NASA Technical Memorandum 103141

An Unsteady Time Asymptotic Flow in the Square Driven Cavity

John W. Goodrich
Lewis Research Center
Cleveland, Ohio

(NASA-TM-103141) AN UNSTEADY TIME ASYMPTOTIC FLOW IN THE SQUARE DRIVEN CAVITY (NASA) 12 p CSCL 20D
N90-22016
Unclas
G3/34 0280229

Prepared for the
IMACS 1st International Conference on Computational Physics
cosponsored by the International Association of Mathematics and Computers in Simulation and the University of Colorado at Boulder
Boulder, Colorado, June 11-15, 1990

NASA

An Unsteady Time Asymptotic Flow in the Square Driven Cavity

John W. Goodrich
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

Summary

This paper presents summary details of an aperiodic time asymptotic numerical solution for the square drive cavity at $Re = 10000$. The data presented is for $6100 < t \leq 7100$, and is representative of the data that characterizes the aperiodic asymptotic state. Complete details are not given in this paper for either the transient evolution or the asymptotic state.

Introduction

A persisting final oscillation in the aspect ratio two driven cavity was shown in [5] at $Re = 10000$ and $t \approx 300$. A periodic time asymptotic flow in the aspect ratio two driven cavity was presented in [3] at $Re = 5000$ and $t \approx 3800$. This paper presents summary details of an aperiodic time asymptotic numerical solution for the square drive cavity at $Re = 10000$ and $t \approx 6000$. Further details are in [2]. Unsteady time asymptotic calculations for the square regularized driven cavity have been presented in [7].

Numerical Method

The velocity field for two dimensional time dependent incompressible fluid flow in a spatial domain Ω may be written in terms of the streamfunction $\psi(\mathbf{x},t)$ as $u(\mathbf{x},t) = \frac{\partial\psi}{\partial y}$ and $v(\mathbf{x},t) = -\frac{\partial\psi}{\partial x}$. The flow equations may be written as

$$\frac{\partial \triangle \psi}{\partial t} = \frac{1}{Re}\triangle^2\psi + \frac{\partial\psi}{\partial x}\triangle\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial y}\triangle\frac{\partial\psi}{\partial x}, \quad \text{for } \mathbf{x} \text{ in } \Omega, \text{ and } t > 0. \tag{1}$$

The data for the impulsively started driven cavity consists of the initial values $\psi(\mathbf{x},0) = 0$ for $\mathbf{x}$ in Ω and $t = 0$, and the boundary conditions $\psi(\mathbf{x},t) = 0$ for $\mathbf{x}$ in $\partial\Omega$ and $t > 0$, and $\frac{\partial\psi}{\partial\eta}(\mathbf{x},t) = 1$ for $\mathbf{x}$ on the cavity lid and $t > 0$, and $\frac{\partial\psi}{\partial\eta}(\mathbf{x},t) = 0$ for $\mathbf{x}$ on the cavity walls and $t > 0$, where $\frac{\partial}{\partial\eta}$ is differentiation in the exterior normal direction at the boundary, and where the lid is moving from left to right for $t > 0$.

Let a discrete approximation for the streamfunction on a square uniform grid be $\tilde{\mathbf{z}} = \{\tilde{\mathbf{z}}^m : m = 0,1,\ldots\}$. If La is the conventional five point centered difference approximation to the Laplacian, if Bi is the conventional thirteen point centered difference approximation to the Biharmonic operator, and if δ_x and δ_y are the conventional centered difference operators, then a discretization of equation (1) is

$$\begin{aligned}
&\mathrm{La}(\tilde{\mathbf{z}}^{n+1}) - \frac{\Delta t}{2Re}\mathrm{Bi}(\tilde{\mathbf{z}}^{n+1}) \\
&\quad = \mathrm{La}(\tilde{\mathbf{z}}^{n}) + \frac{\Delta t}{2Re}\mathrm{Bi}(\tilde{\mathbf{z}}^{n}) - \frac{3\Delta t}{2}\left[\delta_x\left(\delta_y(\tilde{\mathbf{z}}^{n})\mathrm{La}(\tilde{\mathbf{z}}^{n})\right) - \delta_y\left(\delta_x(\tilde{\mathbf{z}}^{n})\mathrm{La}(\tilde{\mathbf{z}}^{n})\right)\right] \\
&\qquad + \frac{\Delta t}{2}\left[\delta_x\left(\delta_y(\tilde{\mathbf{z}}^{n-1})\mathrm{La}(\tilde{\mathbf{z}}^{n-1})\right) - \delta_y\left(\delta_x(\tilde{\mathbf{z}}^{n-1})\mathrm{La}(\tilde{\mathbf{z}}^{n-1})\right)\right].
\end{aligned} \tag{2}$$

This algorithm is second order accurate in both time and space. Standard finite difference approximations are used to incorporate the boundary condition. The velocity components are directly recovered from the discrete streamfunction solution using central difference approximations for the $\frac{\partial\psi}{\partial y}$ and $\frac{\partial\psi}{\partial x}$. The velocity solution is exactly discretely divergence free with respect to a central difference formulation of the mass conservation equation. Further details are given in [2] and [4].

The solution at time t_{n+1} of the implicit equation (2) is obtained by a multigrid method, as in [2]. The Biharmonic operator is factored as two Laplacians, point Gauss-Seidel relaxation is used for the smoothing operator, and linear restriction and prolongation operators are used. A V-cycle iteration scheme is used, with 3 iterations per grid level while coarsening, and none while refining. At each time step, 10 to 15 iteration cycles are used to reduce residuals to less than 5.0×10^{-12}.

Numerical Results

Computations are reported for the impulsively started square driven cavity at $Re = 10000$ on a uniform 128×128 grid with $\Delta t = \frac{1}{160}$. These computations show that the time asymptotic state for this combination of algorithm, data, and grid is an aperiodic laminar flow. Among the indicators that have been used [3] for tracking convergence to an asymptotic state are the relative L_1 norm of the streamfunction change per time step $\sum_{i,j} |\psi_{i,j}^{n+1} - \psi_{i,j}^{n}| / \sum_{i,j} |\psi_{i,j}^{n+1}|$, the total kinetic energy $\frac{1}{2}\Delta x \Delta y \sum_{i,j} \|\mathbf{u}_{i,j}^{n+1}\|_2^{\,2}$, and the minimum streamfunction value, each of which is obtained from the data on the interior grid.

The relative L_1 norm for the streamfunction change per time step is $O(10^{-5})$ for $t \geq 100$, so that virtually the entire time evolution of the system and its time asymptotic state are in a range of very small change per time step with respect to the streamfunction solution surface as a whole. The initial transients completely dominate the time evolution of this system until at least $t \approx 500$. The relative L_1 norm for the streamfunction change per time step seems to approach its asymptotic pattern by $t \approx 1600$, with persisting variation in the range $[0.00006, 0.00009]$. The total kinetic energy of the flow on the interior grid in the cavity is greater than 0.036 by $t \approx 600$, but there is clearly evident transient change until at least $t \approx 1900$ when the asymptotic evolutionry pattern is approached. Persisting variation for the total kinetic energy is in the range $[0.03624, 0.03630]$. The minimum streamfunction value approaches its asymptotic time evolution pattern and range by $t \approx 1900$, with persisting variation in the range $[-0.10823, -0.10804]$. The global streamfunction minimum is also point data, since the global minimum for the asymptotic state is at a fixed point in the very center of the primary circulation. The combination these three indicators suggests that the asymptotic attractor for this flow has been approached by $t \approx 2000$.

There were small transient changes in phase portraits for this data, so the computation was run until $t = 7100$ in order to ensure that the asymptotic state had indeed been reached. There is a period of $T \approx 200$ for the asymptotic flow. The resolution of the asymptotic attractor with phase portraits and spectral analysis required at least 1000 nondimensional time units, or data from 160000 time steps. Figure 1 shows the time series data for the relative L_1 norm of the streamfunction change per time step, and Figure 2 shows the streamfunction minimum, both for $6100 < t \leq 7100$. This data shows approximately five complete periods of the asymptotic state. Notice the complicated beating of multiple frequencies. Figure 3 shows the power spectrum for the total kinetic energy, for normalized frequencies less than 5. This spectrum is representative of the complicated spectra for the data from this flow, with multiple discrete spectral lines, with a primary frequency of $c_p \approx 0.237$, with various harmonics, and with a minimal significant frequency of $c_m \approx 0.005$. For the relative L_1 norm of the streamfunction change per time step the ratio of spectral weights for c_p and c_m is 61.3, for the streamfunction minimum it is 132.0, and for the total kinetic energy it is 5.66. It seems likely that there are at least two incomensurate frequencies with c_m approximately equal to their difference. The power spectra for all three of the scalar time series also reflect a complicated substructure in their spectral lines, with the peaks in the power spectrum away from c_p tending to spread into a cluster of subpeaks, with frequency seperations that are approxunately 0.005. Figure 4 shows a phase portrait of the relative L_1 norm of the streamfunction change per time step versus the total kinetic energy, for $6100 < t \leq 7100$. This time interval was required to define the entire phase portrait. This phase portrait clearly shows a complex but regular dynamical process. Figure 5 shows three representative streamfunction contour plots for the asymptotic flow. The unsteady dynamics are laminar, and are visibly concentrated in the relatively weak secondary flow structures. In each of the secondary flow structures weak tertiary circulations form underneath the separated primary flow near the point of separation, and then are convected into the secondary flow as a whole, to interact with and join the focus of the secondary flow.

Discussion

There are well known steady solutions for the square driven cavity at $Re =$ 10000 in [1] and [6]. These steady results do not contradict the current work since they were obtained with steady state codes. Theoretical expectations [8], the unsteady asymptotic results in [7] for the square regularized driven cavity using a Chebyshev-Tau approximation, and the results in [3] for the rectangular driven cavity using the same method as in this work all support these unsteady asymptotic results in the square driven cavity at $Re = 10000$. Continuing work in preparation [2] suggests that the flow in a square driven cavity has a Hopf bifurcation to a periodic asymptotic state for $Re < 9000$.

Bibliography

[1] U. Ghia, K. N. Ghia and C. T. Shin, *J. Comput. Phys.* **48**, 387 (1982).
[2] J. W. Goodrich , in preparation.
[3] J. W. Goodrich, K. Gustafson and K. Halasi, "Hopf Bifurcation in the Driven Cavity," ICOMP Report 89-21, NASA Lewis Research Center, Cleveland, 1989, to appear in *J. Comput. Phys.*, (1990).
[4] J. W. Goodrich and W. Y.Soh, *J. Comput. Phys.* **84**, 207 (1989).
[5] K. Gustafson and K. Halasi, *J. Comput. Phys.* **70**, 271 (1987).
[6] R. Schreiber and H. B. Keller, *J. Comput. Phys.* **49**, 310 (1983).
[7] J. Shen, "Hopf Bifurcation of the Unsteady Regularized Driven Cavity Flows," Report No. 8906, The Institute for Applied Mathematics and Scientific Computing, Indiana University, Bloomington, 1989.
[8] R. Temam, *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*, (Springer-Verlag, New York, 1988).

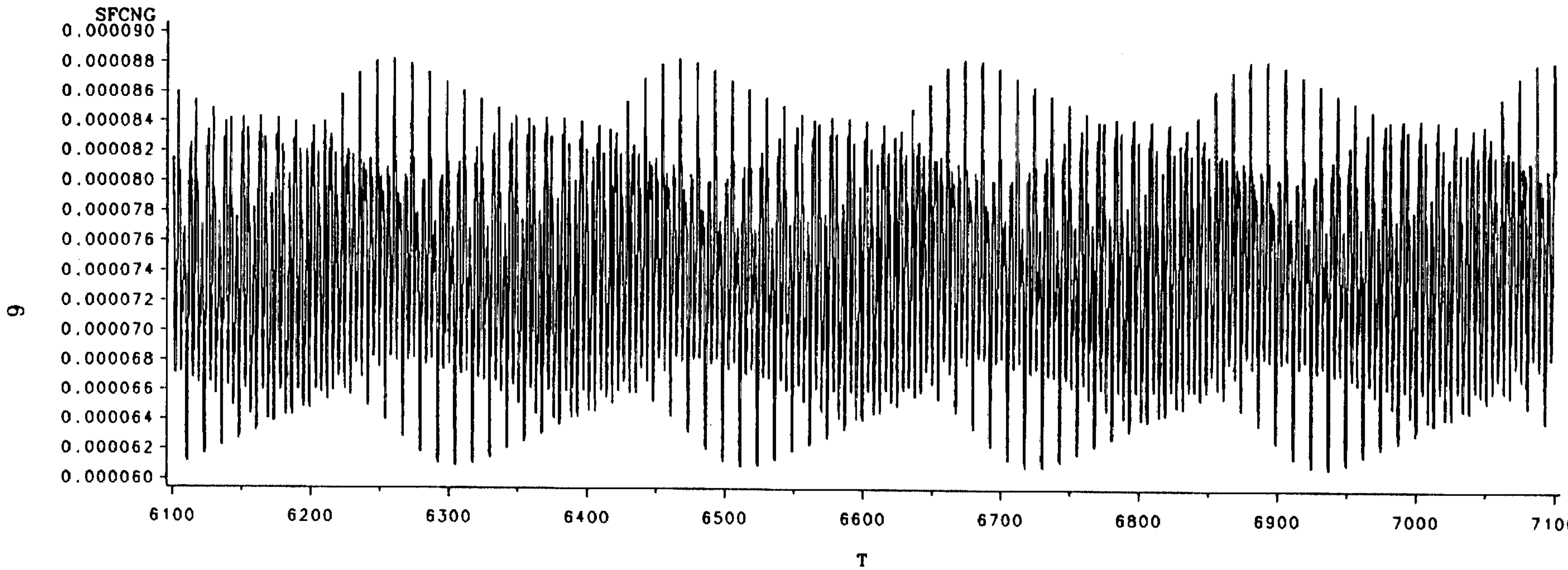


FIGURE 1. - STREAMFUNCTION CHANGE PER TIME STEP, $6100 < t \leq 7100$.

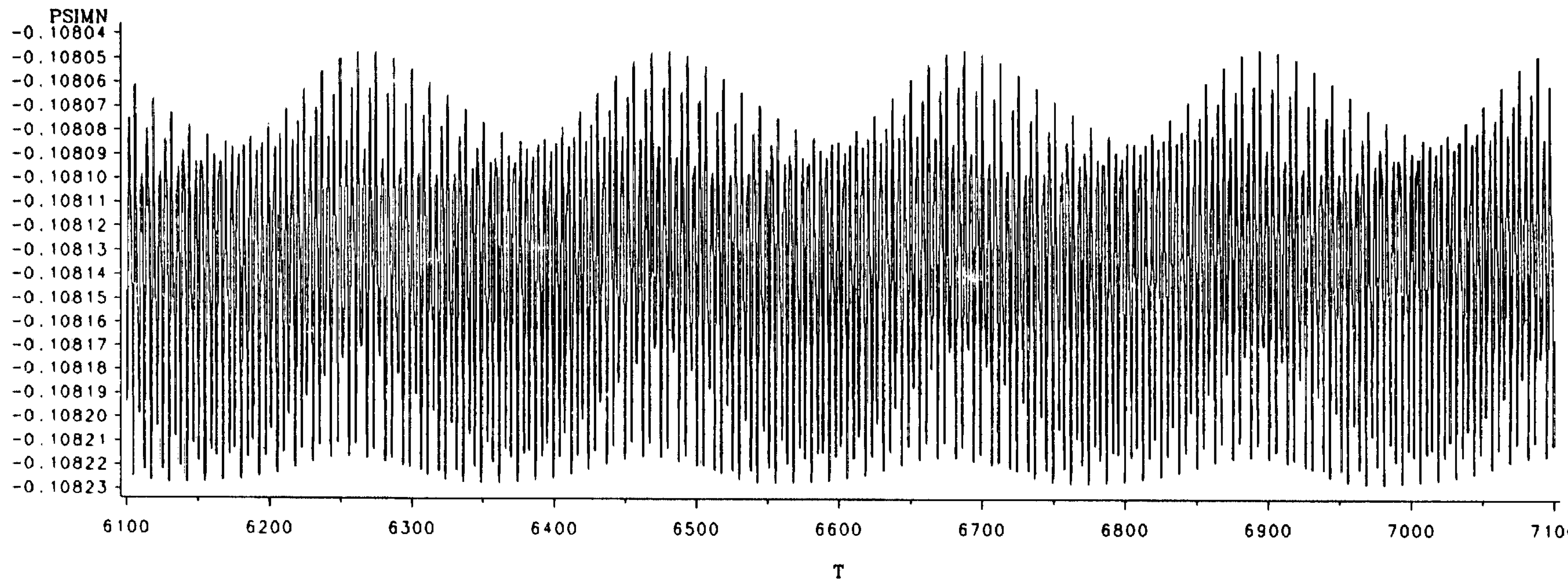


FIGURE 2. - STREAMFUNCTION MINIMUM PER TIME STEP, $6100 < t \leq 7100$.

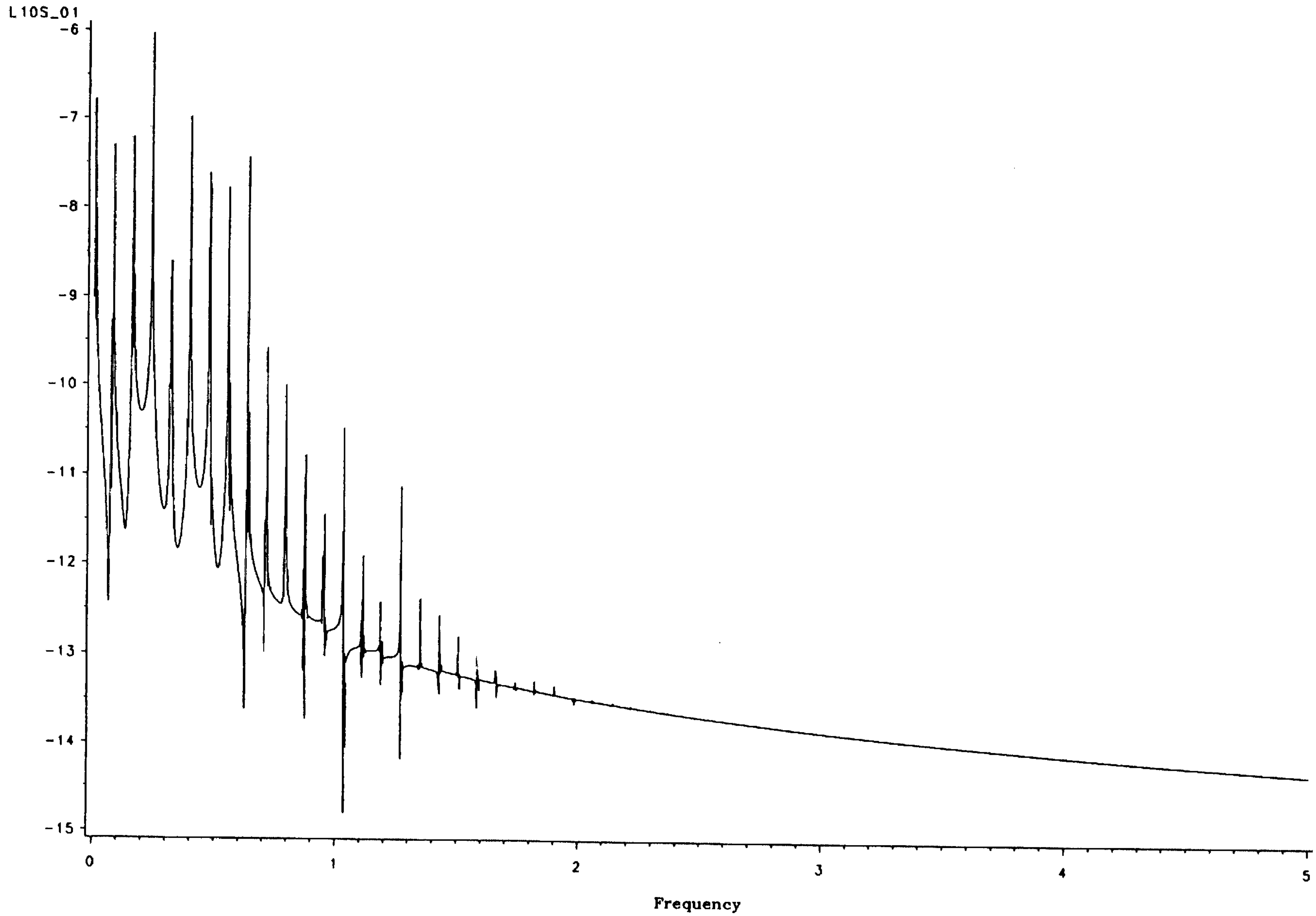


FIGURE 3. - POWER SPECTRUM FOR TOTAL KINETIC ENERGY, $6100 < t \leq 7100$.

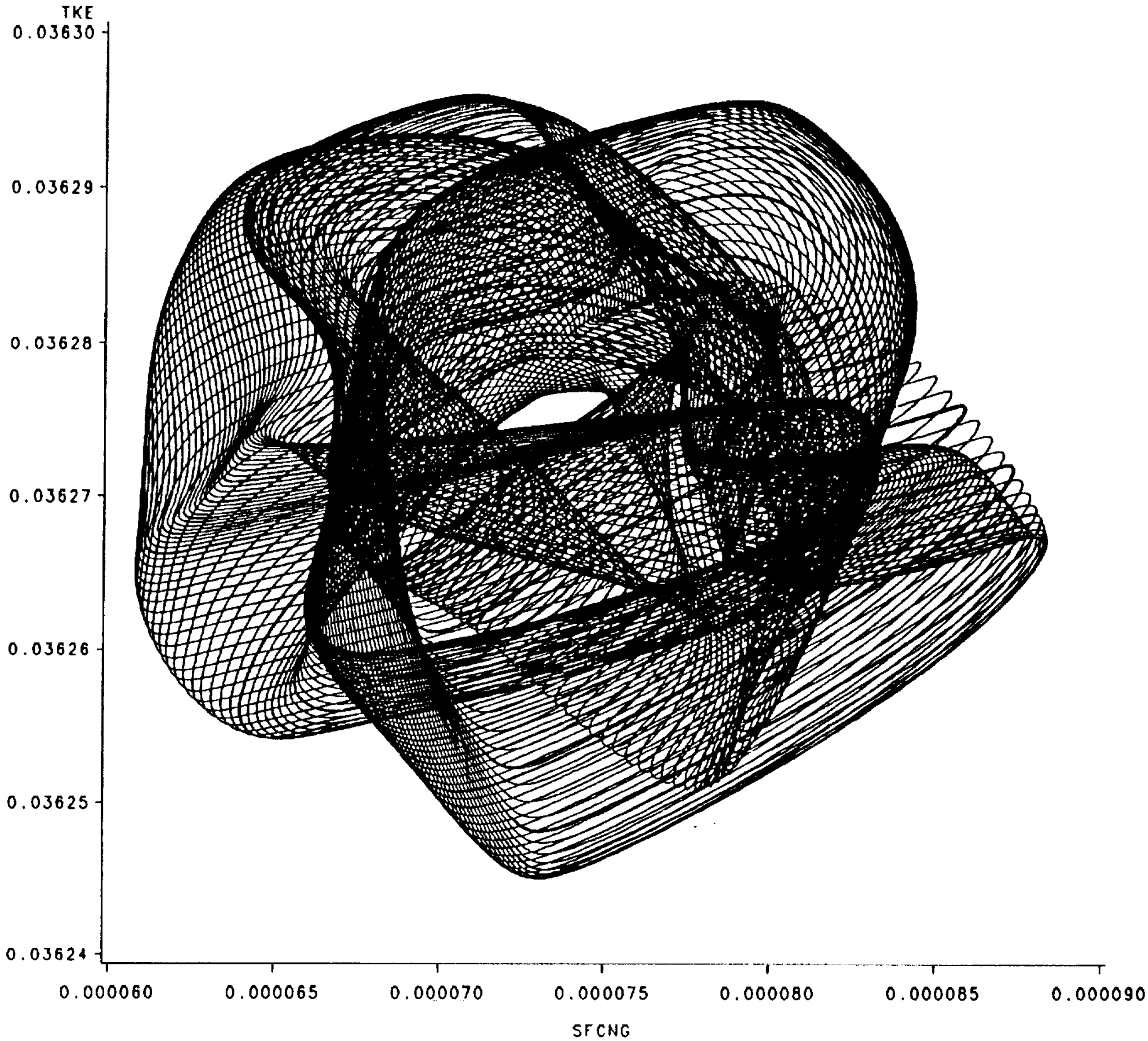
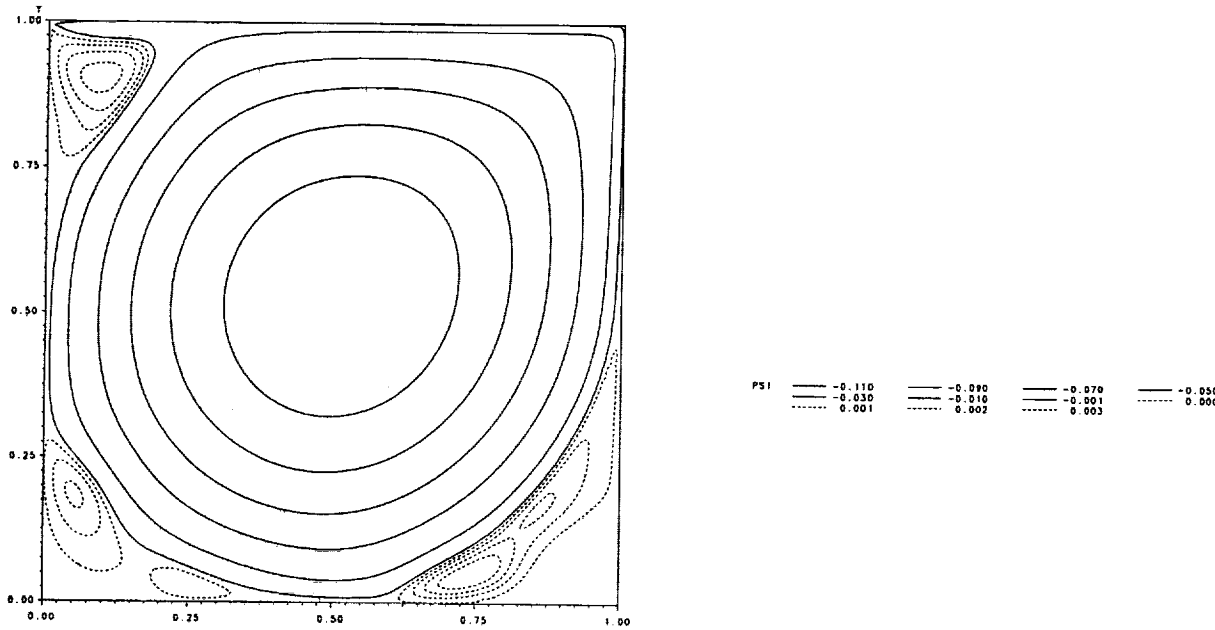
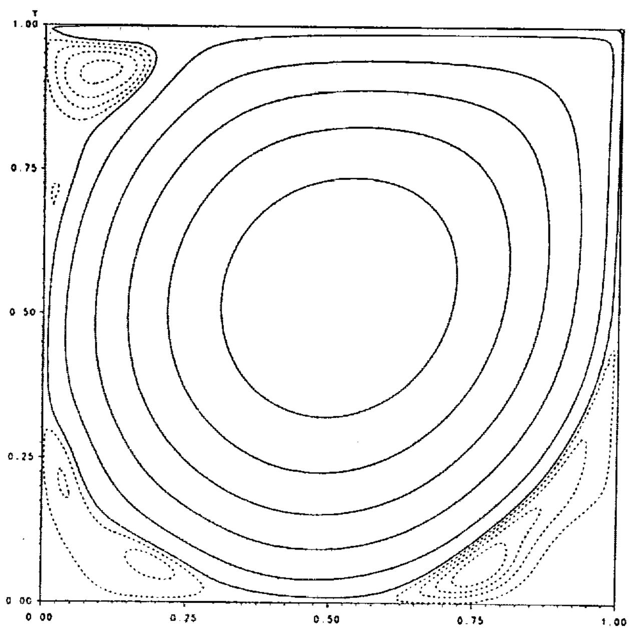


FIGURE 4. - STREAMFUNCTION CHANGE VERSUS TOTAL KINETIC ENERGY, $6100 < t \leq 7100$.

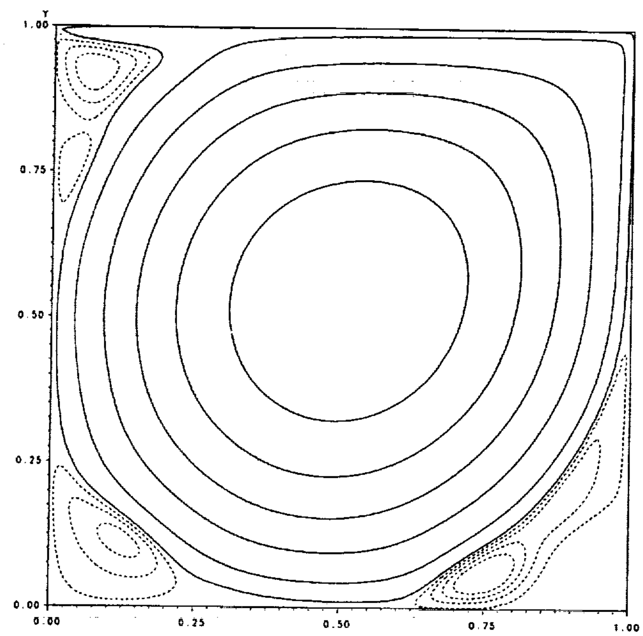
ORIGINAL PAGE IS OF POOR QUALITY



(a) t = 7101.0.



(b) t = 7101.5.



(c) t = 7102.0.

FIGURE 5. - STREAMFUNCTION CONTOURS.

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No. NASA TM-103141	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle An Unsteady Time Asymptotic Flow in the Square Driven Cavity		5. Report Date
		6. Performing Organization Code
7. Author(s) John W. Goodrich		8. Performing Organization Report No. E-5493
		10. Work Unit No. 505-62-21
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the IMACS 1st International Conference on Computational Physics cosponsored by the International Association of Mathematics and Computers in Simulation and the University of Colorado at Boulder, Boulder, Colorado, June 11-15, 1990.

16. Abstract

This paper presents summary details of an aperiodic time asymptotic numerical solution for the square drive cavity at $Re = 10000$. The data presented is for $6100 \leq t \leq 7100$, and is representative of the data that characterizes the aperiodic asymptotic state. Complete details are not given in this paper for either the transient evolution or the asymptotic state.

17. Key Words (Suggested by Author(s)) Incompressible viscous flow; Navier Stokes equations; Time dependent; Unsteady asymptotic state; Driven cavity	18. Distribution Statement Unclassified – Unlimited Subject Category 34

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 12	22. Price* A02

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161