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THE MAKING OF THE MECHANICAL UNIVERSE

James Blinn
JPL Graphics Laboratory
Pasadena, California

EDITORIAL FOREWORD

The *Mechanical Universe* is a two-semester, introductory level, television-based physics course. In the fall of 1985 the first semester of *The Mechanical Universe* was released to the academic community and public broadcasters. The two semesters of the course, *The Mechanical Universe* and *Beyond the Mechanical Universe,* consist of 26 half-hour television lessons and two versions of a text, one for science and engineering majors and the other for nonmajors. The course is scientifically sophisticated and mathematically rigorous, teaching and using calculus. The lecture programs contain computer animation used as a primary tool for the instruction in physics. Each program begins and ends with Caltech Professor David Goodstein providing philosophical, historical, and often humorous comments from his lectures at Caltech.

The television series is not only the basis for a college course, but it also is suitable for a general audience interested in stimulating and challenging science programming. The *Mechanical Universe* television series and college course were funded by The Annenberg/CPB Project and The National Science Foundation (California Institute of Technology, 1986).

The following sections excerpt a number of design considerations regarding the dynamic computer graphics used to communicate physical phenomena and mathematical principles included in the *Mechanical Universe* (Blinn, 1987). The specific recommendations were not intended to be freely extended to other graphics interface applications, but do represent the considered judgment of a pioneer of computer graphics and certainly identify design issues that are faced in all attempts to use computer graphics as a medium for communication of spatial information.

CHAPTER 1 - OVERVIEW

1.1 INTRODUCTION

The *Mechanical Universe* project required the production of over 550 different animated scenes, totaling about 7 1/2 hours of screen time. The project required the use of a wide range of techniques and motivated the development of several different software packages. This report is a documentation of many aspects of the project, encompassing artistic/design issues, scientific simulations, software engineering, and video engineering.

My interest in *Mechanical Universe* is twofold. One, to produce the material and two, to see what tools need to be developed. It is hard to develop tools if you don't know what they are supposed to do. Having a large animation project provides a lot of experience on what the problems really are, instead of what somebody thinks they might be. This is a somewhat empirical approach to systems design. That is, several special-case systems are built, motivated just by the needs of some particular project. They are then analyzed to see what things they seem to have in common. In doing this sort of examination, it is important to realize that you cannot prove that your assertions are correct in the same sense that you can prove a mathematical theorem. The best that can be said is that the mechanisms described here seem to work well for the problems to which they have been applied.

In this section I will discuss a few ideas on graphical design in general. The emphasis will be on concepts that are not specifically for scientific animation, but those that may be applied to other uses of visual communication.

I haven't learned this by formal training. It has come by practice, intuition, and perhaps genetics (I come from a family of artists). I learned to solve design problems by being presented with them and by being forced to think about the implications of color and shape choices. The results are what made sense to me at the time.

CHAPTER 2 – GRAPHICAL DESIGN (STATIC)

Static design refers to the appearance of a single frame. The concept of motion design is discussed in the next chapter.

2.1 WHAT IS A DESIGN PROBLEM?

Let us begin with the question, "what is a design problem?" It can be likened to pantomime. You must present some information that, perhaps, could be described in words, but you are required to use only pictures.

Some examples:

- The Voyager spacecraft approaches a planet. A moon is off to the side. You must pan across to see it, but still give the viewers some idea of context of where they are now looking, compared with where they were looking before.

- How about a more detailed example? We will take an example from program 5, Vectors. The idea is to list the various types of vector expressions and to give an idea of whether the result is a vector or scalar. New items are added to the list as the program proceeds. The whole list may not fit entirely on the screen. In addition, as a new item is added, some geometric demonstration is needed to show what it is.

Let's look at a solution to this last example. We represent an abstract "space" where the vectors live as a kind of vector land. There is a river running down the middle separating it from scalar land. This allows us to display the lists in perspective receding into the distance. As each new object is introduced, it is added to the front of the list and the list recedes farther into the distance. Old list items may no longer be legible, but the memory of them is enough to remind the viewer of what they are. The key elements are to (1) differentiate between vectors and scalars and (2) give an impression of three-dimensional (3-D) space, but not to make it look too realistic.

An oblique view of the ground plane must appear to recede into the distance. This can be shown by texture. An obvious texture is a grid which shows perspective very well. However, at this point in the academic development, the notion of a coordinate system has not yet been presented. Some other textural effect must be used. Texture mapping a random, say pebbly, texture would be slow. The resolution is to place a randomly scattered group of lines looking like grass across the plane. Just a few such lines can give a very cheap impression of receding ground plane. Also, the color of the plane is made to get bluer and paler as it moves into the distance.

Drop shadows help to bring out the 3-D quality and make the vectors seem to hover above the plane, giving an interesting surreal effect.

Later in the program, when unit vectors and coordinates are introduced, the grid is placed on the plane (but only a small piece of it). Grids are a bit overused in computer graphics, but for much of what we do at *Mechanical Universe,* they are necessary because we are actually plotting graphs.

When we introduce unit vectors $\widehat{\subset}$ and $\widehat{\supset}$, they tip their hats. When we show the construction of a vector product, the term for *vector add* and *vector multiply* are slid down close to the grid.

2.2 DIRECTION OF ATTENTION

It is necessary to direct the attention of the viewer to the important parts of the picture. Scenes are shown on television in fairly brief bursts, so the important parts must stand out. One good trick for doing this is to look away from the screen and look back quickly; determine what you see first when looking back. Is that the important part of the picture? If not, change the picture to make it so.

This means avoiding gaudy backgrounds; the background should not look more interesting than the foreground. In one example I had an equation over a dark blue background that graded into orange, giving a sort of sunset effect. It was very pretty, but the problem was that when you first looked at the screen, all you saw was the orange. I changed the background to a more neutral color and now the first thing you see is the equation.

2.3 AVOIDING INFORMATION OVERLOAD

I consciously avoid trying to "dazzle" the viewers. Dazzling implies an overload or numbing of the senses. The idea is to communicate and draw the viewers in instead of making them tip backwards off their chairs.

For the same reason, I don't use lots of spinning or tumbling of 3-D objects. It's distracting. There is a trade-off here between not giving your audience enough views of an object to be able to understand its 3-D shape versus making it confusing by spinning it around too quickly.

One important trick to encourage simplicity is to arrange for the designs to be done while viewing a monitor from across the room. If the image can be made legible at a distance of 10 ft, it's about right. This discourages putting in too much small detail.

2.4 COLOR SELECTION

Given the color television medium, we have both the opportunity to make scenes in color and the responsibility to make the colors look good. There are a few tricks to use in color selection.

I have favorite colors; I lean toward blues and greens. However, I don't like purple. I once used it purposely to break out of a rut, as a background in the scene on conic sections. I originally wanted to put a red cone in front of it, but I couldn't get a red that didn't disappear into the purple in dark areas (as seen in black and white). Finally, I went to a brighter yellow cone.

2.4.1 Make it Work in Black and White

When designing, look at the picture with the color turned off and see if it "reads" (to use a designer term). Reads in this context means "can you tell what is going on; do the appropriate things stand out?"

While color is important in the *Mechanical Universe* animations, it is not the only thing that differentiates items on the screen. It's not crucial. I have made consistent color decisions, but the viewer is not expected to remember color schemes to understand a scene.

2.4.2 Context

Color selection programs are minimally useful because colors always look different in context. The only real way to see how they look is to make an actual picture of the scene.

2.4.3 Distance Cues

Distance can be represented by making things disappear into a fog. This was done literally in a scene of the molecular arrangement of a salt crystal.

Other color cues: the color of things gets bluer and paler with distance.

Field lines are a complex set of 3-D curves. They can look like a pile of spaghetti if you're not careful. The distance effect is aided by three things: (1) normal depth cueing (things get darker—i.e., less luminance contrast—with distance); (2) drawing them in depth order so a closer (brighter) line will overlay a farther line; (3) making the intensity of the line darker at the edges than in the middle. This gives a slight "cylindrical" solid quality to the lines.

2.4.4 Not Too Many

Don't use too many colors.

There is a problem with running out of colors. There are more physical quantities to represent than there are easily distinguishable colors. You can't use saturation or value to distinguish things because sometimes these need to be adjusted depending on context, e.g., energy.

2.4.5 Consistency

Consistently use color schemes to recall previous results as well as to differentiate things. We will discuss the color scheme later:

- But the color scheme wasn't always consistent
- Paler colors for mass multiplied by something
- Colored backgrounds for two integrations of gravity law
- Colored backgrounds for bringing external equations to prove Kepler's third law
- Blue texture for energy equation

2.5 2-D/3-D CONSIDERATIONS

Two-dimensional diagrams are easier to understand than 3-D, especially when they are in motion. This is partly because labels keep getting in the way of 3-D diagrams in some views. Most of the physics of the first term of *Mechanical Universe* is essentially 2-D problems (like Keplerian orbits). These remain 2-D. The inherently 3-D concepts are torque and angular momentum. The punch line is, use 3-D only when absolutely needed.

In fact, some 3-D situations were simplified to 2-D. For example, I used 2-D for the Lennard-Jones atomic motion simulation and the ideal gas simulation. The actual physics is 3-D, of course, but 2-D shows the phenomena adequately and 3-D would be really confusing.

In the second term there were more inherently 3-D problems. You must use 3-D for electromagnetic fields. Many textbooks use 2-D for fields, but much is lost.

Three dimensions are also used as a trick to put more text on the screen. As the screen tilts back, more text fits. The top row might not remain legible, but we can remember what it was.

2.6 MAKING THINGS STAND OUT FROM THE BACKGROUND

Drop shadows help make things stand out from the background. While they are good for labels on graphs, don't put a drop shadow on the plotted graph line because it detaches it from the grid.

Put 3-D shadows for 3-D vectors even if there are abstract shapes with no light source. One can more easily see a 3-D shape by simultaneously having two views of the object, a 3-D view and a projection of that view on the xy plane. This is what the cubists were trying to do—show many views of an object at once. The shadow technique is more the way we are used to seeing and interpreting things.

Make the background a different value; use pale colors.

2.7 REALISM VERSUS ABSTRACTION

Images representing some real, physical object are often overlaid with labels, vectors, etc. For such scenes, the real object is rendered with a simulated light source and shading (usually with a simple polygon rendering program). The mathematical abstractions are overlaid with a line drawing program (lines don't change thickness as they get closer or farther from viewer).

CHAPTER 3 – GRAPHICAL DESIGN (DYNAMIC)

From reading Thomas and Johnson's book (1981), you are left with the impression that animation is the highest form of human art. It encompasses all aspects of static art and adds timing and motion, too. Motion design may well be the next great research topic in computer graphics. Results shown here are very preliminary.

3.1 INTERPOLATION

It is the popular wisdom in animation that spline interpolation is better than linear interpolation. It is smoother. Most of the animations were done with splined motion. However, later in the series I began experimenting with linear interpolation and found it quite pleasing. Let's face it, the algebraic motions represent mechanical operations, so why not make them mechanical looking? In this case non-natural (jerky) motion sometimes looks more interesting than smooth motion because it's different and contains more high frequencies at the key frames.

3.2 INCORPORATION OF "CLASSIC" TECHNIQUES

There are various classic techniques that are found in "conventional" animation that apply here.

3.2.1 Squash/Stretch

Squash and stretch refer to a distortion applied to the shape of an object when it undergoes acceleration. This is easily done by animating the x and y scale factor of an object. Before it begins to move, it gathers itself up by shrinking in x, then it stretches out in x as it is moving, and when it stops it shrinks briefly and returns to its normal size. This wasn't done in the *Mechanical Universe* as much as it should have been.

3.2.2 Overlapped Motion

The concept of overlapped motion states that motion 2 should start before motion 1 is completed. This works well with character animation, but I found it of limited use in algebraic animation. In algebra there is just too much to follow as it is, without having the individual steps of a derivation merge into each other. Making the steps disjoint in time gives the viewer a chance to absorb one step before another begins. I did make the x and y motion of an object overlap, but this just rounds off the corners of the motion.

3.3 PERCEPTIONS OF SPEED

I found it interesting to discover how limited our perception of velocity is. Given two successive scenes, where an object moves, say, one and a half times as fast in the second scene, it is very hard to tell which is which. This was proven because we were showing velocity changes in a lot of the physics. Most of the solutions involved representing velocity spatially as well as temporally by adding streaks or velocity vectors to moving objects.

Another interesting speed-perception discovery concerns double framing. One would think that all animation is ideally single framed. Double framing is just an economy measure if you don't have the computer time to do all the frames. Double framing looks jerkier. But there's another perceptual effect of double framing—double-framed motion looks faster than single-framed motion.

That is, if an object moves across the screen in 1 sec, it will look like it is moving faster if it is animated as 15 frames double-framed rather than 30 frames single-framed. This was alluded to in Thomas and Johnson's book (1981) on Disney animation. They said that motion was sometimes purposely double-framed to give it a "jaunty" look.

3.3.1 Audience

When doing something of this nature, it is important to keep the audience in mind. I had a very specific audience in mind when I designed these animations before I understood the concepts. For the most part, these are the explanations that I would have liked to have had, and that would have made the most sense to me when I was learning physics.

3.3.2 Roots

We are all products of our environment. I would like to mention some previous experiences that have affected my design motions here.

Lillian Lieber and Hugh Lieber are a mathematician/artist team that produced a series of charming books in the 1940s. Hugh, the artist, had a very surreal sense of making mathematical symbology visually interesting.

Various Disney animations were produced for science and mathematics. Among these were "Man in Space" and "Donald Duck in Mathemagic Land."

George Gamow (1967) wrote several books popularizing physics. His best creation is the Mr. Thompkins series. In these books, Mr. Thompkins attends a physics lecture and falls asleep. In his dreams the physical point of the lecture is illustrated, usually by exaggerating the effects so they were more noticeable in daily life. Particularly memorable was a scene in the "Old Woodcarver's Shop" where a sculptor makes atoms out of little green marbles (electrons) and little red marbles (protons).

The "Chem-studies" series of films were made for high school use. These had several conventionally done animations of molecular dynamics during chemical reactions. The motion of the atoms in these animations beautifully gives a sense of the energetics of atomic bonding. These were produced by David Ridgeway, who is on the national advisory committee to the *Mechanical Universe.*

The Bell Labs produced science films such as *The Unchained Goddess* and *Our Mr. Sun.* These were directed by Frank Capra, a Caltech graduate, and also a member of the advisory committee to the *Mechanical Universe.*

Finally, a telecourse from the past: "Continental Classroom"; this was a for-credit course offered on television in about 1960. It had classes in mathematics, physics, and chemistry. When I was young I was interested in this stuff, but I didn't know where to go for information. When I found this course I got up religiously each morning at 6 a.m. to watch it. I understood only about half of it, but it kept my interest in the subject alive. I hope that, with the *Mechanical Universe,* I might be making a series that generates similar interest in a new generation of students.

CHAPTER 4 – VISUAL METAPHORS (DESIGN)

In this chapter I will discuss visual metaphors for physics, grouped by design concepts.

4.1 COLOR

A normal textbook diagram has shapes, lines, and text. In video we have, in addition, color and motion. The challenge is using them. Motion usage is, for the most part, more obvious than color usage. Where there is some previous convention for color assignment, I tried to use it. Where there was none, I had to invent one.

When referring to explicit color values, I will use the notation developed by Alvy Smith. Color is three numbers representing

1. Value or brightness (0...1).

2. Hue going around the color wheel. Numerical quantities go from 0-5 for one cycle: 0 = red, 1 = yellow, 2 = green, 3 = cyan, 4 = blue, 5 = magenta.

3. Saturation. 0 = neutral, 1 = fully saturated.

Written, as an expression, (i,j,k) (i.e., (1,0,1)) would be a red of maximum brightness and saturation.

Many different ideas were keyed to colors. Much of this was subtle, and the animations never relied solely on the color to be understandable. I was left with the impression, however, that there simply aren't enough colors to have a unique one for *everything*.

4.1.1 For Dimensional Analysis

When physical abstractions such as acceleration or torque are represented in vector diagrams or algebraic labels, there must be some color. Rather than just making all vectors and labels white, I chose to institute a color scheme that is keyed to the units in which the quantity is measured. These color schemes are maintained throughout the series. This provides for a sense of continuity and also gives the viewer a sense for dimensional analysis.

Also, I tried to avoid the temptation to get overly cute with the colors. Colors are used primarily for labels. Terms in equations are usually white; otherwise, the equation tends to look like confetti. A term is shown in color only if the dimensions are important for a particular derivation.

Position, velocity, and acceleration are the most commonly used quantities. Position was a green = (1, 1.8, 1); velocity was a yellow = (0.7, 1.2, 1); and acceleration was a red = (1, 0.2, 1).

There are several motivations for this general color scheme. As successive derivatives are taken, the color shows a smooth progression along the color wheel from green to red so there is a visual progression between the colors. (Actually, the reddening applies not so much to derivatives as to the division by time.)

Acceleration is the most "active" of the three concepts. But red means "stop," not a very dynamic idea (although it takes deceleration to stop). This might be a counter argument for the use of this color. But red is also the most exciting, attention-getting color. It shows that something is going on, and thus looks dynamic.

Green (as in grass) shows a static "place-like" effect.

This color scheme worked well when applied to a scene showing an abstract bicycle rider. The intent was to show elevation and slope. The normal color for informational traffic signs (green) was used to label the elevation. The normal color for warning traffic signs (yellow) then labeled the slope.

Note that the colors chosen are not pure; the hue values are not integers. The exact hues were selected visually to look nice together. Exact primary colors tend to look boring.

Mass times acceleration gives force. Mass times velocity gives momentum. Force and momentum were given the same colors as acceleration and velocity except that the saturation was reduced. I think of mass as a sort of dark grey color, looking solid, like lead or iron. So adding grey to the above colors desaturates them.

Energy is a dark blue color. This was chosen to look sort of like a lightning bolt. Energy's color is (0.2, 4, 1).

Angular momentum is a sort of rotational concept. I toyed with the idea of giving angular momentum vectors a sort of barber-pole effect, but it seemed too busy. Angular momentum is also mass times velocity times distance. Maybe a sort of pale yellowish-green? But that would not make it distinguishable enough from the other two. Finally, I decided to take off in a new direction and make it a pale blue. Torque, the derivative of angular momentum, is lavender (blue with red added to it).

Area and volume were made variants on the green color. Area is a slightly bluer shade. Volume is a still bluer shade. Maybe I was getting too subtle here, but you have to pick *some* color, and it might as well be for *some* reason.

Actually this choice was not entirely conscious, and as a result, the color for area is not exactly consistent through the entire series. For example, the color of Gaussian surfaces in the electricity programs was the position color, not the area color. This led to some problems when showing surface integrals. You do your best, but sometimes mistakes creep in.

4.1.2 Solid, Liquid, Gas

In the thermodynamics discussion there is a section on the states of matter. In particular, a PVT diagram is separated into regions where a substance is a solid, a liquid, and a gas. These regions were colored as follows.

Solid – medium brown; an earth color, designates the solidity of ground.

Liquid – bluish; like the color of water.

Gas – white; a transparent color.

In the PVT diagram there is a region above the critical point where the distinction between liquid and gas disappears. Van der Waals' equation was used to find the degree of liquidity and to calculate a saturation value smoothly grading from blue to white for this region.

4.1.3 Electric Charge

Positive and negative charges are shown in many scenes. There has been a sort of convention for some time in engineering to make the positive leads red. In addition, the books by George Gamow represented electrons as green marbles. So a similar color scheme was chosen for the *Mechanical Universe.*

But there are two problems here. First, not everyone has a color television set. So the colors were chosen so that, in black and white, they would still have enough difference in brightness to be distinguishable. Second, although red and green are complementary colors visually, in video it is red and cyan (a sort of pale blue). In some instances a neutral charge (e.g., for neutrons) is shown as, obviously, white. It would seem best to make the plus and minus colors add up to white. So a more bluish hue was chosen for negative charge. The exact value was actually changed during the second half of the series to be exactly cyan. This seemed necessary to make plus and minus add up to neutral, but I'm not sure it was a good idea in retrospect.

4.1.4 Electric and Magnetic Fields

I've always thought of magnetic fields as blue, and many published diagrams have shown it as blue. In fact, in an earlier project showing the magnetic field of Jupiter, I made the field lines blue. The question is, what color are electric fields? Since they are lines between positive (red) and negative (greenish blue), I decided to make it the color halfway between them, yellow. Note again that this is a different yellow than is used for velocity.

4.1.5 Relativity Coordinate Systems

There were many scenes in the relativity section that illustrated events as seen from two different reference frames. The two frames were usually those of a cartoon Albert Einstein and a cartoon Henry Lorentz. When they first appear, Albert is wearing a tan suit and Henry is wearing a blue suit. Thereafter, any algebraic or pictorial reference to Albert's frame is drawn in tan and any reference to Henry's frame is blue. These colors were initially selected as typical colors that suits come in, but they were fine-tuned to show up distinctly in black and white and when placed on a common background. Actually, when I first decided to do this, I had made Henry's suit dark grey. But dark grey didn't look good as a comparison color to tan—tan and blue are more balanced complementary colors. I had to remake one of the first animations just to change the color of Henry's suit. The production people probably thought I was nuts.

4.1.6 Wave/Particle Duality

The last three programs of the series begin to touch on quantum mechanics. Several of the scenes depicted wave-particle duality. Complementary background colors were selected to represent particles and waves. All particle equations appeared over dark pale green; all wave equations and plots of wave functions appeared with a dark pale magenta background.

4.2 LITERAL VERSUS SCHEMATIC

My tendency is to be too literal. The sizes and timings of some phenomena sometimes have too big a range to make this easy. But, because this is computer animation, the viewer expects precision and accuracy. When sizes or timings must be distorted into schematic diagrams, it is important to give some visual cues that this is being done. One way to do this is to have the schematic scenes drawn with sketchy or irregular lines. This removes the precision effect of perfect lines.

4.2.1 Literal

Some things were done geometrically correctly, even though it was difficult. For example, the radii of the orbits of the Bohr atom are proportional to the perfect squares (1, 4, 9, 16, ...). To see as many as four orbits, the scale must be too small to make the first orbit clear. This was usually solved by having the camera pull back when discussing the larger and larger orbits. This is a useful general principle, as it was described in an earlier chapter concerning a list receding in perspective. If some things are too small, start close up and pull back.

4.2.2 Schematic

When force laws are introduced, we needed to show the operation of gravitational and electric forces. At this point, the magnitudes weren't important, only the signs. Crude schematic faces were used as mass particles (grey faces) and as positive and negative charges (red and cyan faces). The motion was sketchy, showing only attraction versus repulsion, and the faces were sketchy, with irregular and comical lines. This visual signaling was not done enough in the series.

Other scenes with schematized motion included:

- A depiction of resistance in metals. The normal velocity of electrons in a metal is far greater than the drift velocity, which is the electric current. Therefore, an accurate depiction of current wouldn't look much different than random thermal motion. The relative velocities were made more equal for illustration purposes. Also, resistance is caused by collisions of electrons with imperfections and thermal motions of the atoms in the metal lattice. These are usually too few and far between to be easily noticeable. They were made more obvious by flagging some metal atoms a different color and having the electrons bounce off them elastically, while not being affected by the positions of all of the nonflagged atoms.

- An electrical spark is generated by a chain reaction. Electrons are accelerated by an electric field and build up enough kinetic energy to knock other electrons off atoms. Again the typical spacing and frequency of the real situation would not fit on the screen. Some exaggeration was done.

CHAPTER 5 – VISUAL METAPHORS (PHYSICS)

Here are some more visual metaphors, this time grouped by subject matter, rather than by design issues.

5.1 ALGEBRAIC BALLET

To make the science respectable we had a lot of algebra to present. Algebra, however, can be a bit draggy. We decided to liven it up by animating the algebraic transformations that the equations go through. These animations usually go by quickly. In fact, it is unlikely that the viewer will be able to follow all the steps upon first viewing. The speed was a concern, but we felt that making it slower would slow down the programs too much. The idea is to get the feel for what is going on and be able to look at a videotape slower to get the detail later if desired.

Transforming algebraic operation into motion proved to be an interesting exercise. Many of the motions seemed pretty obvious to me, but they are listed here for completeness.

5.1.1 Term Labeling

It's easy to lose track of what different symbols in an equation represent. This was addressed by having the symbols identify themselves with English words popping out and shrinking back into them.

5.1.2 Balancing Act

Simple algebraic operations to move terms around were animated literally.

- Terms moving to the opposite side of the = sign. Adding on one side means subtracting on the other so a + or - sign flips its identity as the term hops over the =.

- Factors moving to the opposite side of the = sign. Multiplying on one side means dividing on the other. When a factor jumps over the =, it lands below or above a division bar according to whether it came from above or below.

- Distribution: $a(b + c)$ becomes $ab + ac$ by having the a jump up, split in two, and each copy land next to the appropriate term.

- Squaring: Either two 2s come down from above and land on each side of the =, or a 2 on one side of an = sails over and changes to a $\sqrt{}$ sign on the other side.

5.1.3 Canceling

This applies to the removal of identities like $a - a$ or a/a. Some ways used to depict this were:

- A lightning bolt zaps the two terms and they disappear.

- An eraser appears and erases the terms.

- The two terms turn red and fall off the bottom of the screen together.

- A video-game-style spaceship flies in and fires a missile to explode the term.

- A Monty Python-style foot stomps out the terms.

- The Hand of God touches the term and it becomes a puff of smoke. This was used in the program that derived Kepler's first law (orbits are ellipses) from Newton's laws. The program made comparisons between the accomplishments of mathematics and physics and the accomplishments of art, drama, and music. Art was represented by the Sistine Chapel of Michaelangelo with the Hand of God giving life to Adam. The essential cancellation in the math that makes the derivation work is r^2/r^2; this is done by the Hand of God, too.

- Multiplication sign snipping out a term: The expression **v x v** is equal to 1. When this appears, the cross product sign magnifies around the surrounding **v**'s and then squashes rapidly in y, snipping out the terms.

- Simply fading the terms out: This, of course, was the simplest and was done the most often.

5.1.4 Recalling Old Results

When a result from a previous program, or from a previous course is introduced, some effort was made to indicate to the viewer where it came from. Some examples are:

- A trigonometry book flies in, opens, and trig identities fly out.
- A head with a hinged lid opens to receive some intermediate results; later it returns and the intermediate results fly out.
- A hand pulls down a window shade with old energy equations.
- An entire scene is reprised from a previous program.
- Some results were derived against a background image of some distinctive color. Later, when the results are needed, a slide comes in containing the equation with the same background as old scene.

5.1.5 Substitution

Substitution involves taking an equation defining some variable and replacing occurrences of that variable into another equation. Some examples:

- Vertical shrinking. A term is replaced with a number by shrinking the term vertically to zero and having the number expand up from zero in its place.
- Vacuum cleaner. The identity equation appears above the main equation. The replaced term from the lower equation moves up to the identity to merge with its copy there. The other side of the identity equation moves down to the empty spot left in the original equation.
- Several calculus identities (such as turning dr/dt into v) were shown by rotating the dr/dt about the y axis and having it become v when the other side appeared.

5.1.6 Jokes

The program on wave motion shows some approximate relations between wave speed and various physical parameters. The $\approx$ sign ripples like a propagating sine wave while these equations appear. This was done by modeling the lines of the $\approx$ sign with a one-cycle helix. Rotating it about x and then scaling by 0 in z made it ripple.

5.1.7 Calculus

A few algebraic operations on calculus notation:

- *d/dt* flies in from left and impacts f to form *df/dt*.
- The f slides up and down to form $(d/dt)\, f$ from *df/dt*.
- The two symbols $\int$ and *dt* move in on either side of f and clamp it together to form $\int f(x)dt$.
- A simple differential equation like $(dx/dt) = y$ is solved by moving *dt* to the other side to make $dx = y\, dt$. Then the left-hand d hops over the equal sign and changes into a $\int$ sign, to make $x = \int y\, dt$.
- Integration is done by the $\int$ sign ratcheting across an expression, sort of like a credit card imprinter.
- $\oint$ is formed by drawing the circle on the $\int$ as the path of integration is traced out in a geometric diagram in the background.
- $\oint\!\!\oint$ is formed by revealing the circle on $\iint$ as a Gaussian surface is spread out around a volume in a parallel diagram.

5.2 CALCULUS

5.2.1 Limits

Use explosion to express the limiting process when Δ turns into d. The explosion was generated by a simple 2-D pattern scaled up and faded out simultaneously.

5.2.2 Symbolic Derivative Machine

Because we evaluate derivatives and integrals symbolically many times in the series, we developed a quick way to do it—the derivative machine.

5.2.2.1 Design– The derivative machine is an expression transformer. It has two functions—differentiation and integration. An expression goes in one end and comes out the other end, so it needed to be thin in the x direction so there would be plenty of room on each side to show the inputs/outputs. When the derivative machine is first introduced, it comes in a crate marked "ACME Derivative Machine" (a hat tip to the old Chuck Jones Roadrunner movies). A crowbar shaped like an integral sign opens the crate.

Some random wheels and lights made it look Rube Goldbergish. The sides are not exactly straight and the wheels are not exactly round.

5.2.2.2 Internals– When the derivative machine is introduced in program 3, the internals are shown two ways:

1. As various elementary operations are introduced, they shrink down into a sort of circuit board that is plugged into the machine, the door slams, and a new light blinks on on the front panel.

2. An alternative view of the internals was given briefly, showing the details of how the elementary operations are applied to take the derivative of the simple expression x^2. This was intended to be somewhat a metaphor on how symbolic derivative computer programs work. The input function comes in on a conveyor belt. An eyeball on a stalk comes down and looks at it. (This is indicated by a dotted line running from the eyeball to the function.) This is the pattern recognizer. The derivative operation is basically one of matching the desired function against a list of known patterns which are pulled down into the scene like window shades. Then the proper pattern is found and checked. There will be some dummy parameters in the pattern which need to be filled in with the specific terms from the equation. The eyeball observes these and some handles come down and simultaneously turn all occurrences of the dummy parameter into the specific term needed. Identities such as $x + 0$ or $x * 1$ are removed by an eraser. The expression $x + x$ is turned into $2x$ by a vise-like adder. The final expression is carried out on a conveyor belt.

5.2.2.3 Operation– The lever on the top controls the operation of the derivative machine. When you throw the lever to the right, it takes an expression in the left hopper and spits the derivative out the right hopper. When you throw the lever to the left it takes an expression in the right hopper and spits out the antiderivative (integral) on the left. Sometimes the expression stays put and the derivative machine passes over it. Note: it doesn't evaluate integral expressions, it just takes the antiderivative (i.e., you don't feed $\int x^2$ in to get $(1/3)x^3$, you just feed in x^2). As it operates, the horizontal and vertical scales cycle up and down a bit to give it a squash and stretch look.

REFERENCES

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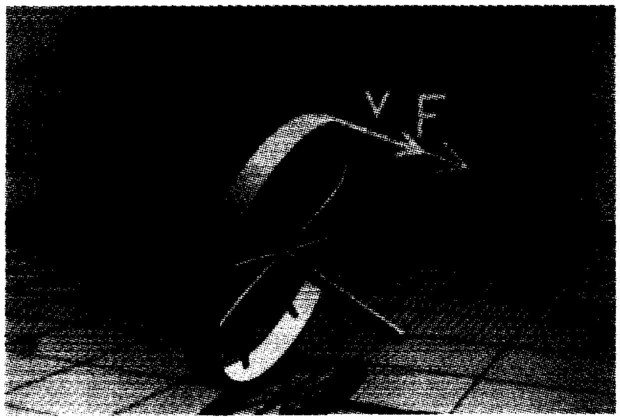
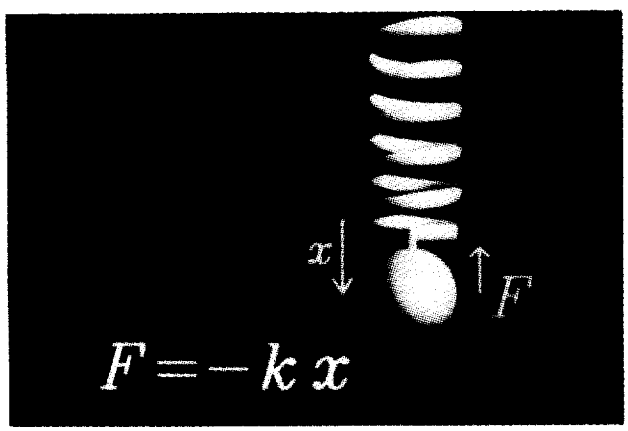
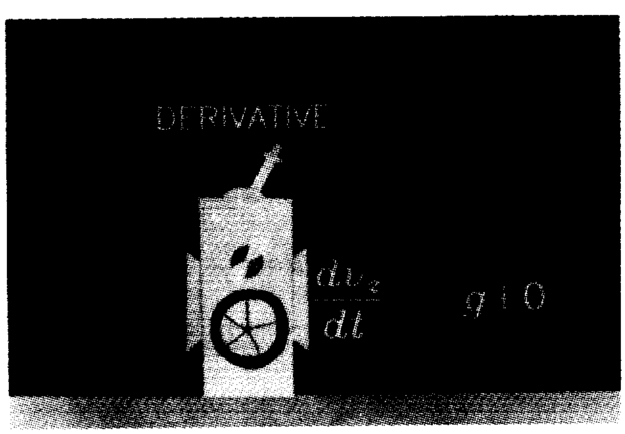


Computer animation dissects the forces and motions that make a gyroscope do its tricks.



The spring force, or Hooke's law, is described in this animated scene from the Harmonic Motion episode.



The *Mechanical Universe* derivative machine has become a legend in its own time.