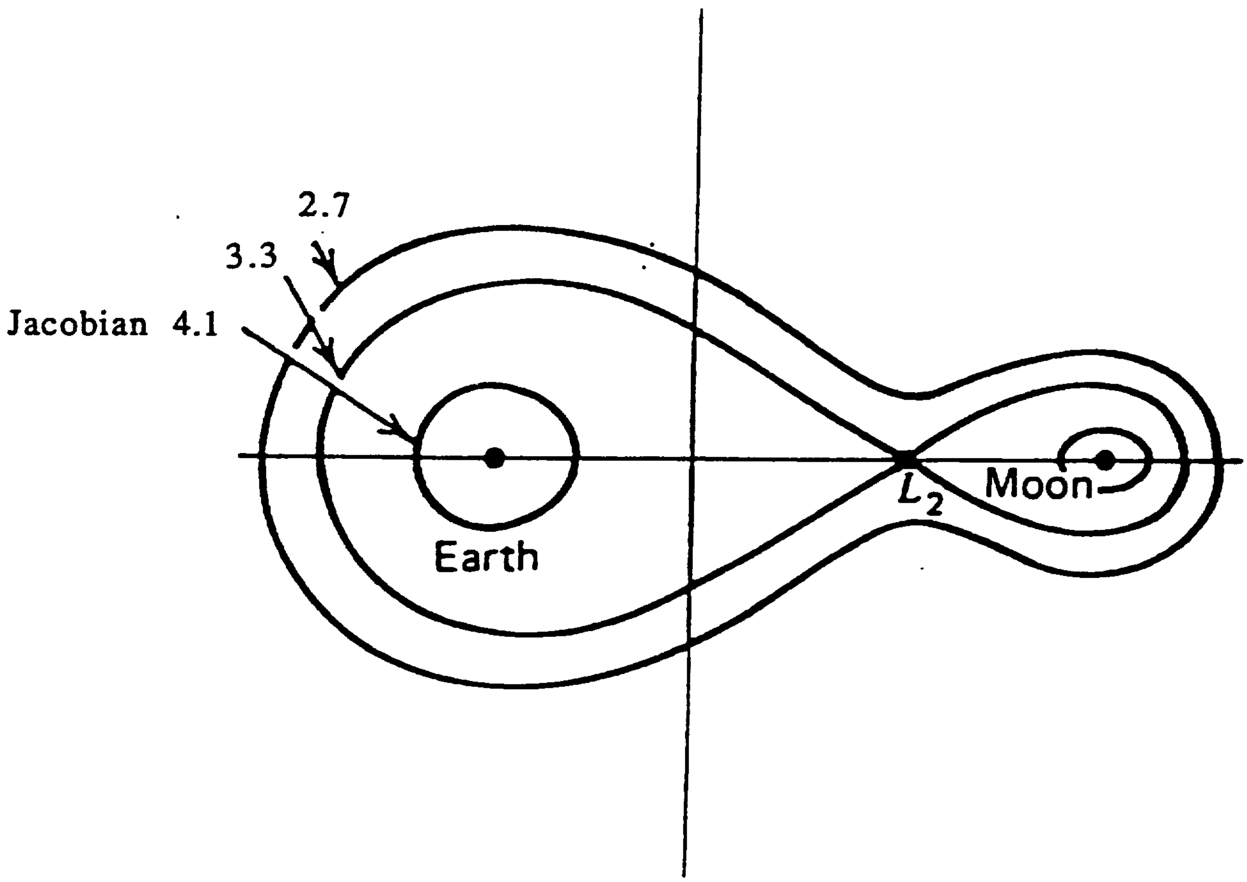


Figure 10 - Midcourse Targeting Zero Velocity Curves

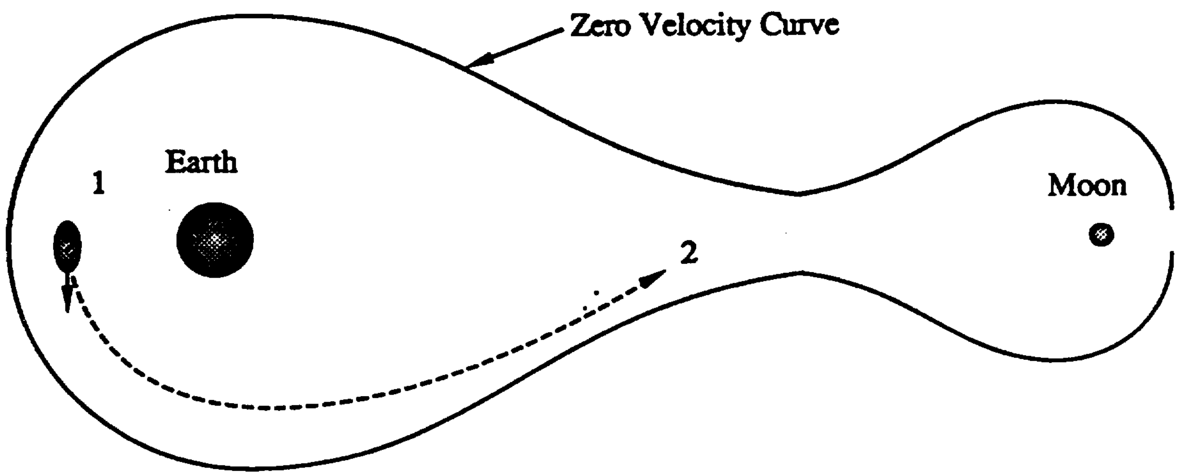


Figure 11 - Midcourse Guidance using the Velocity Curves

parametrically derived as a function of the spacecraft's acceleration. This phase of the trajectory generation is user iterative. If the spacecraft gains too much energy, it will escape the Earth-Moon system. The user is prompted for a response, after 87 days of integration time has passed, or if the integration step size falls below the nominal value. The program gives the user the options of continuing the trajectory generation or regenerating it with a new control after Earth escape when either of two conditions is met. On trajectories from the Moon to the Earth, the same methodology is used.

The necessary energy for the spacecraft to obtain would ideally be the value corresponding to the first zero velocity curve that permits travel between the Earth and Moon. In practice, however, this is not always the case. The progression of zero velocity curves is shown in figure 12. The zero velocity curves start about the Earth and Moon as near circular boundaries, C_1. As the energy increases the two curves meet at a point in space known as the first Lagrange point, curve C_2. At this position, L_2, if a spacecraft is placed there with zero velocity, with respect to the restricted three-body rotating coordinate system, and there are no perturbations, it will remain without need for station-keeping. As the energy level increases further the two curves join to become a single curve, C_3, surrounding both the Earth and Moon. The shape of this curve is similar to the outline of a figure eight. If the energy level increases slightly further, the curve, C_4, surrounding the Earth-Moon system forms an opening behind the Moon at L_1. This opening means that a spacecraft with the appropriate value of the Jacobian constant could leave the Earth-Moon system.

The shape of the zero velocity curves, and the corresponding values of the Jacobian constant require consideration when developing guidance algorithms in Earth-Moon space. The difference between the value of the Jacobian constant where the curves first join together and the value of the Jacobian constant when the curve permits escape from the Earth-Moon system is not very large. This

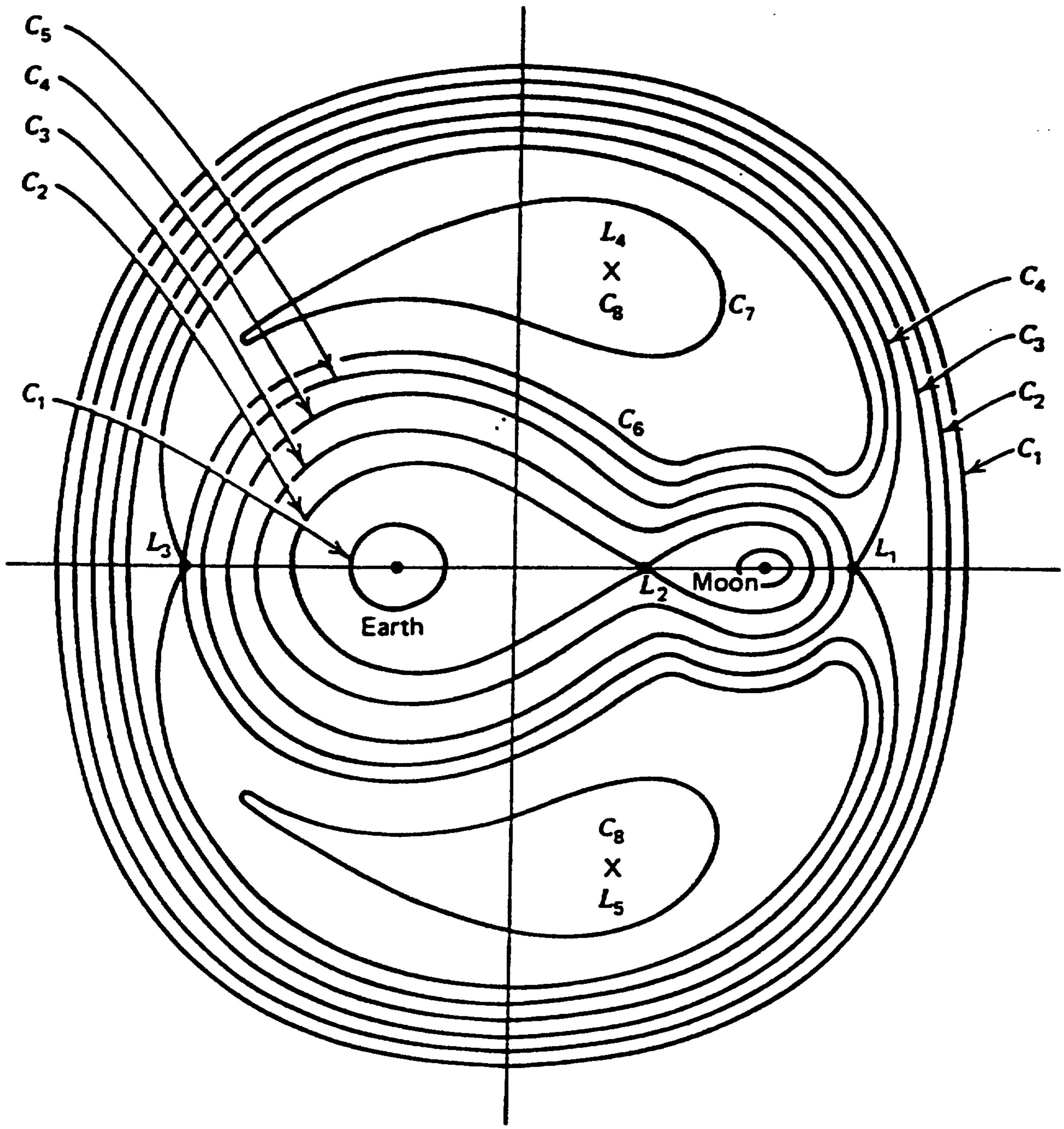


Figure 12 - Zero Velocity Curves in the Earth-Moon System

implies that for Earth to Moon trajectories it would be quite possible for the spacecraft to obtain a value of the Jacobian constant that permits escape from the Earth-Moon system and have the appropriate velocity vector to pass through the opening in the zero velocity curve behind the Moon. The sensitivity of the trajectory generation to the midcourse control value (Jacobian constant) during Earth to Moon trajectories is a direct result of this phenomena. For Earth to Moon trajectories the difference between the Jacobian constant for cislunar transfers and the Jacobian constant corresponding to an open zero velocity curve behind the Earth is relatively large. Therefore, the sensitivity of the Moon to Earth trajectory to the midcourse control value is relatively low.

3.3.3 Capture and Circularization

The subroutine CAPTURE controls the capture and circularization algorithms during the trajectory generation. The capture algorithm is given preference until the two-body orbital energy describing a Keplerian orbit about the target planet is negative. Then the circularization algorithm is used to lower the eccentricity of the orbit. After the eccentricity is less that 0.1, the capture algorithm takes over to continue lowering the orbit. If the eccentricity of the orbit exceeds the 0.1 limit of circularization control again takes control to lower the eccentricity. The two algorithms trade control as necessary until the desired final orbit is reached.

As the vehicle approaches the Moon the capture guidance phase of the trajectory is initiated. In the absence of impulsive thrust, the approach to and capture by the target body are critical and must not necessitate maneuvering beyond the limited capabilities of the propulsion system. The problem of low-thrust spacecraft guidance and trajectory determination between the Earth and the Moon was addressed in a study by Richard H. Battin and James S. Miller in the late 1950's and early 1960's. The concept for the spacecraft guidance during capture used in this study is derived from Battin and Miller's work[10].

The guidance scheme is relatively simple and straightforward. The operation of the capture phase guidance is illustrated in figure 13. The velocity of the spacecraft relative to the target body (i.e., the Earth or the Moon) is compared with a parametric velocity profile for a reference spiral capture. The parametric velocity profile is a function of the radial distance from the target body and the magnitude of the thrust acceleration. The difference between the spacecraft's velocity and the parametric velocity profile is used in combination with the nominal acceleration to determine the direction and magnitude of the spacecraft's thrust vector during capture.

In order to calculate the parametric velocity profile as a function of the radial distance from the capturing body the desired reference trajectory must be generated. The desired reference trajectory is a spiral trajectory that starts at escape conditions relative to the target and achieves circular orbit at the desired final altitude about the target body. The subroutine SPIRAL performs the generation of the reference trajectory and the calculation of the parametric velocity profiles. For a detailed discussion of the problem formulation used to generate the spiral trajectory see Appendix B.

To determine this reference spiral path and the velocity vectors that accompany it, the spacecraft starts in a circular orbit at the desired final altitude about the target body. The mass of the spacecraft at the final altitude is determined by estimating the final mass of the spacecraft at the completion of the mission. The spacecraft's trajectory is integrated using negative mass flow from the propulsion system to spiral out from the target planet using tangentially directed thrust. The mass of the spacecraft increases thereby decreasing the acceleration as the integration progresses. During the generation of the reference trajectory only the gravitational field of the target planet is considered. The spiral trajectory is otherwise without perturbations and consequently remains planar. The calculation of the trajectory continues until the

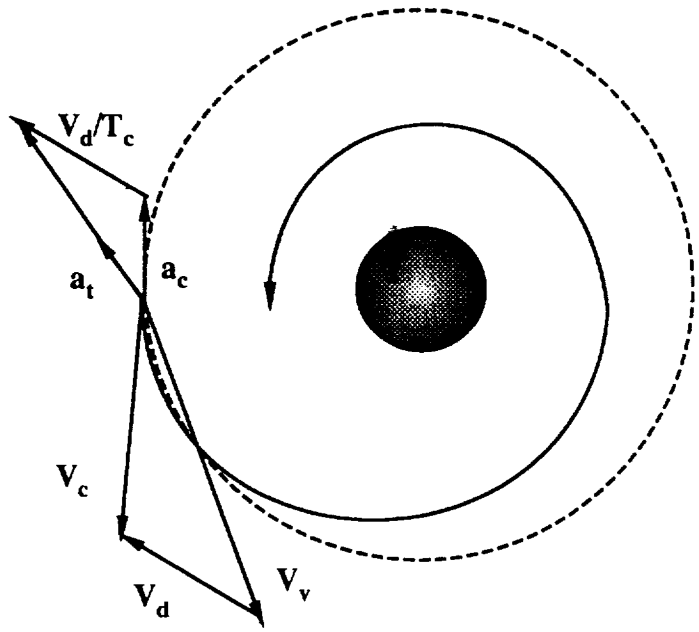


Figure 13 - Capture Phase Thrusting Guidance

vehicle is on a parabolic trajectory. The associated radial and tangential components of spacecraft's velocity are recorded at steps during the integration as functions of the radial distance from the central body. The velocity components are functions of the radial distance from the target body. They are obtained by fitting the recorded velocity components to polynomial and power curves. Figure 14 and 15 are example graphs of the velocity profiles for tangential and radial velocity at the radial distance.

A simplified derivation of the thrust guidance control algorithm is presented as follows. This control algorithm determines the necessary acceleration vector required for the spacecraft to match a desired reference trajectory. The actual velocity of the spacecraft, $\mathbf{V_v}$, at a given radial distance, $\mathbf{r}$, is compared with the parameterized reference capture velocity, $\mathbf{V_c}$, at $\mathbf{r}$. The difference between these velocity vectors is then determined as $\mathbf{V_d}$, where

$$\mathbf{V_d} = \mathbf{V_v} - \mathbf{V_c}, \qquad \{14\}$$

For a spacecraft flying on the reference trajectory, the incremental change in the velocity vector over time, $\Delta\mathbf{V_c}$, can be approximated as the sum of the spacecraft's acceleration vector due to its thrust and the gravitational attraction of the planet acting over some small time increment, Δt. This implies

$$\Delta\mathbf{V_c} = (\mathbf{a_c} + \mathbf{g})\,\Delta t \qquad \{15\}$$

where $\mathbf{a_c}$ is nominal acceleration vector of the spacecraft, and $\mathbf{g}$ is the gravitational acceleration vector of the capture planet.

The actual change in the velocity vector of the spacecraft, $\Delta\mathbf{V_v}$, can also be approximated similarly as

$$\Delta\mathbf{V_v} = (\mathbf{a_t} + \mathbf{g})\,\Delta t \qquad \{16\}$$

where $\mathbf{a_t}$ is the controllable thrust acceleration vector of the spacecraft.

It can be easily seen from equation {14} that the incremental change in the difference between the parametric and actual velocity vectors, $\Delta\mathbf{V_d}$, is simply

$$\Delta\mathbf{V_d} = \Delta\mathbf{V_v} - \Delta\mathbf{V_c}. \qquad \{17\}$$

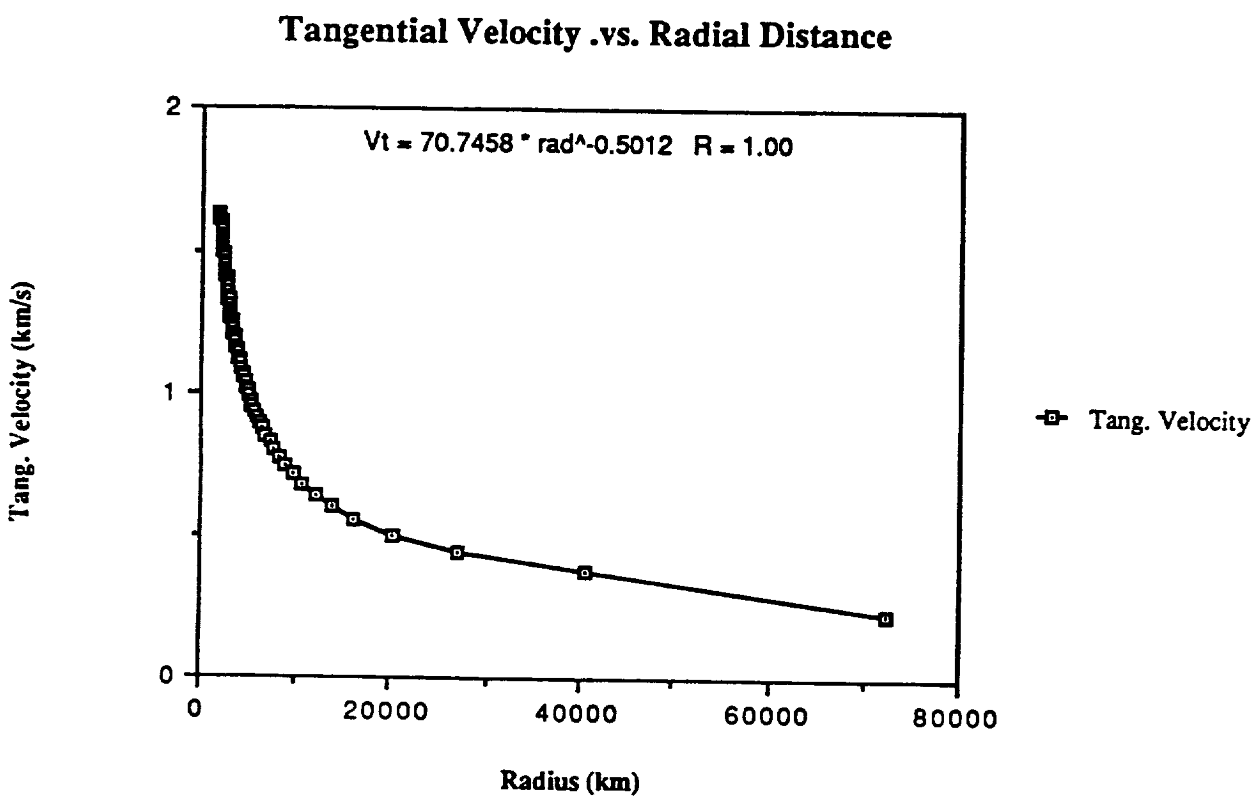


Figure 14 - Tangential Velocity as a function of Radial Distance

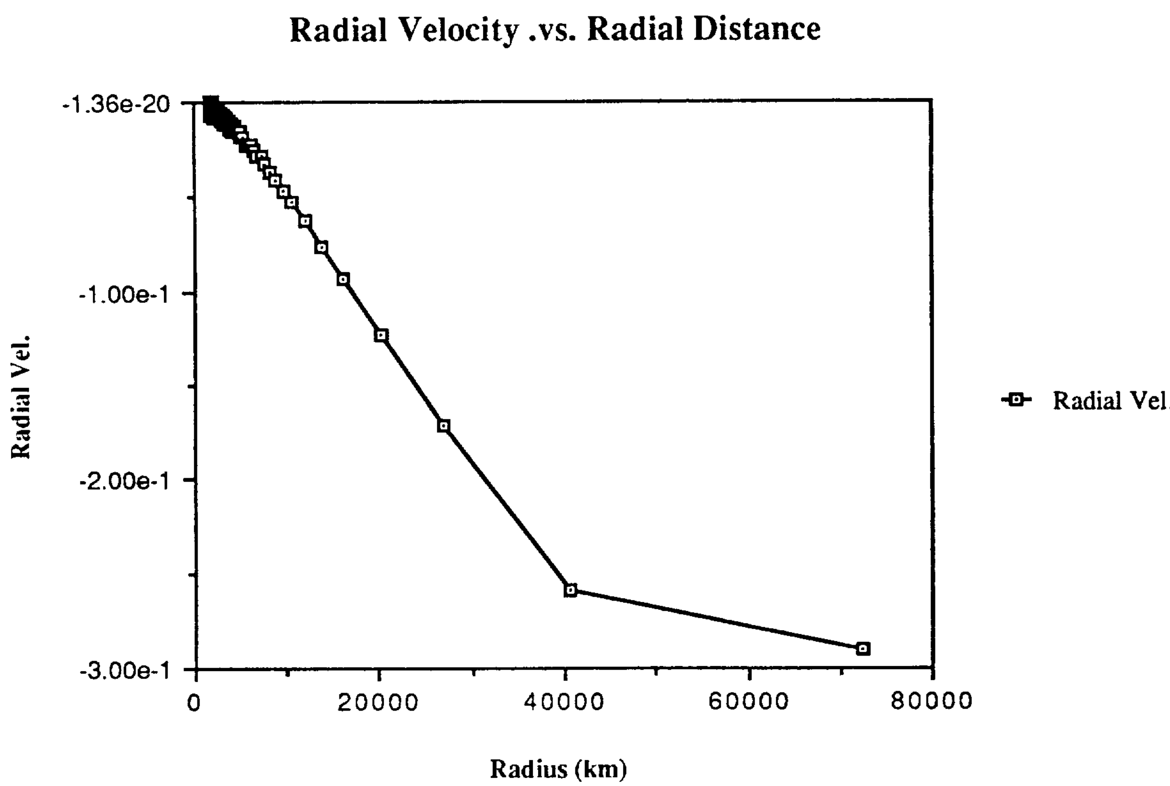


Figure 15 - Radial Velocity as a function of Radial Distance

Combining equations {15}, {16}, and {17} and rearranging to solve for $\mathbf{a}_t$, the controllable thrust acceleration vector, equation {18} is obtained,

$$\mathbf{a}_t = \mathbf{a}_c - \frac{\Delta \mathbf{V}_d}{\Delta t} \cdot \qquad \{18\}$$

The thrust acceleration is then chosen so that the rate of change of the velocity vector $\mathbf{V}_d$ is proportional to $\mathbf{V}_d$ itself. This results in

$$\frac{\Delta \mathbf{V}_d}{\Delta t} = - \frac{\mathbf{V}_d}{\mathrm{T}_c} \qquad \{19\}$$

where T_c is an empirically determined time constant. With this formula the appropriate thrust acceleration can be determined in both magnitude and direction simply with the knowledge of the vehicle's position, velocity, and nominal thrust acceleration $\mathbf{a}_c$.

In the application of the guidance it is reasonable to assume that the direction of the thrust acceleration can be varied at will, but the magnitude of the thrust is limited by the capabilities of the propulsion system. The spacecraft thrust is never required to deliver greater than the nominal thrust. The possibility of a reduction in the thrusting acceleration is not precluded as a desirable effect of the thrusting algorithm. Figure 16 is a graphical representation of the acceleration vectors $\mathbf{a}_t$ and $\mathbf{a}_c$. The radii of the circles are determined by the nominal acceleration of the spacecraft. Then equation {18} becomes,

$$\mathbf{a}_t \leq \mathbf{a}_c + \frac{\mathbf{V}_d}{\mathrm{T}_c} \cdot$$

When the appropriate magnitude of the thrust algorithm is greater than the nominal capabilities of the engine, a less than nominal thrust is used in the appropriate direction.

After the spacecraft has been captured by the target body, the capture algorithm continues to lower the spacecraft's energy level to bring it in closer to the desired orbit. One distinct difficulty of the capture algorithm is lowering the eccentricity of the capture orbit.

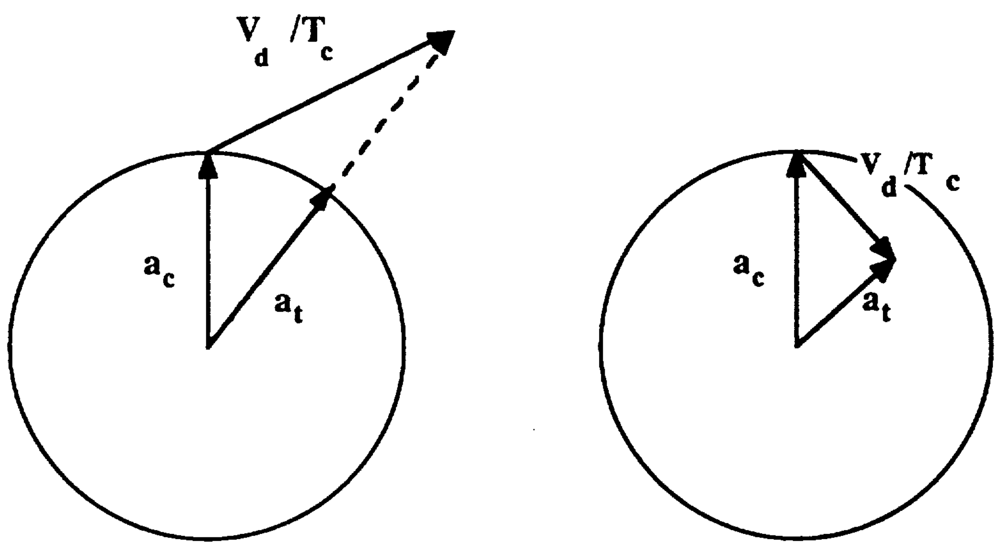


Figure 16 - Thrust Guidance Algorithm

For this reason a separate circularization algorithm was developed to control and lower the spacecraft's eccentricity during capture.

The concept behind the circularization algorithm is very straightforward. The spacecraft needs to be in a near circular orbit for the capture algorithm to be most effective. When a orbit is nearly circular the velocity at each portion of the orbit is approximately the same magnitude. For eccentric orbits the velocity of the spacecraft is highest at periapsis and lowest at apoapsis. The goal of the algorithm then is to drive the spacecraft's velocity to a uniform value along the entire orbit. At periapsis the spacecraft needs to increase its velocity, at apoapsis it needs to decrease its velocity. In order to achieve this, the spacecraft's thrusting vector is pointed opposite its velocity vector at apoapsis and along the velocity vector at periapsis. This thrusting behavior translates into the following controls for the radial $\mathbf{a}_{tr}$, and transverse $\mathbf{a}_{ts}$, acceleration components. Then

$$\mathbf{a}_{tr} = \mathbf{a_c} \sin(v)$$

and

$$\mathbf{a}_{ts} = \mathbf{a_c} \cos(v)$$

where v is the true anomaly of the orbit. Figure 17 shows what the acceleration vector would look like at various points along the orbit.

A potential restriction on nuclear-powered OTVs is the proposed nuclear safe orbit (NSO)[11]. This would be a designated altitude above the Earth, below which the nuclear powered spacecraft would be prohibited. The spacecraft would be unable to descend below the restricted altitude at any point of its trajectory. This limits the types of trajectories available for cislunar transfer.

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1 Advanced Space Analysis Office-Sverdrup/NASA-LERC, "Evaluation of Advanced Propulsion/Power Concepts," presented to Advanced Space Propulsion Workshop, April 12-13, 1988.

2 Riehl, J., Mason, L., Gilland, J., Sovey, J., and Bloomfield, H., "Power and Propulsion Parameters for Nuclear Electric Vehicles," NASA Lewis Research Center, Space Flight Systems Directorate, Version 1, Release 1, July, 1988.

3 Tsien, H. S., "Takeoff from Satellite Orbits," Journal of the American Rocket Society, pg 23, July - August, 1953.

[4] Keaton, Paul W., "Low-Thrust Rocket Trajectories," Los Alamos National Laboratory Report LA-10625-MS, January, 1986, page 4.

[5] Hill, P.G., and Peterson, C.R., Chapter 10.

[6] Moulton, Forest Ray, An Introduction to Celestial Mechanics, Dover Publications, Inc., New York, 1914, 2nd Revised Edition.

[7] Bahls, D.L., and Paris, S.W., "A Simplified Method for Obtaining Near Minimum Time Low Thrust Transfers", AAS 79-166.

[8] Korsmeyer, David J., "Trajectory Determination and Characterization of Cislunar Low-Thrust Spacecraft," AIAA/LPI 2nd Symposium on Lunar Bases and Space Activities of the 21st Century, Houston, Tx., April, 1988, LBS-88-080.

[9] Szebehely, Victor, Personal Communication, November, 1988.

[10] Battin, R. H. and Miller, J.S.

[11] Galecki, Diane L., and Patterson, Micheal J., "Nuclear Powered Mars Cargo Transport Mission Utilizing Advanced Ion Propulsion," AIAA/SAE/ASME/ASEE 23rd Joint Propulsion Conference, San Diego, Ca., 1987, AIAA-87-1903.

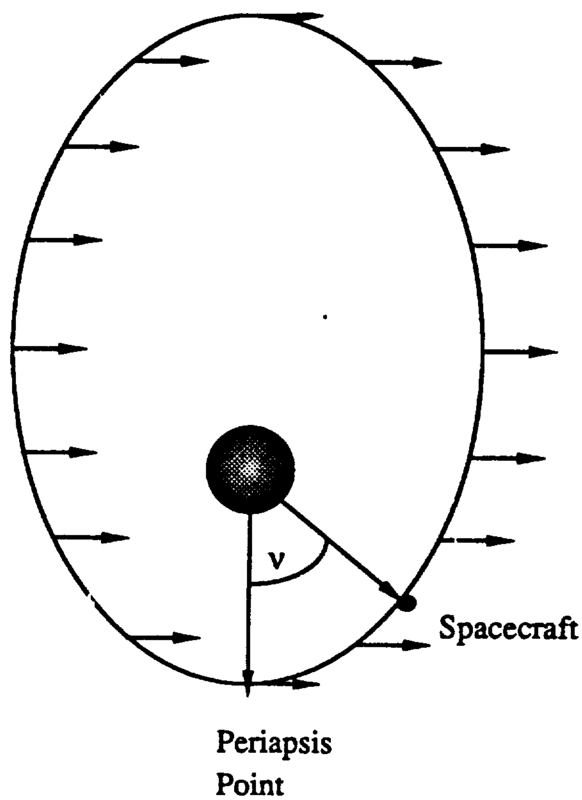

Figure 17 - Circularization Thrusting Control

Appendix A

Variation of Parameters

This is a useful method when a solution is sought to a homogeneous differential equation under a perturbation. The homogeneous equation under consideration is the equation of motion of a spacecraft about the Earth. The equation of motion is commonly written as,

$$\ddot{\vec{x}} = -\frac{\mu \vec{x}}{x^3}, \qquad \{A1\}$$

where μ is the gravitational parameter of the Earth, $\vec{x}$ is the position vector of the spacecraft, and $\ddot{\vec{x}}$ is the resultant acceleration of the spacecraft. This equation has a homogeneous solution that derives the six classical orbital elements $a_\alpha = (a, e, i, M, \omega, \Omega)$. An alternate, but equally valid solution derives the equinoctial elements $p_\alpha = (a, h, k, \lambda, p, q)$. This means the position and velocity of the spacecraft is a function of time only, so $\vec{x} = \vec{x}(t)$, and $\dot{\vec{x}} = \dot{\vec{x}}(t)$.

If there is a perturbing force, $\vec{F}$, to this orbit, the equation of motion can be written as

$$\ddot{\vec{x}} = -\frac{\mu \vec{x}}{x^3} + \vec{F}, \qquad \{A2\}$$

this is an associated inhomogeneous differential equation and the solution to equation {A2} can be found through variation of parameters.

The basic idea behind variation of parameters is to replace the constants of the homogeneous solution (the orbital elements) with time-varying functions. This implies the position is a function of both the orbital elements and time, $\vec{x} = \vec{x}(p_\alpha, t)$. Then to determine $\dot{\vec{x}}$, the chain rule for differentiating is used. First the partial of $\vec{x}$ with respect to p_α is multiplied by the partial of p_α with respect to time, and then the partial or $\vec{x}$ with respect to time is taken, so

$$\dot{\vec{x}} = \frac{\partial \vec{x}}{\partial p_\alpha} \frac{\partial p_\alpha}{\partial t} + \frac{\partial \vec{x}}{\partial t}.$$

However, this equation is constrained by setting

$$\frac{\partial \vec{x}}{\partial p_\alpha}\frac{\partial p_\alpha}{\partial t} = 0 \quad , \qquad \{A3\}$$

this forces the velocity to be the same for the perturbed and unperturbed case. In effect, this constraint says that the position of the orbit is not changed at the time the perturbation acts. To determine the acceleration, $\ddot{\vec{x}}$, the chain rule is again used,

$$\ddot{\vec{x}} = \frac{\partial \dot{\vec{x}}}{\partial p_\alpha}\frac{\partial p_\alpha}{\partial t} + \frac{\partial \dot{\vec{x}}}{\partial t}.$$

However, equation {A1} already gives a solution to $\ddot{\vec{x}}$. This implies,

$$\frac{\partial \dot{\vec{x}}}{\partial p_a}\frac{\partial p_\alpha}{\partial t} = \vec{F} \quad . \qquad \{A4\}$$

The six unknowns of equations {A3} and {A4}, p_α, can be found by inverting the 6 x 6 matrix of partials. If equations {A3} and {A4} are put in a matrix, the inverted 6 x 6 matrix of equations would yield.

$$\dot{p}_\alpha = \begin{bmatrix} \frac{\partial p}{\partial \vec{x}} & \frac{\partial p}{\partial \dot{\vec{x}}} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \vec{F} \end{bmatrix}$$

If the matrix equation is multiplied through, it is found,

$$\dot{p}_\alpha = \frac{\partial p_\alpha}{\partial \dot{\vec{x}}} \cdot \vec{F}, \qquad \{A5\}$$

where the right hand side of {A5} is the dot product of the partials with respect to velocity together with the rectangular components of the perturbing acceleration $\vec{F}$. To solve for $\dot{p}_\alpha$ it is first necessary to know the explicit form of the partial derivatives of the orbital elements with respect to the velocity components of the reference orbit. These partial derivatives are obtained by using the Poisson Brackets[1]:

$$\frac{\partial p_\alpha}{\partial \dot{\vec{x}}} = -\sum_{\beta=1}^{6} (p_\alpha, p_\beta)\frac{\partial \vec{x}}{\partial p_\beta}. \qquad \{A6\}$$

To use the Poisson Brackets to find the partials of the equinoctial elements with respect to the velocity, the partial derivatives of position and velocity vectors with respect to the equinoctial elements must first be found. The initial step to this is to determine the position and velocity of the spacecraft from the equinoctial elements of the orbit. The quantities (X_1, Y_1, 0) are the coordinates of the spacecraft relative to the equinoctial frame. These coordinates must be found and then transformed into the traditional (x, y, z) of the inertial coordinate system.

It is necessary to recall the coordinate system for equinoctial elements. The equinoctial coordinate frame is defined by unit vectors $\hat{f}$, $\hat{g}$, and $\hat{w}$ illustrated in Figure A1 and defined by

$$\hat{f} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 1 - p^2 + q^2 \\ 2pq \\ -2p \end{bmatrix},$$

$$\hat{g} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 2pq \\ 1 + p^2 - q^2 \\ 2q \end{bmatrix},$$

$$\hat{w} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 2p \\ -2q \\ 1 - p^2 - q^2 \end{bmatrix}.$$

In order to have a variation of parameters program which is valid for all eccentricities and inclinations it is necessary to use a formulation which is free of singularities. The key to this is to have the angles describing the orbit defined for all eccentricities and inclinations. The equinoctial formulation uses angles called the mean longitude (λ), and the eccentric longitude (F), rather than the more classical angles of mean anomaly (M), eccentric anomaly (E), and true anomaly (ν):

$$\lambda = M + \omega + \Omega, \qquad \{A7\}$$

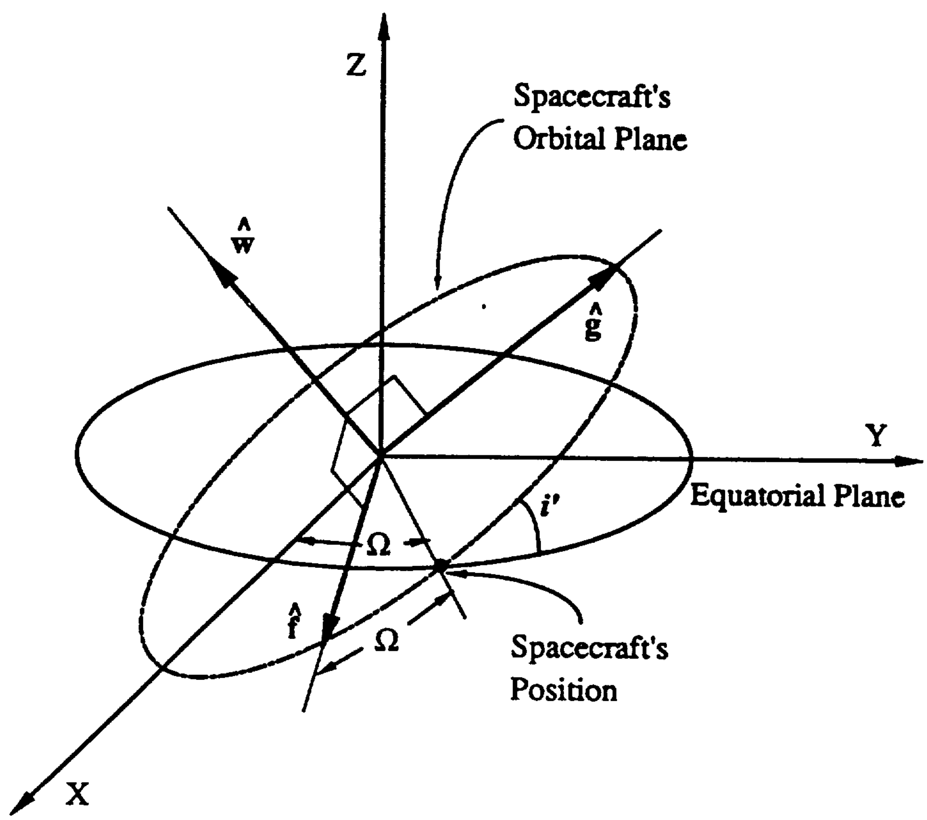

Figure A1 - Equinoctial Coordinate Frame

$$F = E + \omega + \Omega, \qquad \{A8\}$$

The position in the orbit can be indicated by F. However, in order to compute the position vector of the spacecraft from the equinoctial elements, it is necessary to solve Kepler's equation. It is advantageous to write Kepler's equation in terms of the eccentric longitude rather than the eccentric anomaly because the eccentric longitude is defined for all eccentricities and inclinations. Some elementary manipulations show the Kepler's equation and the expression for the radial distance, r, from the Earth can be written in terms of the eccentric longitude,

$$\lambda = F + h \cos F = k \sin F, \qquad \{A9\}$$

$$r = a\,[1 - h \sin F - k \cos F]. \qquad \{A10\}$$

Kepler's equation {A9} can be solved for F with the standard Newton-Raphson procedure (or any other iteration method) once the value of λ has been determined from {A7}. Once Kepler's equation has been solved, the three coordinates (x, y, z) of the inertial system are obtained with the following matrix equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 1 - p^2 + q^2 & 2pq & 2p \\ 2pq & 1 + p^2 - q^2 & -2q \\ -2p & 2q & 1 - p^2 - q^2 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ 0 \end{bmatrix}.$$

The quantities $(X_1, Y_1, 0)$ are the coordinates of the spacecraft relative to the equinoctial frame. They can be expressed in terms of F by:

$$X_1 = a\left[\cos F - k + \frac{h(\lambda - F)}{1 + \sqrt{1 + e^2}}\right],$$

$$Y_1 = a\left[\sin F - h + \frac{k(\lambda - F)}{1 + \sqrt{1 + e^2}}\right]. \qquad \{A11\}$$

Also of note are the following time derivations:

$$\dot{F} = \frac{na}{r},$$

$$\dot{X}_1 = \frac{-na^2}{r}\left[\sin F - \frac{h\,(k \cos F + h \sin F)}{(1 + \sqrt{1 - e^2}\,)}\right],$$

$$\dot{Y}_1 = \frac{na^2}{r}\left[\cos F - \frac{k\,(k \cos F + h \sin F)}{(1 + \sqrt{1 - e^2}\,)}\right].$$

The above expressions are valid for all eccentricities and inclinations. In addition, all the expressions are functions of the equinoctial rather than the classical elements. Due to the lack of singularities for circular or zero inclination orbits the equinoctial formulation is more appropriate for the integration of low-thrust trajectories.

The partial derivatives

In the variation of parameters derivation it is necessary to have several partial derivatives of the two-body equations. First, all of the partials of X_1 and Y_1 with respect to h and k are needed. The following expressions were taken from work by Broucke[2]:

$$\frac{\partial X_1}{\partial h} = a\left[-(\lambda - F)\,(\beta + \frac{h\beta^3}{1 - \beta}) - \frac{a}{r}\cos F\,(h\beta - \sin F)\right],$$

$$\frac{\partial X_1}{\partial k} = -a\left[(\lambda - F)\,\frac{hk\beta^3}{1 - \beta} + 1 + \frac{a}{r}\sin F\,(\sin F - h\beta)\right],$$

$$\frac{\partial Y_1}{\partial h} = a\left[(\lambda - F)\,\frac{kh\beta^3}{1 - \beta} - 1 + \frac{a}{r}\cos F\,(k\beta - \cos F)\right], \text{and}$$

$$\frac{\partial Y_1}{\partial k} = a\left[(\lambda - F)\left(\beta + \frac{k^2\beta^3}{1 - \beta}\right) + \frac{a}{r}\sin F\,(\cos F - k\beta)\right].$$

The quantity β which has been introduced here is defined as follows:

$$\beta = \frac{1}{1 + \sqrt{1 - h^2 - k^2}}\,.$$

With the use of the above expressions the following partial derivatives of the position vector $\vec{x}$ are easily derived:

$$\frac{\partial \vec{x}}{\partial a} = \frac{1}{a}\left[\vec{x} - \frac{3}{2}\dot{\vec{x}}\,(t - t_o)\right],$$

$$\frac{\partial \vec{x}}{\partial h} = \frac{\partial X_1}{\partial h}\,\vec{f} + \frac{\partial X_1}{\partial h}\,\vec{g},$$

$$\frac{\partial \vec{x}}{\partial k} = \frac{\partial X_1}{\partial k}\,\vec{f} + \frac{\partial Y_1}{\partial k}\,\vec{g},$$

$$\frac{\partial x}{\partial \lambda_0} = \frac{\dot{X}_1\,\vec{f} + \dot{Y}_1\,\vec{g}}{n},$$

$$\frac{\partial \vec{x}}{\partial p} = \frac{2}{1 + p^2 + q^2}\left[q\,(Y_1\,\vec{f} - X_1\,\vec{g}) - X_1\,\vec{w}\right], \text{and}$$

$$\frac{\partial \vec{x}}{\partial q} = \frac{2}{1 + p^2 + q^2}\left[p\,(X_1\,\vec{g} - Y_1\,\vec{f}) - Y_1\,\vec{w}\right].$$

Finally the partials of the equinoctial elements with respect to the velocity are obtained by using the Poisson Brackets shown in equation {A5}. The results are then:

$$\frac{\partial a}{\partial \dot{\vec{x}}} = \frac{2\dot{\vec{x}}}{n^2 a},$$

$$\frac{\partial q}{\partial \dot{\vec{x}}} = \frac{(1 + p^2 + q^2)\,X_1}{2na^2\sqrt{1 - h^2 - k^2}}\,\vec{w},$$

$$\frac{\partial p}{\partial \dot{\vec{x}}} = \frac{(1 + p^2 + q^2)\,Y_1}{2na^2\sqrt{1 - h^2 - k^2}}\,\vec{w},$$

$$\frac{\partial h}{\partial \dot{\vec{x}}} = \frac{k(qY_1 - pX_1)}{na^2\sqrt{1 - h^2 - k^2}}\,\vec{w} + \frac{\sqrt{1 - h^2 - k^2}}{na^2}\left[\left(\frac{\partial X_1}{\partial k} - h\beta\frac{\dot{X}_1}{n}\right)\vec{f} + \left(\frac{\partial Y_1}{\partial k} - h\beta\frac{\dot{Y}_1}{n}\right)\vec{g}\right],$$

$$\frac{\partial k}{\partial \dot{\vec{x}}} = \frac{-h(qY_1 - pX_1)}{na^2\sqrt{1 - h^2 - k^2}}\,\vec{w} - \frac{\sqrt{1 - h^2 - k^2}}{na^2}\left[\left(\frac{\partial X_1}{\partial h} + k\beta\frac{\dot{X}_1}{n}\right)\vec{f} + \left(\frac{\partial Y_1}{\partial h} + k\beta\frac{\dot{Y}_1}{n}\right)\vec{g}\right],$$

$$\frac{\partial \lambda_o}{\partial \dot{\vec{x}}} = \frac{-2}{na^2}\left[\vec{x}\,\frac{3}{2}\dot{\vec{x}}\,(t - t_o)\right] + \frac{\sqrt{1 - h^2 - k^2}}{na^2}\beta\left[\left(h\frac{\partial X_1}{\partial h} + k\frac{\partial X_1}{\partial k}\right)\vec{f} + \left(h\frac{\partial Y_1}{\partial h} + k\frac{\partial Y_1}{\partial k}\right)\vec{g}\right],$$

$$+ \frac{1}{na^2\sqrt{1 - h^2 - k^2}}[qY_1 - pX_1]\,\vec{w} \quad .$$

These partials are calculated at each integration time step and used with equation {A5} to find the time rate of change of the equinoctial elements due to the perturbing force $\vec{F}$. This permits the integration of the equinoctial orbital elements under perturbation.

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[1] Broucke, R., and Cefola, P. "The Equinoctial Orbit Elements," JPL Technical Memorandum 391-238, September 28, 1971.

[2] Broucke, R., and Cefola, P. "Numerical Integration of Satellite Orbits with Equinoctial Elements," JPL Technical Memorandum 391-248, November 5, 1971.

Appendix B

<u>Generating Two-Dimensional Spiral Trajectories for a Low-Thrust Spacecraft</u>

The equations of motion of a powered spacecraft in a two-dimensional orbit can be derived the spacecraft's kinetic energy, potential energy, and from the force equation, F = M*a (total force = mass * acceleration).

If the magnitude of the acceleration of the spacecraft due to the propulsion system is a, then

$$T = M*a, \qquad \{B1\}$$

where M is the mass of the spacecraft. The spacecraft's acceleration , a, is then determined from equations {B1} to be

$$a = \frac{T}{M}.$$

The acceleration of the spacecraft due to the thrust of the propulsion system changes as the total spacecraft mass, M, changes. As mass is expelled from the spacecraft the total spacecraft mass decreases. So,

$$M = M_0 - \dot{m}*t ,$$

where t is the time period during which the spacecraft is losing mass at $\dot{m}$, the mass flow rate of the spacecraft's propulsion system.

To determine the equations of motion of the spacecraft in polar coordinates the radial and transverse components of the acceleration due to the thrust need to be calculated from the thrusting force. Figure B1 shows an arbitrary thrust vector with respect to the radial and transverse vector. Assuming that the thrust vector makes an angle ϕ with the local horizon, (measured from the direction of the motion), the radial, R, and transverse, S, components of the acceleration are:

$$a_R = \frac{T}{M}\sin\phi, \; a_S = r\frac{T}{M}\cos\phi \; . \qquad \{B2\}$$

In an unperturbed orbit the only external force present is the Earth's gravity force which in polar coordinates has the radial

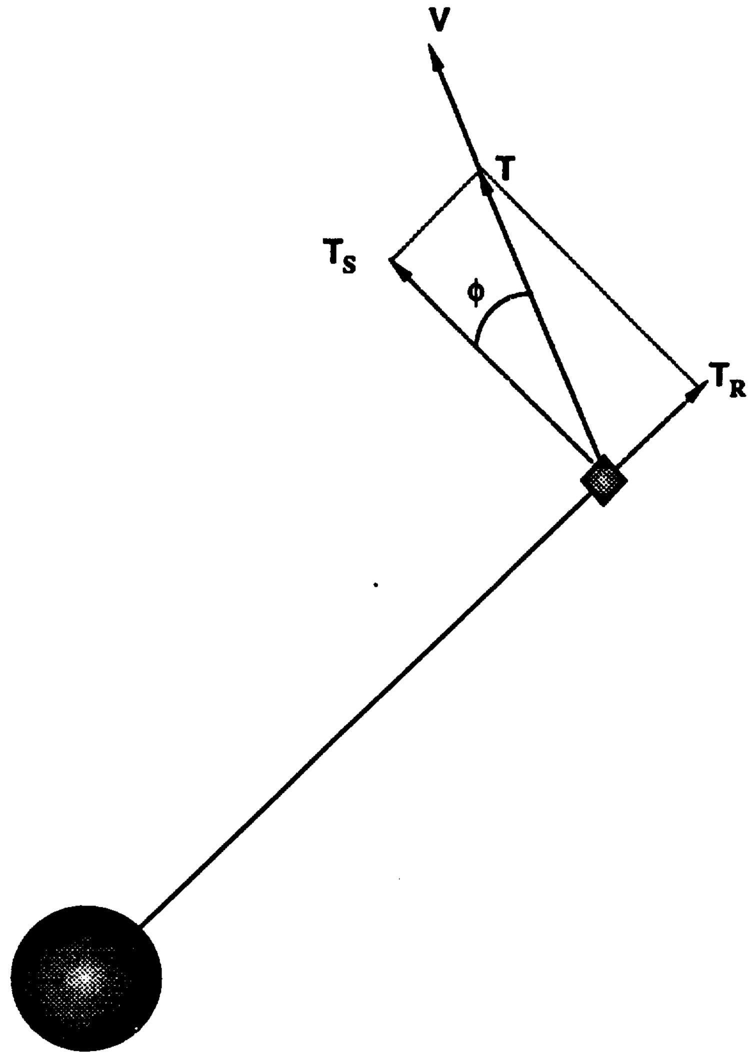

Figure B1 - Tangential Thrust Components component. The equations of motion in polar coordinates can be derived quickly from the Lagrangian which is the sum of the kinetic and potential energy,

$$L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GM}{r}.$$

Using the Euler-Lagrange equations, the equations of motion are obtained as,

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2},$$

and

$$\frac{d(r^2\dot{\theta})}{dt} = 0. \qquad \{B3\}$$

If the acceleration components due to the spacecraft's thrust from equation {B2} are added to the equations of motion {B3}, then

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} + \frac{T}{M}\sin\phi,$$

$$\{B4\}$$

$$\frac{d(r^2\dot{\theta})}{dt} = r\frac{T}{M}\cos\phi$$

The above system {B4} can be replaced by a new system of four first-order equations. This is done by introducing two new variables, the radial velocity component $u = \dot{r}$, and the transverse velocity component, $\upsilon = r\dot{\theta}$. Through the change of variables, {B4} can be rewritten as,

$$\dot{u} - \upsilon\dot{\theta} = -\frac{GM}{r^2} + \frac{T}{M}\sin\phi,$$

$$\{B5\}$$

$$\frac{d(r\upsilon)}{dt} = r\frac{T}{M}\cos\phi.$$

The two second order differential equations, {B5}, can be broken into four first-order coupled equations:

$$\dot{r} = u,$$

$$\dot{\theta} = \frac{\upsilon}{r},$$

$$\dot{u} = \frac{\upsilon^2}{r} - \frac{GM}{r^2} + \frac{T}{M}\sin\phi ,$$

$$\dot{\upsilon} = \frac{1}{r}(-u\upsilon + r\frac{T}{M}\cos\phi) .$$

For the numerical integration of these four equations, the state vector is defined to be (r, θ, u, υ), (in this order). These equations were used in the subroutine SPIRAL to perform generation of the velocity component profiles for the reference trajectory for spiral capture. This formulation was especially useful because the tangential and radial components of the velocity were required to be save to an array as a function of the radial distance from the planet. Using this formulation eliminates the need for any cumbersome calculation of the velocity components and radial distance as the integration progressed. The necessary information was inherent in the formulation of the problem.

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16. Abstract

The documentation of the Trajectory Generation and System Characterization Model for the Cislunar Low-Thrust Spacecraft is presented in Technical and User's Manuals. The system characteristics and trajectories of low-thrust nuclear electric propulsion spacecraft can be generated through the use of multiple system technology models coupled with a high-fidelity trajectory generation routine. The Earth to Moon trajectories utilize near Earth orbital plane alignment, midcourse control dependent upon the spacecraft's Jacobian constant, and capture to target orbit utilizing velocity matching algorithms. The trajectory generation is performed in a perturbed two-body equinoctial formulation and the restricted three-body formulation. A single control is determined by the user for the interactive midcourse portion of the trajectory. The full spacecraft system characteristics and trajectory are provided as output.

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