AN ASTROPHYSICS DATA PROGRAM INVESTIGATION OF

CLUSTER EVOLUTION

p-25

NASA Grant NAG5-1200

Semiannual Report No. 1

For the Period 1 August 1989 through 31 January 1990

Principal Investigator
Dr. Edwin M. Kellogg

June 1990

Prepared for:

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20771

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

The Smithsonian Astrophysical Observatory
is a member of the
Harvard-Smithsonian Center for Astrophysics

The NASA Technical Officer for this grant is Dr. Donald K. West, NASA, Code 684, Laboratory for Astronomy and Solar Physics, Space Sciences Directorate, Goddard Space Flight Center, Greenbelt, Maryland 20771.
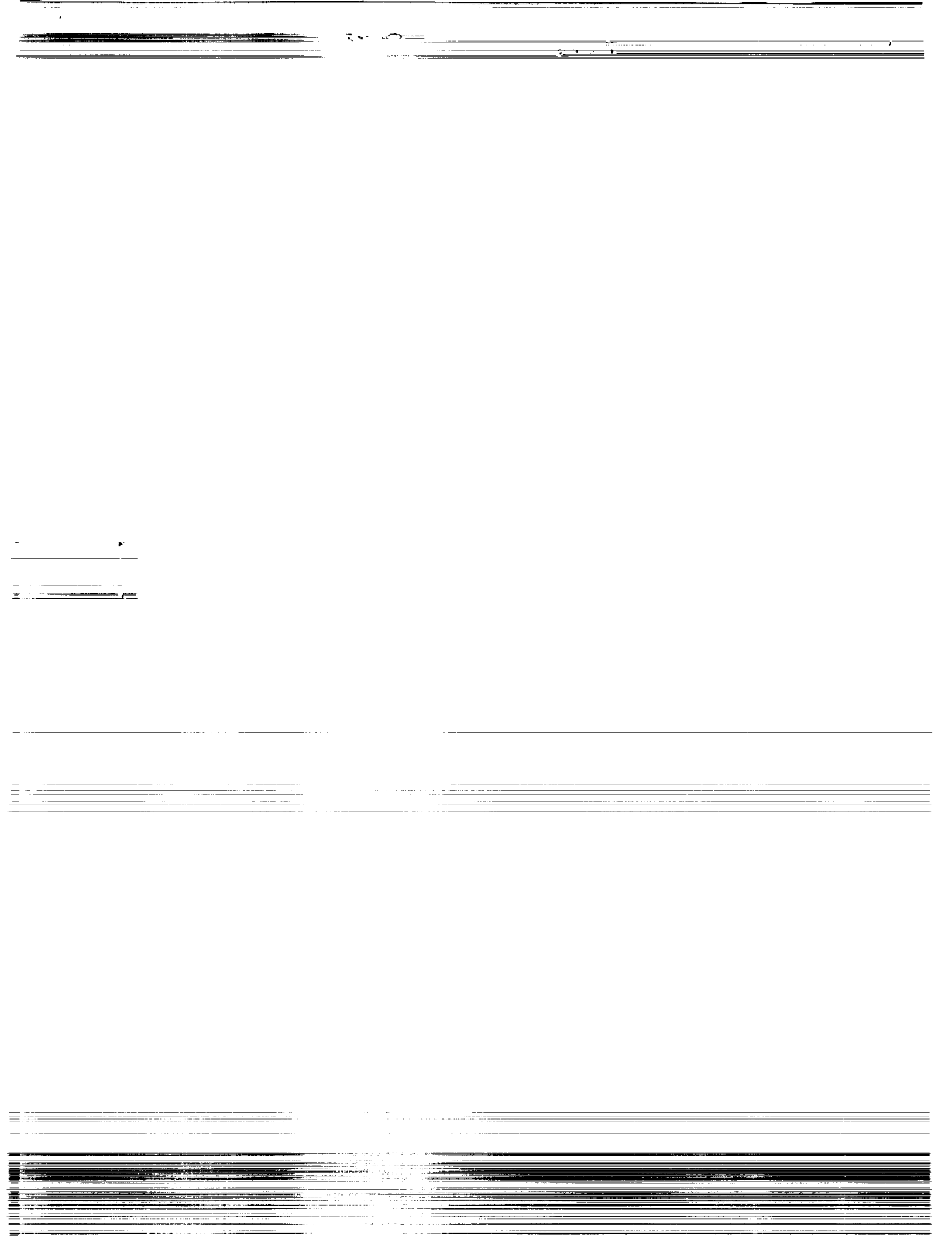
Work during the first half of the year concentrated on study of the distant X-ray clusters associated with gravitational lenses. the attached manuscript describes this work.

In addition, a general catalog of 35 radial distributions of the counts from the IPC from distant clusters was compiled. This will be subjected to further analysis.

X-RAY OBSERVATIONS OF GRAVITATIONAL LENSES

E. Kellogg, E. Falco, W. Forman, C. Jones and P. Slane

Harvard-Smithsonian Center for Astrophysics

Cambridge, Massachusetts

ABSTRACT

A preliminary status report is given on studies using the *Einstein* X-ray observations of distant clusters of galaxies that are also candidates for gravitational lenses. The studies will determine the location and surface brightness distribution of the X-ray emission from clusters associated with selected gravitational lenses. The X-ray emission comes from hot gas that traces out the total gravitational potential in the cluster, so its distribution is approximately the same as the mass distribution causing gravitational lensing. Core radii and X-ray virial masses can be computed for several of the brighter *Einstein* sources, and preliminary results are presented on A2218. Preliminary status is also reported on a study of the optical data from 0024+16. A provisional value of 1800 to 2200 $km\,s^{-1}$ for the equivalent velocity dispersion is obtained. The ultimate objective is to extract the mass of the gravitational lens, and perhaps more detailed information on the distribution of matter as warranted. A survey of the *Einstein* archive shows that the clusters A520, A1704, 3C295, A2397, A1722, SC5029-247, A3186 and A370 have enough X-ray counts observed to warrant more detailed optical observations of arcs for comparison. Mass estimates for these clusters can therefore be obtained from three independent sources: the length scale(core radius) that characterizes the density dropoff of the X-ray emitting hot gas away from its center, the velocity dispersion of the

galaxies moving in the cluster potential, and gravitational bending of light by the total cluster mass. This study will allow us to compare these three techniques and ultimately improve our knowledge of cluster masses.

INTRODUCTION

Gravitational lensing behavior in distant clusters of galaxies was discovered by Lynds and Petrosian (1986) - Soucail et al (1987). Faint arcs and rings in deep blue CCD exposures were found to be "mirage" images caused by bending of light from blue galaxies at very high redshift by the gravitational effect of clusters at smaller redshifts. The redshift of an arc in A370 was first measured by Soucail et al, 1988, confirming its nature as a gravitational lens image, and the first detection of the faint blue mini-arcs in A370 was reported by Fort et al, 1988. The information contained in these optical images of the arcs and rings in clusters of galaxies permits an estimate of the mass contained in the gravitational lens producing the arcs and rings.

This paper is a preliminary report on studies using the *Einstein* X-ray data on some of these distant clusters. We determine the location and surface brightness distribution of the X-ray emission from clusters associated with selected gravitational lenses. Since the X-ray emission comes from hot gas that traces out the total gravitational potential in the cluster, its distribution should be approximately the same as the mass distribution that gives rise to gravitational lensing effects. While most of the very distant clusters in the *Einstein* data are too weak to permit studying the brightness distribution with high resolution, we can obtain core radii and compute X-ray virial masses for several of the brighter sources.

The optical data on 0024+16 are also being analyzed to extract the mass of the gravitational lens. In many cases, optical observations have been made of individual galaxy velocities in a given cluster. From these, a velocity dispersion can be calculated, and a virial mass derived.

2

Mass estimates for these clusters can be obtained from three independent sources: the length scale(core radius) that characterizes the density dropoff of the X-ray emitting hot gas away from its center, the velocities of the galaxies moving in the cluster potential, and the bending of light by the total cluster mass. This study will allow us to compare these mass estimates for each cluster. If they are all consistent with a single value of the mass for a given cluster, we will have a strong statement on the total mass, including dark matter, contained in the cluster. If the estimates do not agree, further study of the differences will be indicated.

These studies will be of immediate interest in the planning of ROSAT (and ultimately AXAF) pointed observations of gravitational lenses. Detailed studies of these cluster X-ray sources will also be useful to optical astronomers investigating gravitational lensing. Centroid positions for the extended X-ray emission, which represent an improvement over those available in the *Einstein* production data processing, are derived and peculiarities in the X-ray brightness distribution are noted. This information will be a useful guide to planning and interpreting the optical observations.

Since the discovery of cluster X-ray sources in the Uhuru data (Gursky et al. 1971, Kellogg et al. 1972, Forman et al. 1972, Kellogg et al. 1973), when the brightest few clusters were observed, a large number of cluster X-ray sources have been detected and some of these have been studied in detail (Jones et al. 1979, Forman and Jones 1982, Jones and Forman 1984). Spectral analysis (Kellogg, Baldwin and Koch 1975) shows that the emission comes from hot gas with a significant concentration of iron series elements (Mitchell et al. 1976, Serlemitsos et al. 1977), although the abundances are $\sim$ 2-4 times below solar (Mitchell and Mushotsky 1980, Mushotsky 1980).

The gas can be used as an improved tracer of dark matter, because the gas is in hydrostatic equilibrium with the total mass, both dark and luminous. Thus, the gas distributes itself in the gravitational potential well formed by all the matter in the lens, including dark matter (David et al. 1989). Previously, galaxies have been used as test particles to trace the cluster's potential well. Hot gas atoms are much better

tracers than galaxies; since their collision times are much shorter, the associated velocity distribution is much closer to thermal equilibrium.

The *Einstein* observatory obtained the first two-dimensional images of X-ray emitting clusters of galaxies at $z > 0.2$ (White, Silk and Henry 1981, Henry and Lavery 1984). A systematic study of the morphology of these distant cluster X-ray sources has recently begun (Kellogg et al. 1989). The investigation involves a search for evolution effects, by comparing the class of clusters with $z \leq 0.2$ and the class with $0.2 < z \leq 1$. These sources are very difficult to study because they are very faint. Therefore, fewer counts have been accumulated in their images than for the more intense sources at lower z. As a result, not only are the statistical uncertainties greater, but the background subtraction has been a problem. The combination of these two effects has discouraged detailed investigation of large redshift clusters in the *Einstein* data. In this study the most recent, corrected background estimates are used. In this study, a careful optimization of the removal of the telescope and detector point response is being done, to obtain the best possible interpretation of the observations. Further an intensive effort is going on at CfA to extract better quality data from the Monitor Proportional Counter (MPC) for faint sources, which may give X-ray spectra out to higher energies for some of these clusters.

During the past year, there have been significant optical observations of gravitational lens behavior in clusters of galaxies. At the Toulouse Workshop on Gravitational Lenses, a list of 11 clusters that show this behavior was circulated, in addition to about 25 other objects that exhibit rings, or multiple magnified images. There is a feeling among observers that roughly 1/4 to 1/3 of all compact clusters exhibit some obvious form of mirage behavior. A recent paper by Nemiroff and Dekel (1989) presents quantitative arguments in support of this notion. Recent results by Tyson (1989) argue that every cluster will show such behavior if a sufficiently deep exposure is taken so that background objects with $z \geq 1$ are detected.

In the past several years, various investigators (Schramm and Kayser 1987, Kochanek

et al.1989) have developed techniques to analyze extended gravitational images such as arcs or rings, with the ultimate goal of estimating the mass distribution of gravitational lenses and possibly of estimating cosmological parameters. Such estimates are affected by uncertainties due to various problems, such as the difficulty in measuring redshifts for very faint, extended sources (*viz.* the arcs and rings) and the possible presence of unseen inhomogeneities of the cosmological medium itself, along the line of sight to the clusters (Gorenstein, Falco and Shapiro 1988). These uncertainties currently give a latitude of factors of a few to the best estimates of the mass in clusters. The analysis software that is currently implemented at CfA was written to treat unresolved gravitational images; it is currently being extended by Falco to treat extended images. Our hope is to define clearly the uncertainties in known analysis techniques by studying the uniqueness of models for the mass distribution in clusters, and to apply them to as many different clusters as possible. Then, we may be able to develop statistical measures of the power of the techniques to achieve the important cosmological goals indicated above.

EINSTEIN OBSERVATIONS OF GRAVITATIONAL LENS CLUSTERS

To begin the investigation of X-ray properties of lens candidates, radial count rate distributions and detailed enlarged contour maps of a 20 arc min region about each X-ray source were plotted for approximately 30 distant clusters. Examples of these for some of the better known gravitational lens candidates are given in Figures 1 to 4.

The examples shown illustrate the varying quality of the data, which is a function of the total counts observed and the intensity of the X-ray source.

We have been using image processing and analysis systems developed at CfA to analyze the optical image of 0024+16 and to derive a description of the underlying gravitational lens. A preliminary examination of the *Einstein* IPC and HRI data indicates that point sources will not make a major contribution to the total emission, so
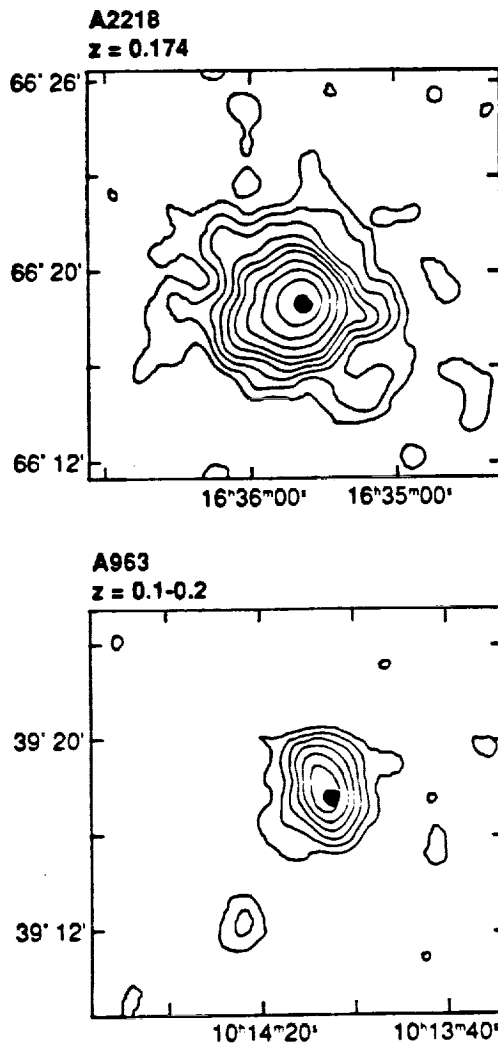
Figure 1: *Einstein* X-ray contour maps of A2218(203 X-ray counts) and A963(42 X-ray counts), both gravitational lenses in clusters of galaxies. IPC data. The filled circle is the optical position of the cluster lens. The IPC maps have a position accuracy of ±30 arc sec.

0024+16
z = 0.39
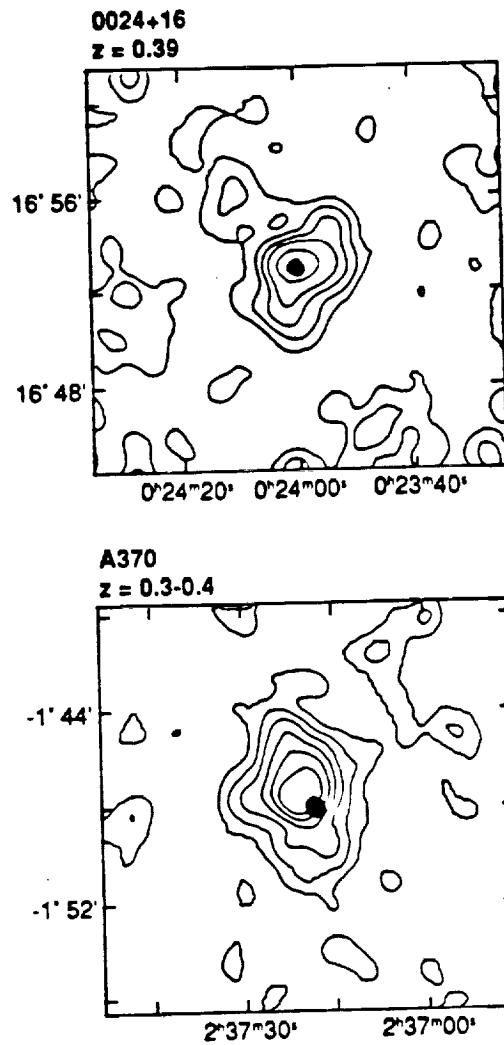
A370
z = 0.3-0.4

Figure 2: *Einstein* X-ray contour maps of Cl0024+16(120 X-ray counts) and A370(109 X-ray counts), both gravitational lenses in clusters of galaxies. IPC data. The filled circle is the optical position of the cluster lens. The IPC maps have a position accuracy of ±30 arc sec.
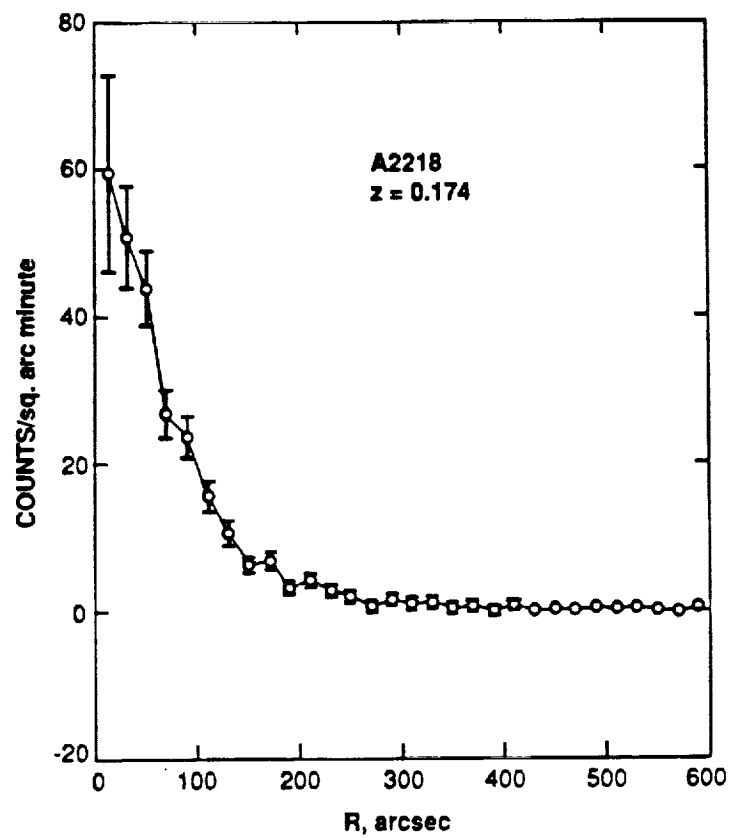
Figure 3: Radial X-ray count distribution for A2218. IPC data. Telescope response not removed.
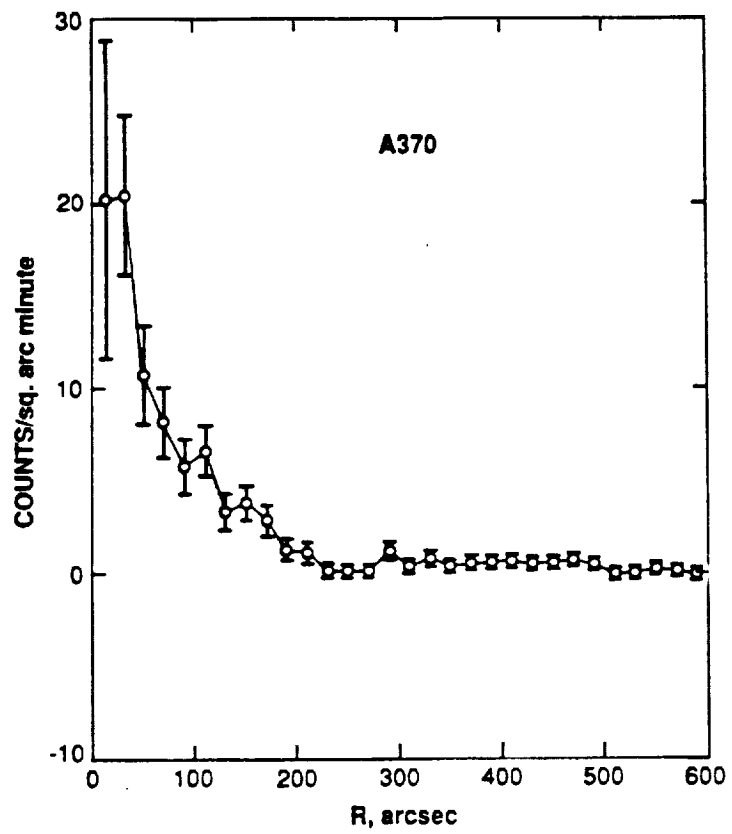
Figure 4: Radial X-ray count distribution for A370. IPC data. Telescope response not removed.

that the IPC data will be usable for deriving the X-ray virial mass.

We have generated a list of 19 cluster X-ray sources that are good candidates for optical lensing searches. The list includes centroid positions of the X-ray emitting region, accurate to about 30 arc seconds (see Table 1).

The first column of Table 1 gives the name of the source. The second column gives the redshift if one has been measured and the third column gives a reference to the redshift measurement. The fourth column gives the total counts observed (after background subtraction) and can be used to evaluate the statistical precision of the data. The fifth and sixth columns give the position of the X-ray source, and the seventh column gives the position uncertainty.

The statistical quality of the data is a function primarily of the total number of counts observed, although it also depends on the intensity of the source relative to the background counting rate. The sources with the greatest number of counts are those that will permit the best determination of the X-ray core radius and virial mass, and so should be the ones most useful for obtaining optical data for comparative analysis of the mass determinations.

This list will be useful to optical observers who are searching for gravitational lens behavior in clusters, because it points to the location of the majority of the mass in these clusters. This position is not always in the same place as the concentration of brightest galaxies. Therefore, the location of the X-ray source is probably the best center about which to search for arcs and rings with deep CCD exposures.

CLUSTER MASS DETERMINATIONS

The goal of this study is to determine the mass contained in clusters of galaxies in three different ways. First, mass estimation from the *Einstein* X-ray data; second, analysis of the gravitational lensing images in the visible spectrum, modelling the mass distribution

10

Table 1: Distant Cluster X-ray Sources Detected by *Einstein* Observatory

Name	z	Ref.	X-ray Counts	R.A.(1950)	Dec	Position Error (arc min)
A520	0.203	1	~253	04 51 36.0	02 50 42.5	32.0
A1704	0.220	2	239	13 12 37.84	64 50 36.1	31.0
3C295	0.464	4	195	14 09 33.36	52 26 22.0	32.0
A2397	0.224	1	158	21 53 36.0	01 09 18.2	32.0
A1722	0.328	2	~137	13 18 35.92	70 20 18.4	35.0
SC2059-247	0.188	3	113	20 59 14.92	-24 43 47.9	32.0
A3186	-	-	~112	03 53 12.76	-74 11 06.7	32.0
A370	0.373	1	~109	02 37 21.85	-01 46 17.2	36.0
A913	0.3 - 0.4	-	92	09 59 40.66	20 45 40.6	36.0
A1246	0.216	1	62	11 21 22.5	21 45 39.7	35.0
A2111	0.229	1	60	15 37 44.67	34 35 02.3	30.0
A1557	0.210	2	54	12 30 15.48	63 09 33.5	36.0
3C 19	0.483	4	52	00 38 13.64	32 53 14.3	36.0
A2645	0.246	1	45	23 38 42.0	-09 17 58.9	35.0
PHL1093	0.270	-	40	01 37 23.39	01 16 39.7	36.0
A1655	0.234	2	38	12 56 37.0	65 38 19.0	36.0
A41	0.275	1	35	00 26 22.16	07 34 08.4	36.0
4C74.13	-	-	34	07 35 38.48	74 21 38.0	35.0
Cluster 249	-	-	26	03 46 40.05	-45 23 45.4	37.0

1 = Sarazin et al 1982.
2 =Huchra et al 1990.
3 = White et al 1981.
4 = Giacconi and Seward, 1979

required to explain the observed distorted images of background galaxies; third, use of available redshift data for galaxies in these clusters to find a corresponding virial mass.

X-ray Virial Mass Estimates

In cases where sufficient counts are detected, say ≥ 175, detailed fitting of brightness functions is justified.

Pulse height spectroscopy can be done with the *Einstein* IPC. There is also some potential for using the *Einstein* Monitor Proportional Counter for sources that do not have another source of comparable strength within the field of the 3/4 degree collimator. The MPC can give an estimate of the gas temperature to higher energy because it does not suffer from the 4 keV upper energy cutoff of the *Einstein* mirror; it has reasonable efficiency up to ≥ 15 keV. For some sources, it may be possible to obtain spectra with the Ginga satellite.

The IPC images, the spectral hardness in at least two energy bands for some sources, and the comparison of IPC and HRI data can be analyzed for a few sources to check for compact sources. Fits are computed to specific model brightness distributions as appropriate. The degree to which the analysis of cluster brightness distributions is pushed depends upon the number of photons detected.

The hot gas pervading a typical cluster may be used to trace the density profile of the cluster. Given this density distribution, the virial mass of the cluster may be calculated and compared with that determined by velocity dispersion measurements from cluster galaxies or by modeling of gravitational lens images associated with the cluster.

We may treat the cluster gas as an ideal gas in hydrostatic equilibrium in a (spherically symmetric) cluster potential. Thus, we have

$$P = \frac{kT}{\mu M_H} \rho$$

and

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

where $M(r)$ is the mass interior to the radius r. We may combine these to find

$$M(r) = -\frac{kTr}{G\mu M_H}\left[\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right]$$

or, using an isothermal approximation,

$$M(r) = -\frac{kTr}{G\mu m_p}\frac{d\ln\rho}{d\ln r}.$$

Typically, the X-ray surface brightness distribution may be represented by:

$$I(r) = I_0\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2}.$$

For an isothermal distribution, the density profile may then be deprojected:

$$\rho(r) = \rho_0\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta/2}.$$

Differentiating, we may substitute to find

$$M(r) = \frac{3\beta kT}{G\mu M_H}\frac{r}{[1 + \left(\frac{r_c}{r}\right)^2]}.$$

Thus, we need only determine r_c, β, and T from the X-ray data in order to derive the mass distribution of the cluster. The core radius r_c and β may be determined by fitting the X-ray data to trial expressions for $I(r)$ (properly convolved with the instrument point response function) as β and r_c are varied. A temperature value for the cluster may be determined if a sufficiently long IPC observation exists. For observations lacking such spectral information, typical values of $T \sim 5$ keV may be assumed without serious loss of accuracy. Similarly, for sparse data sets, the average value $\beta \approx 2/3$ (Jones and Forman, 1984) may be used to facilitate determination of r_c. In this manner, the mass contained within the image-producing portion of the cluster-lens may be calculated (given the lens radius inferred from optical observations) and compared with calculations based upon the optical lensing models.

We have begun such an analysis for A2218. From fits to the *Einstein* data (Figure 3) we find an X-ray core radius of 0.16 − 0.36 Mpc for $H_0 = 50$, and $\beta = 0.55 - 0.75$. Using $kT = 6.7^{+0.5}_{-0.4}$ keV as determined by Ginga observations (McHardy et al, 1989), we obtain the curve in Figure 5 for the contained X-ray virial mass as a function of angular distance from the center of the cluster. The confidence limits in the curve plotted are obtained by plotting a curve for each combination of the extreme values of core radius, T and β obtained from the fits to A2218 data, and estimating the envelope of curves corresponding to all such values within the extremes.
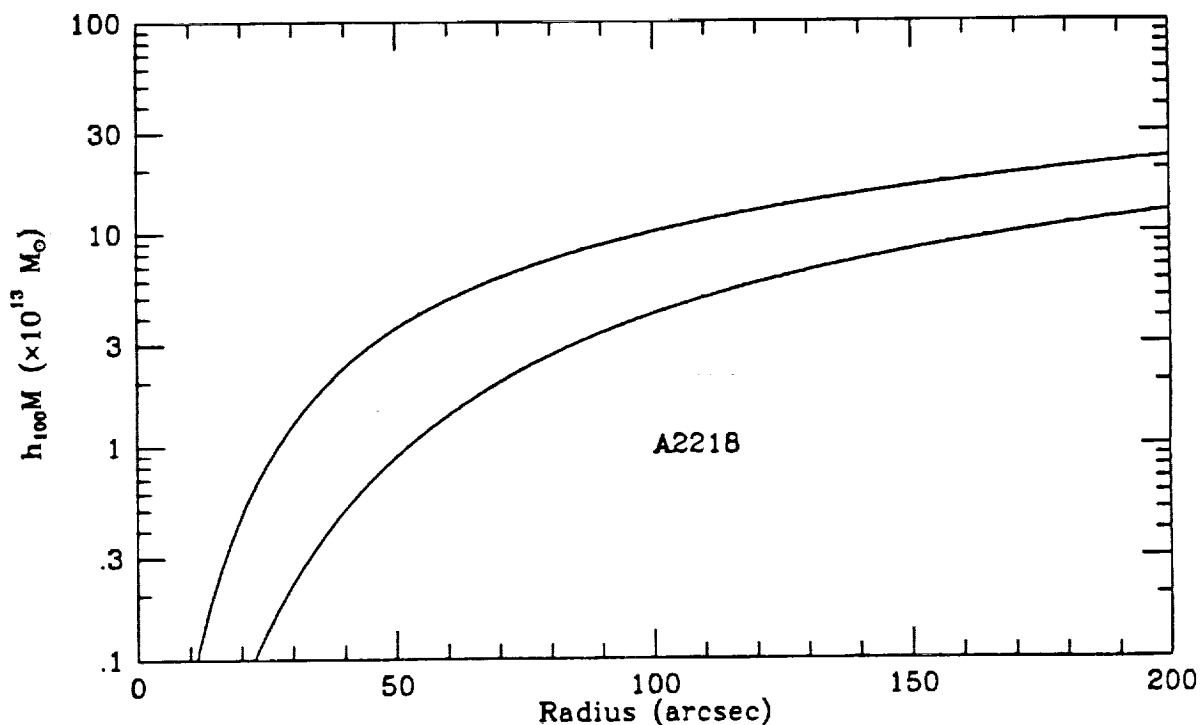

Figure 5: Contained mass of X-ray emitting hot gas versus angular radius for A2218. The two curves represent the approximate extrema of the confidence region, although a precise assignment of level of confidence has yet to be done.

Cluster Mass Estimates by Gravitational Lens Analysis of Optical Images

Giant luminous arcs are now a nearly common feature formed by compact clusters. Now that such features are expected, more are certain to be found. The potential database of candidates is very large. Therefore, the analysis of optical views of gravitational images is a promising technique for cosmological studies. One of the critical possible results is a determination of the total mass in clusters, including dark matter, because the gravitational deflection of light is sensitive to all matter, regardless of its emissivity. It is expected that most of the matter in clusters of galaxies is non-luminous. Therefore, using gravitational bending of light is a promising technique to obtain a new type of mass estimate, to be compared with optical and X-ray virial masses. Such new estimates, if sufficiently accurate and numerous, will provide new constraints, not only on the mass-to-light ratios in clusters, and therefore, on types of dark matter therein, but also on the mean matter density of the universe.

There are two regimes of interest for clusters acting as gravitational lenses. In the first regime, the surface mass density is too low to yield multiple images of backgound galaxies, and no large arcs can form. In such a case, the images of background galaxies will be elongated, preferentially in a tangential direction with respect to the cluster's center. Deep CCD images of the cluster field then may best be analyzed statistically (Tyson, Valdes and Wenk 1989) to determine the mass distribution of the cluster. In the second regime, the surface mass density suffices to yield multiple images of background galaxies, and large arcs can appear. The presence of large arcs is due to chance alignment of caustics of the lens and background galaxies. The larger the mass of the cluster, the larger the cross-section for such chance alignments. The shapes of large arcs (from 30 degrees to a full ring) provide very strong constraints on the mass distribution of the lensing cluster, because these shapes are very sensitive to the separation between the caustics and the background galaxies. Of course, the two regimes mentioned overlap, because of the granularity of the mass distribution in clusters, as evinced by

15

luminous galaxies. Each case requires careful accounting of the individual galaxies that are observed.

What optical observations are needed once a candidate lensing cluster has been identified? Deep CCD imaging and spectrophotometry are necessary for two reasons: first, the shape of arcs must be mapped carefully to minimize geometrical uncertainties; second, a determination of the redshift of the galaxies that are stretched into arcs is critical to remove degeneracies in estimates of the mass distribution of lensing clusters. Sufficient interest has been generated in the lensing properties of clusters that we can safely expect observations of these sources to be constantly pushed to their limits.

Images of the cluster 0024+16 show many arcs that are attributed to gravitational deflection (Turner, 1988, 1989. Also see Koo, 1988). The arcs are thought to be produced by gravitational lensing due to a foreground cluster of red galaxies, at z $\cong$ 0.39 (Dressler, Gunn, and Schneider 1985). We have started modeling efforts to account for the presence of the arcs. As a first approximation, we assumed the main mass component of the cluster to be a smooth surface mass distribution with a King-type profile, parametrized by a line-of-sight velocity dispersion and a core radius, and centered at the approximate center of brightness of the X-ray emission from the cluster. The values of the model parameters that account for the arc separations are in the range 1800 to 2200 km s^{-1} for the velocity dispersion, and 30 to 40 arcsec for the core radius. These are preliminary values that indicate simply that reasonable values of the parameters will eventually suffice to account for the properties of the arcs.

We plan to model the surface mass distribution of the cluster in greater detail, based on a more refined description of the distribution of X-ray brightness, and on accounting for the presence of individual galaxies that affect the shapes of the arcs.

Comparisons of Mass Estimates

The optical virial mass of a cluster is determined by measuring the velocities of a sufficiently large number of galaxies. The X-ray virial mass of a cluster is determined by calculating the volume mass density in the cluster that is necessary to produce the observed X-ray emission, and then integrating the density. The lensing mass of a cluster is determined by analyzing the properties of the optical images produced by the cluster. If the only matter in clusters were of the luminous variety, these three mass estimates would coincide. Since the X-ray and optical virial masses disagree by at least an order of magnitude with mass estimates obtained by accounting only for the luminous matter, we know that dark matter must be found in clusters. Thus, the third proposed type of estimate will add to the evidence for dark matter, and may yield more accurate values for the amounts in which it is present in clusters. Each of the individual techniques can be in error for its own reasons. Therefore, the three techniques used jointly can compensate for the individual errors of each technique.

The optical virial mass estimates are susceptible to errors caused by misclassifying individual galaxies as cluster members, when they are really background or foreground objects. In that case, their redshift is associated with the Hubble flow, not with their virial velocity in the cluster. Optical virial estimates are also distorted by substructure in the cluster; if there is subclustering, some of the galaxies within the cluster are also members of groups within the cluster and have their own peculiar velocity field distortions as a result.

X-ray virial masses are presently limited by the small number of X-ray photons detected, with resultant poor counting statistics. This makes measurements of the core radius, β, and temperature uncertain. As the number of photons detected increases with future X-ray astronomy observations from ROSAT, BBXRT, Astro-D, Spectrum X-γ, and AXAF, the statistical uncertainties will decrease and other sources of confusion and error will emerge. The X-ray virial mass determinations will also have problems from foreground and background source confusion. Substructure within the cluster will result

17

in temperature, density, and velocity inhomogeneities that cause errors in measuring the total mass. Some of these errors can be mitigated by measuring the redshift of the gas that emits the X-rays, using advanced spectrometer detectors such as the CCD and the bolometer on AXAF. However, some confusion and inaccuracy will remain in the X-ray mass determinations.

SUMMARY AND DISCUSSION

This paper has given a preliminary report on studies using the *Einstein* X-ray data on some of these distant clusters. Examples of the X-ray radial brightness distribution have been given, and the core radius and β have been derived for A2218.

A preliminary value of the mass of the gravitational lens in 0024+16 has been obtained. The value, 1800 to 2200 km s^{-1}, is somewhat higher than the value of 1287 km s^{-1} obtained from optical redshift measurements on the galaxies (Dressler, Gunn, and Schneider 1985), although no formal error limit is quoted for their value. The work for deriving an X-ray virial mass is still under way, so no value is yet available. For A2218, we have derived a contained X-ray mass vs. radius, which can be compared with future determinations of mass from the optical velocity dispersion and the gravitational lens analysis. As this paper was being finalized, we learned that a preliminary value of (1050 +700 -200) km s^{-1} has been obtained (Birkinshaw, 1989), with suggestion that the correct value is closer to 1300.

Further work will focus on obtaining and comparing mass estimates from three independent sources in each cluster: the length scale(core radius) that characterizes the density dropoff of the X-ray emitting hot gas away from its center, the velocities of the galaxies moving in the cluster potential, and the bending of light by the total cluster mass. We urge the optical observers to concentrate their efforts on obtaining data for the clusters listed in Table 1, especially those at the top of the list for which well in excess of 100 counts were detected in X-rays.

18

REFERENCES

Birkinshaw, M. (1989) private communication.

David, L., Arnaud, K., Forman, W., Jones, C. (1989) submitted to Ap. J.

Dressler, A., Gunn, J., and Schneider, D. 1985 Ap. J. 294 70.

Forman, W. and Jones, C. (1982) Ann. Rev. Astron. Astrophys. 20 547.

Forman, W., Kellogg, E., Gursky, H., Tananbaum, H. and Giacconi, R. (1972) Ap. J. 178 309.

Fort, B., Prieur, J.L., Mathez, G., Mellier, Y., Soucail, G., 1988, Astron. Astrophys. 200, L5

Giacconi, R. and Seward, F. (1979) CFA/HEA 79-285, Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.

Gorenstein, M. V., Falco, E. E., and Shapiro, I. I. (1988) Ap. J. 327 693.

Gursky, H., Kellogg, E., Murray, S., Leong, C., Tananbaum, H. and Giacconi, R. (1971) Ap. J.(Letters) 167 L81.

Helfand, D., Ku, and Abramopoulos, F. (1980) Highlights of Astronomy 5 747.

Huchra, J., Henry, J. P., Postman, M., and Geller, M. 1990 Ap. J. in press.

Henry, J. P. and Lavery, R. J. (1984) Ap. J. 280 1.

Hughes et al, 1987(8)

Jones, C., Mandel, E., Schwarz, J., Forman, W., Murray, S., and Harnden, F. (1979) Ap. J.(Letters) 234 L21.

Jones, C. and Forman, W. (1984) Ap. J. 276 38.

Kellogg, E., Gursky, H., Tananbaum, H. and Giacconi, R. (1972) Ap. J.(Letters) 174 L65.

Kellogg, E., Murray, S., Giacconi, R., Tananbaum, H. and Gursky, H. (1973) Ap. J.(Letters) 185 L13.

Kellogg, E., Baldwin, J. and Koch, D. (1975) Ap. J. 199 299.

Kochanek, C. S., Blandford, R. D., Lawrence, C. R., and Narayan, R. (1989) M.N.R.A.S. 238 43.

Koo, D.C. (1988) "Large-Scale Motions in the Universe: A Vatican Study Week. V.C. Rubin and G.V. Coyne, eds. Princeton U. Press. p. 253.

Lavery, R. J., and Henry, J. P. (1988) Ap. J. (Letters) 329 L21.

Lynds, R., and Petrosian, V. (1986) Bull. Am. Astr. Soc. 18 1014.

Mitchell, R. J., Culhane, J.L., Davison, P.J. and Ives, J.C. (1976) Mon. Not. Roy. Astron. Soc. 189 329.

Nemiroff, R. and Dekel, A. 1989 Ap. J. 344 51.

Sarazin, C., Rood, H., and Struble, M. 1982 Astron. and Ap. 108, L7.

Serlemitsos, P.J., Smith, B.W., Boldt, E.A., Holt, S.S., and Swank, H. (1977) Ap. J. (Letters) 211 L63.

McHardy, I., Stewart, G., Edge, A., Cooke, B., Yamashita, K. and Hatsukade, I. 1989 (preprint).

Mitchell, R.J. and Mushotsky, R. (1980) Ap. J. 236 730.

Mushotsky, R.F. (1980) in *X-ray Astronomy*, ed. R. Giacconi and G Setti, Reidel, Dordrecht, p. 171.

Nemiroff, R.J. and Dekel, A. (1989) Ap. J. 344 51.

Sarazin, C. L., Rood, H. J. and Struble, M. F. (1982) Astron. and Ap. 108 L7.

Schramm, T. and Kayser, R. (1987) Astr. Ap. 174 361.

Soucail, G., Mellier, Y., Fort, B., Picat, J.P., 1987, Astron. Astrophys. 172, L14

Soucail, G., Mellier, Y., Fort, B., Mathez, G., and Cailloux, G. (1988) Astr. Ap. (Letters) 191 L19.

Turner (1988) Proc. 14th Texas Symposium on Relativistic Astrophysics.

Turner (1989) private communication.

Tyson (1989) Proc. Toulouse Workshop on Gravitational Lenses, to be published

Tyson, J. A., Valdes, F., and Wenk, R. A. (1989) preprint.

White, R.A., Sarazin, C.L., Quintana, H. and Jaffe, W.J. (1981) Ap. J. (Letters) 245 L1.

White, R. H., Sarazin, C. L., Quintana, H. and Jaffe, W. J. (1981) Ap. J. Lett. 245 L1.

White, S.D.M., Silk, J. and Henry,J.P. (1981) Ap. J. (Letters) 251 L65.