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DETERMINATION OF JOINT DRIVES FOR STABLE END-EFFECTOR MOTION IN FLEXIBLE ROBOTIC SYSTEMS

SITKI KEMAL IDER
Assistant Professor
Department of Mechanical Engineering
Middle East Technical University, Ankara, Turkey

ABSTRACT

The prescribed tasks in high speed robotic systems are severely deteriorated because of their manipulator dynamic deflections. On the other hand conventional dynamic modeling techniques fail to reveal appopriate control forces in flexible systems. In this paper the conventional dynamic equations of motion for systems subject to kinematical constraints are modified by a new concept of control force representation. The directions of the control forces are selected such that they correspond to the joint degrees of freedom. Then the joint control forces and torques that yield unperturbed prescribed motions are solved simultaneously with the system motion. A flexible manipulator is presented to illustrate the methods proposed.

1. INTRODUCTION

The operation of high speed robots is severely limited by their manipulator dynamic deflection. The vibrations deteriorate the accuracies of the prescribed tasks assigned to certain points and significantly reduce the robot arm production rate. Hence determination of the appropriate control forces and torques at the joints that yield stable prescribed motions is an important control problem.

In this paper geometrical constraints represent geometrical restrictions such as closed loops and physical guides. On the other hand kinematical constraints represent prescribed desired paths or prescribed motions of certain points or bodies. Such prescribed motions are to be realized by control forces applied by the actuators in the system which are usually placed at the joints.

In the conventional approach, constraints in the system are modeled by constraint reaction forces whose directions are perpendicular to the constraint surfaces. (See Yoo and Haug [1], Shabana [2], Kamman and Huston [3], Hemami and Weimer [4], Nikravesh [5].) Using conventional methods, when the prescribed motions are treated as constraint equations and embedded into the governing equations of motion, the corresponding generalized constraint reaction forces can be determined. However these forces cannot be utilized as physically possible control forces due to the presence of components that correspond to the elastic coordinates. For this reason previous solution procedures involved feedback and adaptive control algorithms which in turn increase the complexity of the problem considerably.

In this paper the kinematical constraints are modeled by general direction control forces. The modified equations of motion for flexible multibody and robotic systems subject to geometrical and kinematical constraints are developed. The directions of the control forces are selected such that they correspond to the joint degrees of freedom. By this way joint control forces and torques that achieve the unperturbed prescribed motions are solved simultaneously with the corresponding system motion. The modeling of flexible multibody systems in joint space based on finite element method and component mode synthesis, as developed in references Ider [6], Ider and Amirouche [7] is also outlined. In the equations of motion all nonlinear interactions between the rigid body and elastic coordinates are automatically incorporated.

This paper is divided into seven sections. The first section provided an introduction. In the second section kinematics and constraint equations in flexible multibody systems are outlined. The conventional equations of motion for constrained systems are presented in the third section. In section four the problems with the conventional approach are discussed. In the fifth section the modified equations of motion with general direction control forces are developed. Sixth section presents the simulations of a flexible manipulator by the proposed method. Conclusions form the last section.

2. FLEXIBLE MULTIBODY KINEMATICS AND CONSTRAINT EQUATIONS

In a multibody system each joint connection can be described by a total of six degrees of freedom. The constrained joint coordinates are eliminated in the analysis, hence all possible joint types are allowed. The system may contain closed loops and any selected points may have prescribed motions. First the recursive dynamical equations are developed for a tree configuration which is obtained by cutting the closed loops open (using any arbitrary joint in the loop). Closed loops and prescribed motions are then imposed as a set of constraint equations.

In Figure 1, a typical deformable body B_k and its lower connecting body B_j are shown. The joint between B_k and B_j is the lower joint of B_k and is defined by points Q_k and Q_k^* and axis frames n^k and n^{k*} fixed at these points. The elastic deformations are modeled by finite element method with respect to a body reference axis frame denoted by N^k. N^k, in general, is not fixed to a point on the body. It follows the rigid body motion of B_k in a manner consistent with the specified boundary conditions [6,7].

The position of the system can be described by the relative joint coordinates of each body and the modal coordinates of the flexible bodies. Translation of n^k with respect to n^{k*} is denoted by vector z^k. For the relative rigid body rotation degrees of freedom, successive Euler angles in transforming n^k to n^{k*} can be used. The modal coordinates η_j^k, j=1,..,m^k

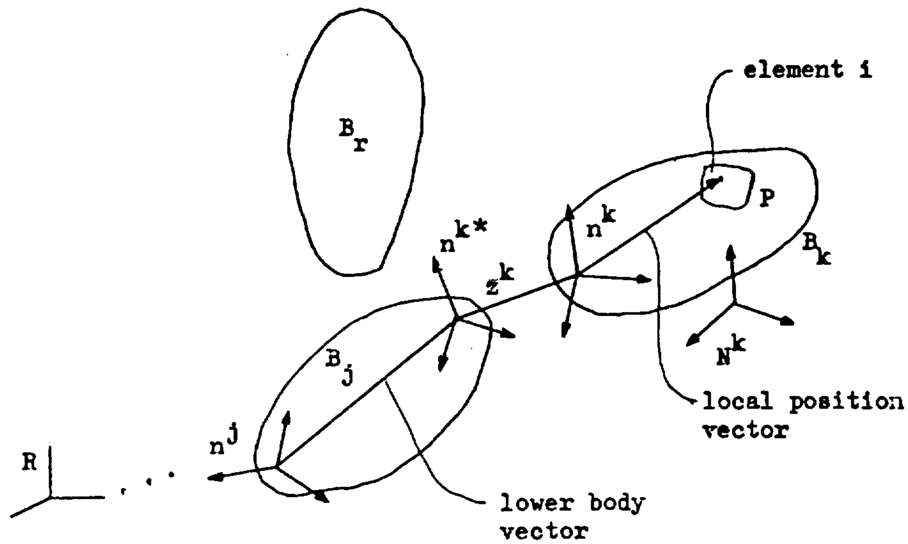

Figure 1. A multibody system

of each body B_k represent the normal modes of deformation obtained by component mode synthesis and m^k is the number of the modes considered.

Let vector $\hat{w}^k$ represent the angular velocity of n^k with respect to n^{k*}. Then the generalized speeds of the system can be conveniently selected as the relative angular velocity components, the relative translational velocity components and the modal coordinate derivatives. The vector of the system generalized speeds y can be defined as

$$y = [\hat{w}^T, \dot{z}^T, \dot{\eta}^T]^T \tag{1}$$

where

$$\hat{w} = [\hat{w}_1^1, \hat{w}_2^1, \hat{w}_3^1, \ldots, \hat{w}_1^N, \hat{w}_2^N, \hat{w}_3^N]^T \tag{2}$$

$$\dot{z} = [\dot{z}_1^1, \dot{z}_2^1, \dot{z}_3^1, \ldots, \dot{z}_1^N, \dot{z}_2^N, \dot{z}_3^N]^T \tag{3}$$

and

$$\dot{\eta} = [\dot{\eta}_1^1, \ldots, \dot{\eta}_{m^1}^1, \ldots, \dot{\eta}_1^N, \ldots, \dot{\eta}_{m^N}^N]^T \tag{4}$$

For the dynamical equations we need the velocity, in fixed frame R, of an arbitrary point P in finite element i of body B_k. To this end, the angular velocity of N^k in R, w^k, is obtained by summing successive relative angular velocites and can be compactly expressed as

$$w^k = \nu^k \hat{w} + \mu^k \dot{\eta} \tag{5}$$

where ν^k and μ^k are the partial angular velocity matrices composed of the coeffficients of the generalized speeds $\hat{w}$ and $\dot{\eta}$, respectively.

It can be shown [6,7] that the velocity of point P in R could be written as

$$v^{ki} = a^{ki} \hat{w} + b^k \dot{z} + c^{ki} \dot{\eta} \tag{6}$$

where a^{ki}, b^k and c^{ki} are the partial velocity arrays associated with $\hat{w}$, $\dot{z}$ and $\dot{\eta}$, and are functions of displacements only.

Depending on each joint type, the constrained joint coordinates are then eliminated in the generalized speed vectors $\hat{w}$ and $\dot{z}$ (equations (2), (3)). When the corresponding columns of the partial velocity matrices are eliminated, ν^k and a^{ki} are $3xn_1$ matrices, and b^k is a $3xn_2$ matrix, where n_1 is the total number of the free joint rotation degrees of freedom and n_2 is the total number of the free joint translation degrees of freedom. The remaining arrays μ^k and c^{ki} are 3xm ($m=m^1+...+m^N$).

Let the tree structure have n degrees of freedom ($n=n_1 + n_2 + m$), and let the total number of closed loop and prescribed motion type of constraints be c. Then the system's degrees of freedom reduce to n-c.

The constraint equations can be generated using the partial velocity matrices. For example, if a point say A in B_r has a prescribed motion, and the prescribed velocity vector of that point is given by g(t) with respect to R, then denoting the local undeformed vector from O_r to A by s^r,the resulting three constraint equations are

$$a^{ri} \hat{w} + b^r \dot{z} + c^{ri} \dot{\eta} = g \tag{7}$$

where a^{ri} and c^{ri} correspond to s^r.

Similarly if the refence axis frame of B_r has a prescribed angular velocity h(t), we have

$$\nu^r \hat{w} + \mu^r \dot{\eta} = h \tag{8}$$

For a closed loop type of constraint, let B_r and B_s connect with each other to form a closed loop in 3-D. Differentiation of the position vector equation expressing loop closure leads to the three velocity level constraint equations,

$$(a^{ri}-a^{si}) \hat{w} + (b^r-b^s) \dot{z} + (c^{ri}-c^{si}) \dot{\eta} = 0 \tag{9}$$

The holonomic and nonholonomic constraint equations can be compactly written as

$$B\ y = g \tag{10}$$

where B is a cxn constraint matrix and g contains prescribed velocities.

3. CONSTRAINT REACTION FORCES AND EQUATIONS OF MOTION

Kane's equations for the constrained system can be written as

$$F + F^* + S + F^c = 0 \tag{11}$$

where F, F^*, S and F^c are respectively the vectors of generalized external, inertia, stiffness and constraint forces.

The generalized inertia forces F^* can be written in the following form,

$$F^* = M\ \dot{y} + Q \tag{12}$$

where individual submatrices of M and Q can be expressed in terms of the partial velocity matrices [7] as,

$$M = \sum_k \sum_i \int_{V_{ki}} \begin{bmatrix} a^{ki^T} a^{ki} & & \text{sym.} \\ a^{ki^T} b^k & b^{k^T} b^k & \\ a^{ki^T} c^{ki} & b^{k^T} c^{ki} & c^{ki^T} c^{ki} \end{bmatrix} \rho\, dV \tag{13}$$

and

$$Q = \sum_k \sum_i \int_{V_{ki}} \begin{bmatrix} a^{ki^T}(a^{ki}\ \hat{\dot{w}} + b^k\ \dot{z} + c^{ki}\dot{\eta}\ ) \\ a^{ki^T}(a^{ki}\ \hat{\dot{w}} + b^k\ \dot{z} + c^{ki}\dot{\eta}\ ) \\ a^{ki^T}(a^{ki}\ \hat{\dot{w}} + b^k\ \dot{z} + c^{ki}\dot{\eta}\ ) \end{bmatrix} \rho\, dV \tag{14}$$

The stiffness vector S is obtained from the structural and geometrical stiffness matrices of each body expressed in modal coordinates.

The generalized constraint forces F^c can be expressed as

$$F^c = B^T\ \lambda \tag{15}$$

where λ is the vector of undetermined multipliers. Since the rows of B are the partial velocity vectors, λ_i, i=1,...,c represent the constraint reaction forces generated at the application of the constraints. A row of

B can also be viewed as the direction of that constraint in the generalized space.

Substitution of equations (12) and (15) into eq. (11) leads to

$$M\ y + S + Q + B^T \lambda = F \tag{16}$$

Our purpose is to find the accelerations for numerical integration. To this end the constraint equations in the acceleration level are

$$B\ y = \dot{g} - \dot{B}\ y \tag{17}$$

Equations (16) and (17) constitute n+c equations from which the accelerations and the undetermined multipliers can be obtained.

The multipliers could be eliminated for computational efficiency. To this end, let C denote a nx(n-c) matrix which is orthogonal complement to B [8]. Premultiplying eq. (16) by C^T, and combining the resulting equation with eq. (17), we obtain the augmented equations for the constrained system as below.

$$\begin{bmatrix} C^T\ M \\ \hline B \end{bmatrix} \dot{y} = \begin{bmatrix} C^T(F-S-Q) \\ \hline \dot{g}-\dot{B}y \end{bmatrix} \tag{18}$$

4. PROBLEMS WITH THE CONVENTIONAL APPROACH

In the conventional approach the constraints in the system are modeled by constraint reaction forces which are perpendicular to constraint surfaces. They represent the reactions of the environment. However kinematical constraints represent desired motions and are meant to be realized by internal control forces. Kinematical constraints are particularly important in robotic systems where certain points are assigned specific tasks that should be realized by joint actuators.

Let c_1 of the constraints in the system be geometric and the remaining c_2 ($c_2=c-c_1$) be kinematical. The matrix of the constraint force directions B and the vector of constraint force magnitudes λ can be partitioned such that

$$B = [B^{G^T} \quad B^{K^T}]^T \tag{19}$$

and

$$\lambda = [\lambda^{G^T} \quad \lambda^{K^T}] \tag{20}$$

where the dimensions of B^G, B^K, λ^G and λ^K are $c_1$xn, $c_2$xn, c_1 and c_2 respectively.

Then eq. (16) can be written in the following form

$$M\,\dot{y} + Q + S + F^G + F^K = F \qquad (21)$$

where the generalized constraint forces F^G and F^K corresponding respectively to geometrical and kinematical constraints are

$$F^G = B^{G^T}\,\lambda^G \qquad (22)$$

and

$$F^K = B^{K^T}\,\lambda^K \qquad (23)$$

The constrained system as given by eq. (18) could be simulated to determine the generalized constraint forces. In view of eq. (21), then control forces numerically equal to F^K would yield the same motion of the system ensuring the realization of the desired motions.

However, in flexible systems F^K contains components that correspond to the elastic coordinates in addition to the components that correspond to the joint coordinates. While the latter can be applied by the joint actuators as control forces, the former cannot be produced by a physical means as control forces. That F^K has components in the direction of the elastic coordinates is apparent from eqs. (7) and (8) where the coefficients of the generalized speeds form the vectors of B^K in eq. (23). Hence, with the conventional approach it is not possible to design a set of control forces that can achieve unperturbed prescribed motions in flexible robotic and multibody systems.

5. CONTROL FORCES FOR KINEMATICAL CONSTRAINTS AND MODIFIED EQUATION OF MOTION

Since kinematical constraints are to be realized by control forces in the system, general direction control forces are introduced to the equations of motion, so that

$$M\,\dot{y} + Q + S + B^G\,\lambda^{G^T} + B^{K^T}\,\lambda^K + A^T\mu = F \qquad (24)$$

where A is a c_2xn matrix of control force directions and μ is a c_2 dimensional vector of control force magnitudes. Let the control force directions are selected such that the constraint reaction forces corresponding to the kinematical constraints become zero. Then eq. (24) can be written as follows, as shown in accompanying paper (Ider [9]).

$$M\,\dot{y} + Q + S + Z^T\nu = F \qquad (25)$$

where

$$Z^T = [B^{G^T} \quad A^T] \qquad (26)$$

and

$$\nu^T = [\lambda^{G^T} \quad \mu^T] \tag{27}$$

The directions A need to be selected from physical considerations and then equations (25) and (17) can be solved together to compute the control force magnitudes and the corresponding generalized accelerations. A should be chosen such that rank of Z is c, so that c_2 kinematical conditions could be controlled.

Let $\bar{C}$ be a cx(n-c) matrix orthogonal complement to Z. Premultiplying eq. (25) by $\bar{C}^T$ and augmenting with eq. (17), we obtain reduced equations

$$\begin{bmatrix} \bar{C}^T M \\ \hline B \end{bmatrix} \dot{y} = \begin{bmatrix} \bar{C}^T(F-S-Q) \\ \hline \dot{g}-\dot{B}y \end{bmatrix} \tag{28}$$

The selected control force directions can realize the prescribed motions if and only if the augmented mass matrix in eq. (28) is non singular. Hence singularity of the augmented mass matrix represents a condition to test the solution. In other words, the directions A should be such that the vector space spanned by the rows of B and the vector space spanned by the rows of $\bar{C}^T$ are nonintersecting [9].

It has been observed that for flexible robotic systems if the control forces are selected in the directions along the corresponding joint degrees of freedom the augmented mass matrix becomes full rank and hence it is possible to realize the kinematical constraints by actuators at the joints. This will be illustrated by the simulations of a flexible manipulator in the next section.

6. SIMULATIONS OF A FLEXIBLE MANIPULATOR

In the planar manipulator shown in Figure 2, link 2 is a flexible link, while link 1 is treated rigid. The data used for link 1 are L_1=1m, m_1=30kg and I_1=10 kg.m^2. Link 2 is modeled by beam elements with deformation displacement and rotation nodal coordinates [10], and L_2=2.7m, a=1.8m, m_2=15kg, E=68.95x10^9 N/m^2 and area A=0.0005 m^2. The longitudinal deformation is neglected due to the axial stiffness and the transverse deflection is described by the first two modes since higher modes were observed to be negligible. Therefore the generalized coordinates of the system are θ_1, θ_2, and modal coordinates η_1 and η_2. The generalized speed vector

$$y = [\dot{\theta}_1, \ \dot{\theta}_2, \dot{\eta}_1, \dot{\eta}_2]^T.$$

Initially the system is at rest, and θ_1=80° and θ_2=-160°. Point A on link 2 is required to deploy from the given position 1.5m horizontally. The prescribed motion of point A is given as

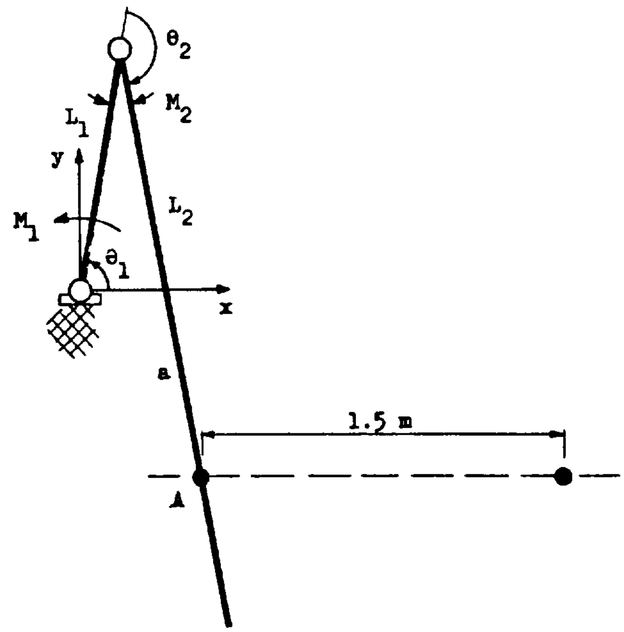

Figure 2. Flexible manipulator

$$x_A = 1.5\,\frac{1}{T}\left(t - \frac{T}{2\pi}\sin\frac{2\pi t}{T}\right) + 0.4862$$
$$y_A = -0.7878 \tag{29}$$

The constraint equations in the system can be expressed as

$$L_1c_1 + L_2c_{12} - s_{12}(\phi_1\eta_1 + \phi_2\eta_2) = x_A$$
$$L_1s_1 + L_2s_{12} + c_{12}(\phi_1\eta_1 + \phi_2\eta_2) = y_A \tag{30}$$

where ϕ_1 and ϕ_2 are the values that correspond to the location of point A in the first and second eigenvectors respectively. $c_1=\cos\theta_1$, $c_{12}=\cos(\theta_1+\theta_2)$, $s_1=\sin\theta_1$ and $s_{12}=\sin(\theta_1+\theta_2)$.
At the acceleration level the constraints are given by eq. (17) where B and g are

$$B = \begin{bmatrix} L_1s_1+L_2s_{12}+c_{12}(\phi_1\eta_1+\phi_2\eta_2) & L_2s_{12}+c_{12}(\phi_1\eta_1+\phi_2\eta_2) & \phi_1s_{12} & \phi_2s_{12} \\ L_1c_1+L_2c_{12}-s_{12}(\phi_1\eta_1+\phi_2\eta_2) & L_2c_{12}-s_{12}(\phi_1\eta_1+\phi_2\eta_2) & \phi_1c_{12} & \phi_2c_{12} \end{bmatrix}$$

and

$$g = [\dot{x}_A\ ,\ \ 0]^T. \tag{31}$$

First the system is simulated using the conventional method. Notice that both constraints in the system are kinematical. Our objective is to determine joint moments M_1 and M_2 that would produce unperturbed desired motion of point A. The generalized constraint forces in eq. (15) are

$$\begin{bmatrix} F_1^c \\ F_2^c \\ F_3^c \\ F_4^c \end{bmatrix} = \begin{bmatrix} \lambda_1\{L_1s_1+L_2s_{12}+c_{12}(\Phi_1\eta_1+\Phi_2\eta_2)]\}+\lambda_2\{L_1c_1+L_2c_{12}-s_{12}(\Phi_1\eta_1+\Phi_2\eta_2)\} \\ \lambda_1\{L_2s_{12}+c_{12}(\Phi_1\eta_1+\Phi_2\eta_2)\} + \lambda_2\{L_2c_{12}-s_{12}(\Phi_1\eta_1+\Phi_2\eta_2)\} \\ \lambda_1\Phi_1s_{12} + \lambda_2\Phi_1c_{12} \\ \lambda_1\Phi_2s_{12} + \lambda_2\Phi_2c_{12} \end{bmatrix} \tag{32}$$

The system is simulated for the deployment motion period T=1sec. The generalized constraint forces obtained are plotted in Figure 3. Notice that if one considers F_1^c and F_2^c as joint control moments, F_3^c and F_4^c will be left unaccounted. They cannot be converted to any set of physically applicable control forces or moments, and a simulation only with F_1 and F_2 as control moments M_1 and M_2 produces perturbations for point A.

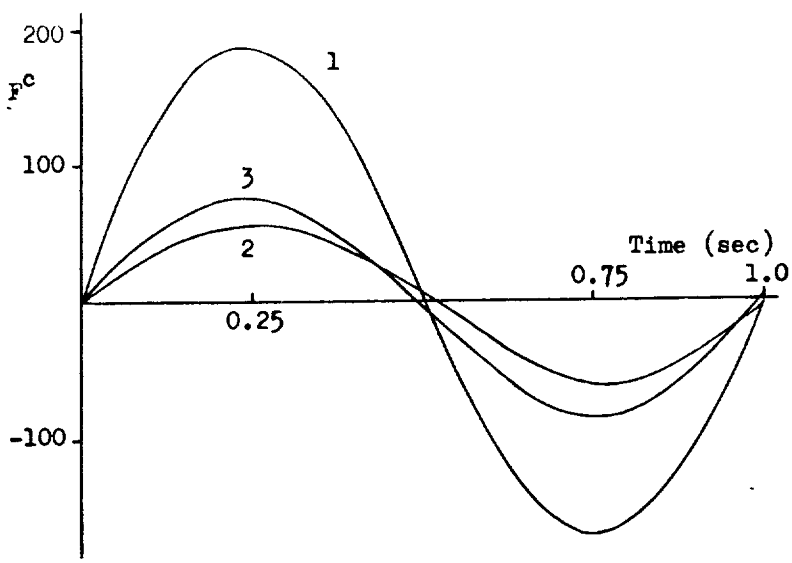

Figure 3. Generalized constraint forces using conventional method: 1. F_1^c , 2. F_2^c , 3. F_3^c , F_4^c

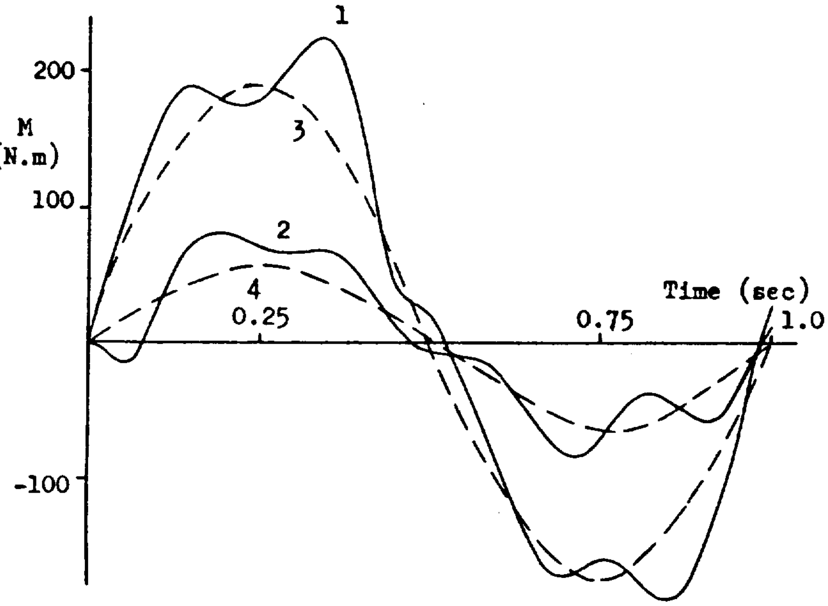

Figure 4. Joint moments for unperturbed motion of A.
Flexible system: 1. M_1 , 2. M_2
Rigid system: 3. M_1 , 4. M_2

The system is then resimulated by the control force approach presented in this paper. The control force directions are selected such that they correspond to the joint coordinates θ_1 and θ_2, i.e.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{33}$$

The control forces $Z^T\nu$ become

$$Z^T\boldsymbol{\nu} = \nu_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \nu_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

This means that the required joint moments for unperturbed motion of point A are $M_1=\nu_1$ and $M_2=\nu_2$.

With the above control force directions, the augmented mass matrix was observed to be full rank, i.e. singularity (or near singularity) did not occur, conforming with physical expectations. The joint control moments M_1 and M_2 that produce unperturbed motion of point A are plotted in Figure 4.

For comparison, the system is resimulated with both bodies considered rigid, and the joint moments corresponding to the rigid system are also shown in Figure 4. The difference in the control moments for the flexible system accounts for the effects of the elastic deformations.

7. CONCLUSIONS

This paper presented a general procedure to determine the joint control forces and torques in flexible robotic systems, that realize prescribed motions in an unperturbed manner. The method is based on a new approach for modeling kinematical constraints by general direction control forces. The control forces have been selected along the directions of the joint degrees of freedom in the generalized space, and the control force magnitudes are solved simultaneously with the corresponding system motion.

It has been shown that with the conventional approach of perpendicular constraint forces a solution to the problem cannot be obtained.

In the analysis the flexible bodies have been modeled by finite element method and all interactions between the rigid and elastic motion have been included. By the procedures presented in this paper the body flexibilities can be controlled by applying forces and torques at the joints.

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