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Implementation of Generalized Optimality Criteria in a Multidisciplinary Environment

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INTRODUCTION

The aerospace industry has begun to incorporate optimization methods into their design procedures in recent years. The Automated Structural Optimization System (ASTROS),[1] is an example of an automated multidisciplinary tool to assist in the preliminary design or modification of aircraft and spacecraft structures. The Air Force has distributed ASTROS to more than 90 organizations in the aerospace community in the last 18 months. The philosophy behind this system is to integrate proven and reliable analysis methods with numerical optimization using modern executive system and database management concepts (Fig 1). The engineering disciplines include structural analysis, aerodynamic loads, aeroelasticity, control response, and structural optimization.

The structural analysis, based on and highly compatible with NASTRAN,[2] uses the finite element method to calculate: deflections and stresses from static, thermal, or gravity loads; normal modes; and transient or frequency response due to time dependent loads including gust loads. The air loads module, an advanced paneling method based on USSAERO-C,[3] calculates flexible loads and determines a trimmed configuration. The aeroelastic module employs the Doublet Lattice method[4] for subsonic unsteady aerodynamics and the Constant Pressure method[5] for the supersonic regime. Flutter solutions are found using the PK method. When response quantities in any of these disciplines are constrained, the sensitivity analysis calculates analytic derivatives for each active constraint. These derivatives are fed to the optimization module which employs the Automated Design Synthesis program[6] to minimize structural weight. The final design's dynamic response in the presence of a given control system can be simulated by the control response module. The ability to simultaneously consider multiple boundary conditions, flight conditions, store loadings, and disciplines uniquely qualify ASTROS for structural design in a production environment.

The structural optimization methodology in ASTROS utilizes design variable linking and approximation concepts[7] to efficiently handle large problems. Even so, the maximum number of design variables that can be handled effectively is no more than a few hundred. In the past the aircraft industry has handled thousands of variables by using optimality criterion methods, *e.g.*, Stress Ratio Method; however, their application in industry has been limited to considering a single discipline at a time, *i.e.*, stresses, or displacements, or flutter alone. Fleury and Schmit demonstrated the equivalence of optimality criterion and mathematical programming methods,[8] and more recently Venkayya formulated a generalized optimality criteria approach for general mathematical functions.[9] Present efforts meld

Fleury and Schmit's dual solution method and Venkayya's compound scaling algorithm in ASTROS to handle multidisciplinary structural design with thousands of variables.

MATHEMATICAL STATEMENT OF THE DESIGN PROBLEM

The objective is to minimize the weight (or equivalently, mass)

$$\min W(\mathbf{v}) \tag{1}$$

subject to m normalized constraints

$$g_j(\mathbf{v}) \leq 0; \qquad j = 1, \ldots, m \tag{2}$$

and side constraints on the n design variables, $\mathbf{v}$

$$v_i^L \leq v_i \leq v_i^u; \qquad i = 1, \ldots, n. \tag{3}$$

The constraint functions are formed by normalizing the response quantities, $z_j(\mathbf{v})$ by their allowable values, z_{bj}.

$$g_j = \pm \left(\frac{z(\mathbf{v}) - z_{bj}}{|z_{bj}|} \right) \tag{4}$$

The finite element cross-sectional properties (areas of rods and thicknesses of membranes), $\mathbf{d}$, are controlled by the design variables, $\mathbf{v}$ through a linking matrix, $\mathbf{T}$.

$$\mathbf{d} = \mathbf{T}\mathbf{v} \tag{5}$$

The approximation concepts develop first order Taylor series for the constraint functions in the reciprocal design variable space.

$$x_i = \frac{1}{v_i} \tag{6}$$

When a row of the linking matrix, $\mathbf{T}$, in eq (5), has only one non-zero element, a single design variable controls one or more finite elements. This *physical linking* can accommodate the use of the reciprocal variables defined in eq (6). When a row of $\mathbf{T}$ has more than one non-zero element, the design variables can be interpreted as coefficients that scale some shape function defined by a column of the linking matrix. Because this *shape function* design variable may be zero, reciprocal variables cannot be used, in which case $x_i = v_i$.

After each complete analysis of the structure, an approximate sub-problem is formed using a first order Taylor series to represent the constraint functions. For the finite elements used in ASTROS the objective function is a non-linear but explicit function of the reciprocal variables.

$$\min W(\mathbf{x}) = \mathbf{x}_o + \sum_{i=1}^{n} \frac{w_i}{x_i} \tag{7}$$

The approximate constraints are linear in the reciprocal variables.

$$\bar{\mathbf{g}} = \bar{\mathbf{g}}_\mathbf{o} + \mathbf{N}^t \mathbf{x} \tag{8}$$

where $N_{ij} = \frac{\partial g_j}{\partial x_i}$ is the gradient matrix and $\bar{g}_{oj} = \mathbf{N}_j^t \mathbf{x}_o$. To stay within the region of validity for the Taylor series, move limits are applied to the design variables

$$\frac{x_i}{f} \leq x_i \leq f x_i \tag{9}$$

where the move limit factor, f, is set to two as a default in ASTROS.

THE DUAL PROBLEM

Optimality criterion methods are derived using the Lagrangian function which augments the objective function with a summation of terms that weight each constraint by a *Lagrangian multiplier* (later referred to as a *dual variable*).

$$L(\mathbf{x}, \lambda) = W(\mathbf{x}) + \lambda^t \mathbf{g}(\mathbf{x}) \tag{10}$$

Application of the well-known *Kuhn-Tucker* conditions to this convex and separable approximate sub-problem results in a min-max optimization problem. The solution of this *dual problem* also defines the global optimum of the original *primal problem*.[8]

$$\max_{\lambda} L(\lambda) = \sum_{i=1}^{n} \min_{\mathbf{x}} \left[w_i / x_i + \lambda^t \bar{\mathbf{g}}(\mathbf{x}) \right] \tag{11}$$

where $\mathbf{x}$ is found explicitly from the condition that $\frac{\partial L}{\partial x_i} = 0$ for any given λ as

$$x_i = \sqrt{\frac{x_i}{\mathbf{N}_i \lambda}} \tag{12}$$

for all free x_i, *i.e.*, those not at their lower or upper bounds.

This dual problem is an unconstrained maximization problem. The approximate constraints are derivatives of the objective (Lagrangian) function.

$$\frac{\partial L}{\partial \lambda_j} = \bar{g}_j \tag{13}$$

The advantage of solving eq (11) in place of eq (7) is that the dimensionality of the problem is reduced from n design variables to m_a dual variables corresponding to only the strictly active constraints. By definition the Lagrange multipliers are zero for inactive constraints and positive for active constraints ($g = 0$). Whenever the number of positive dual variables (active constraints) is fewer than the number of primal (design) variables, the dual problem is more efficient to solve. One of the numeric difficulties, however, is the problem of terminating the optimization when eq (13) is zero. Other termination criteria (*e.g.*. relative change in the objective) in are often satisfied before the approximate constraints are within a tolerance acceptable for the primal problem. Fleury and Schmit accounted for this potential pitfall in two ways. First, their dual solver "does not seek the maximum of the dual function along the [search] direction S, rather it is designed to assure that either: (a) a regular Newton unit step is taken without any change in the set of free primal variables: or (b) the move distance is selected so that the value of the dual function increases. Second, they offer the option of reducing the size of the dual space by using zero order approximations—side constraints on the primal variables based on the element's stress ratio—for some stress constraints." Numeric difficulties occur less often when there are fewer dual variables.

Another approach would be to solve the dual problem as a constrained optimization problem in order to explicitly require the objective's derivatives not be greater than zero. The required derivatives of the approximate constraints with respect to the dual variables were derived in Ref 8.

COMPOUND SCALING ALGORITHM

The approach used here for preventing constraint violoations is different: a compound scaling algorithm[10] to guarantee the approximate constraints are satisfied. In formulating a generalized optimality criteria Venkayya defines the target response ratio as

$$\beta_j \equiv \frac{z_{bj}}{z_j} \tag{14}$$

and a sensitivity parameter,

$$\mu_{ij} \equiv \frac{N_{ij} x_i}{z_j}. \tag{15}$$

For each constraint the design vector is partitioned into groups based on the sign of the constraint derivative.

$$\mu_j^N = -\sum_{i=1}^{n} \min\left[0, \mu_{ij}\right], \quad \mu_j^P = \sum_{i=1}^{n} \max\left[0, \mu_{ij}\right] \tag{16}$$

Each partition of the design vector can be scaled by a factor, Λ, to be determined.

$$\mathbf{x}^N = \Lambda_j^N \mathbf{x}_o^N, \quad \mathbf{x}^P = \Lambda_j^P \mathbf{x}_o^P \tag{17}$$

Substituting these definitions into eq (8) yields an approximation of the target response ratio as a function of the two scale factors.

$$\beta_j(\Lambda_j^N, \Lambda_j^P) \approx 1 - \mu_j^N(\Lambda_j^N - 1) + \mu_j^P(\Lambda_j^P - 1) \tag{18}$$

Contours of the approximate target response ratio can be plotted as a function of the two scale factors. The desired target response lies on the contour line for $\beta = 1$ shown in Fig 2. Selecting a unique pair of scale factors requires a second equation in addition to eq (18). For reference, point S in Fig 2 represents simple scaling where the entire design vector is scaled by a single factor. The original derivation of the scale factors in Ref 10, represented by points A and B, assumed that scaling either partition alone would achieve the target response. The current approach is to select the point M on the scaling line that minimizes the distance to the current design at point O. This is the point closest to the small region about point O where the Taylor series approximation is accurate. The solution for the scale factors is

$$\Lambda_j^N = \left(\beta_j^N\right)^{\frac{-1}{\mu_j^N}}, \qquad \Lambda_j^P = \left(\beta_j^P\right)^{\frac{1}{\mu_j^P}} \tag{19}$$

where the partial target response ratios are defined as

$$\beta_j^N = 1 + \frac{\mu_j^N}{\mu_j^N + \mu_j^P}\left(\beta_j - 1\right), \qquad \beta_j^P = 1 + \frac{\mu_j^P}{\mu_j^N + \mu_j^P}\left(\beta_j - 1\right). \tag{20}$$

Two tables are used to select scale factors for multiple constraints. The Scale Factor Table is simply formed using the scale factor for the corresponding partition of the design vector.

$$\Lambda_{ij} = \begin{cases} \Lambda_j^N & \text{if } \mu_{ij} < 0 \\ \Lambda_j^P & \text{if } \mu_{ij} > 0 \\ 1.0 & \text{if } \mu_{ij} = 0 \end{cases} \tag{21}$$

The Scale Factor Assignment Table is formed from the sensitivity factors, except that the total differential term is normalized by the allowable value instead of the response value.

$$t_{ij} = \left|\frac{N_{ij} x_i}{z_{bj}}\right| = \left|\mu_{ij}\beta_j\right| \tag{22}$$

A scale factor is selected for each design variable according to the following three cases.

$$\begin{array}{lll}
\text{Case 1} & \overset{\text{all}}{j} N_{ij} \leq 0, & \lambda_i = \underset{j, N_{ij} \neq 0}{\max}(\Lambda_{ij}) \\
\text{Case 2} & \overset{\text{all}}{j} N_{ij} \geq 0, & \lambda_i = \underset{j, N_{ij} \neq 0}{\min}(\Lambda_{ij}) \\
\text{Case 3} & \underset{j}{\max}\, t_{ij}, & \Lambda_i = \Lambda_{ij} \begin{cases} \text{if } w_i(\Lambda_{ij} - 1) < 0 \\ \text{or } z_j > z_{bj} \end{cases}
\end{array}$$

The above three cases replace the rules presented in Ref 10 and simplify the scale factor selection procedure by avoiding the special cases of simple scaling and compound scaling with a single constraint. Also, starting from a uniform design is unnecessary.

Each design cycle in ASTROS consists of a complete multidisciplinary analysis followed by redesign based on the approximate problem. The new generalized optimality criterion algorithm begins with a dual solution scheme to find the Lagrange Multipliers and corresponding primal variables, followed by iterative compound scaling until approximate constraint violations are tolerable.

RESULTS

Three design problems with 200 to 1527 design variables were solved to compare primal and dual solution methods for large optimization problems. Although only the second problem is multidisciplinary, the approach is the same for all problems regardless of the disciplines considered. Iteration history plots are shown for each example. The label "Primal" refers to the current algorithm in ASTROS that solves the approximate subproblem directly. The label "Dual" refers to the current generalized optimality criteria being tested. Normalized CPU times for the entire execution (analyses, sensitivity, and redesign for all iterations) are shown as a factor next to each label in the legend of each plot.

200 Member Plane Truss

A 72 node plane truss made of two hundred steel elements subject to five loading conditions[11] (Fig 3 and Table 1) was used to demonstrate the efficiency of a generalized optimality criterion approach for statics. Stress and displacement limits together accounted for 2500 applied constraints. The dual method required one fourth the computational effort compared the primal method (Fig 4).

Intermediate Complexity Wing

The next example considered was an intermediate complexity wing.[12] The structural model has 158 elements and 234 degrees of freedom (Fig 5). The composite cover skins are made of graphite epoxy with the properties given in Table 2. Stress constraints were imposed on all membrane elements and displacement constraints were imposed at the tip of the wing in the transverse direction for two independent static loading conditions. A flutter speed limit of 925 knots corresponding to a flight condition of 0.8 Mach number at sea level was also applied, resulting in 722 constraints and 350 design variables. For comparison, the results from Ref 12 using 22 shape function variables and two physical design variables are shown as well (Fig 6). The slightly higher weight for this case demonstrates the penalty for constraining the skin thicknesses to vary quadratically with the span and the spar web thicknesses to vary linearly with the span. The shape function solution was more efficient since 60 constraints were active at the optimum of the dual problem. Nevertheless, even with an additional iteration, the dual method is twice as efficient as the primal method when all 350 variables are considered. The improvement is less dramatic than for the previous example because the multidisciplinary analysis (statics, modes, and flutter) is more costly relative to the optimization.

High Altitude Long Endurance (HALE) Aircraft

The finite element model for the right wing of a HALE aircraft (Fig 7) is comprised of a truss substructure and metallic cover skins. The mission of this 270 foot span airplane is to patrol for several days at 150 to 250 knots at an altitude of 65,000 feet. Since the ASTROS steady and unsteady aerodynamics models were not yet complete, three static loads were applied to an aluminum version of this wing. Stresses and wing-tip deflections were constrained, producing a total of 6124 constraints (Table 3). All 1527 elements were designed independently. The primal method could not be solved within the memory available to ASTROS, so a Fully Stressed Design (FSD) method was used as the basis for comparison (Fig 8). A design with deflection constraints alone was one order of magnitude more costly than the FSD due to the sensitivity analysis (optimization was negligible). A weight penalty was incurred, of course, when designing for the stress constraints as well.

CONCLUSIONS

A generalized optimality criterion method consisting of a dual problem solver combined with a compound scaling algorithm has been implemented in the multidisciplinary design tool, ASTROS. This method enables, for the first time in a production design tool, the determination of a minimum weight design using thousands of independent structural design variables while simultaneously considering constraints on response quantities in several disciplines. Even for moderately large examples, the computational efficiency is improved significantly relative to the conventional approach.

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Material	Steel
Modulus of Elasticity	$E = 30 \times 10^6$ psi
Weight Density	0.283 lb / cu in
Stress Limits	30,000 psi
Lower Limit on rod areas	0.1 sq in
Displacement Limits on all nodes (horizontal and vertical directions)	0.5 in
Number of Loading Conditions	5
Loading Condition 1	1000 lb acting in +X direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, 71
Loading Condition 2	1000 lb acting in -X direction at nodes 5, 14, 19, 20, 28, 33, 42, 47, 56, 61, 70, 75
Loading Condition 3	10,000 lb acting in -Y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, . . . , 71, 72, 73, 74, 75
Loading Condition 4	Loading Conditions 1 and 2 together
Loading Condition 5	Loading Conditions 2 and 3 together

Table 1: 200 Member Plane Truss Design Conditions

Isotropic Material	Aluminum
Modulus of Elasticity	$E = 30 \times 10^6$ psi
Poisson's Ratio	0.30
Weight Density	0.10 lb / cu in
Tensile Stress Limit Compressive Stress Limit Shear Stress Limit	55,000 psi 55,000 psi 45,000 psi
Lower Limit on Thickness	0.04 in
Orthotropic Material	Graphite Epoxy
Modulus of Elasticity	$E_1 = 18.5 \times 10^6$, $G_{12} = 0.65 \times 10^6$ $E_2 = 1.6 \times 10^6$ psi
Poisson's Ratio	0.25
Weight Density	0.055 lb / cu in
Stress Limits	115,000 psi
Lower Limit on Plies	0.00525 in
Limit on Transverse Tip Displacements	10.0 in
Flutter Speed Limit	925 knots

Table 2: Intermediate Complexity Wing Design Conditions

Material	Aluminum
Modulus of Elasticity	$E = 10.5 \times 10^6$ psi
Weight Density	0.10 lb / cu in
Poisson's Ratio	0.30
Stress Limits	60,000 psi
Lower Limit on Thicknesses	0.021 in
Lower Limit on rod areas	0.10 sq in
Limits on Transverse Tip Displacements	200.0 in
Number of Loading Conditions	4

Table 3: HALE Design Conditions

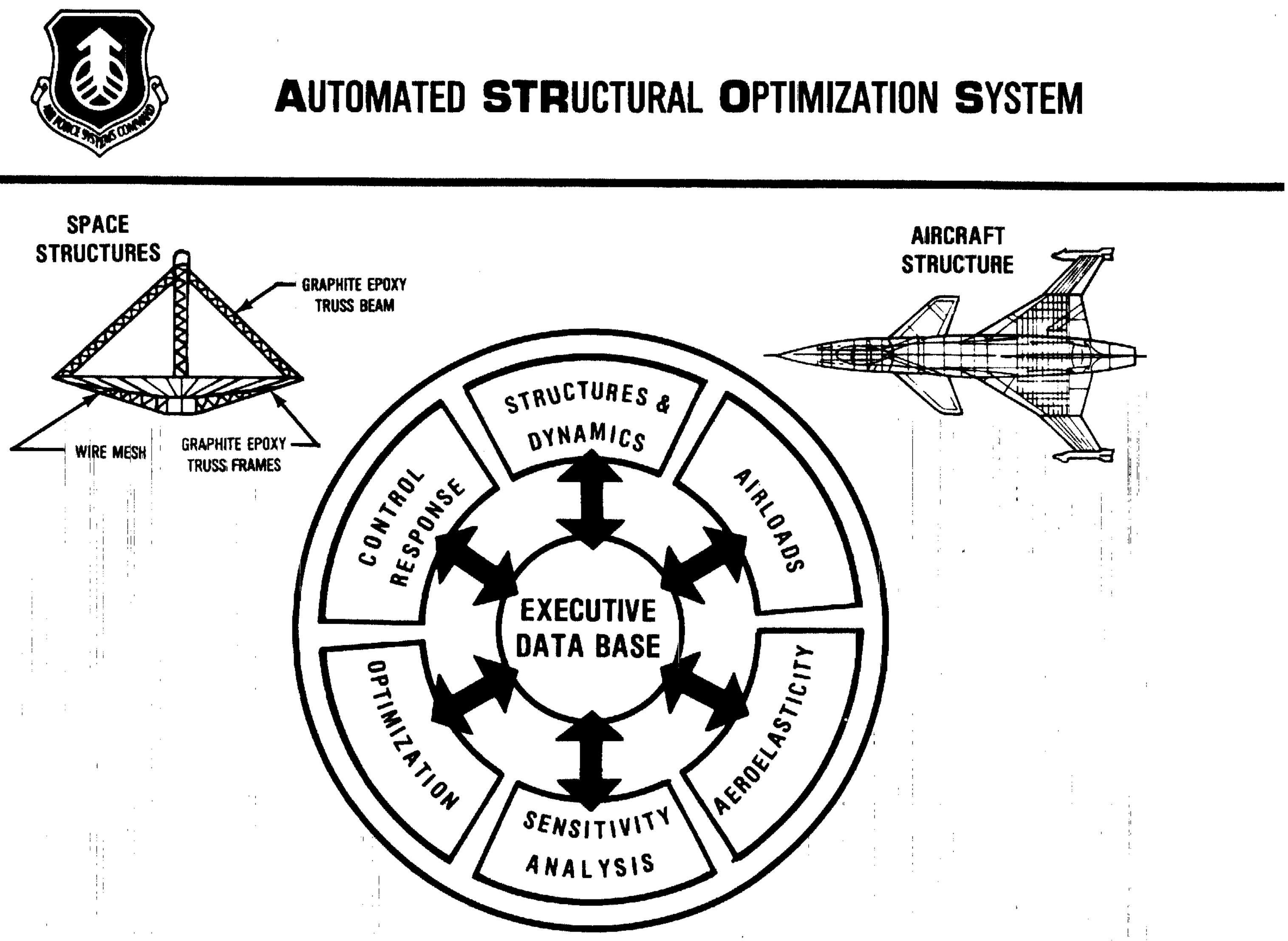
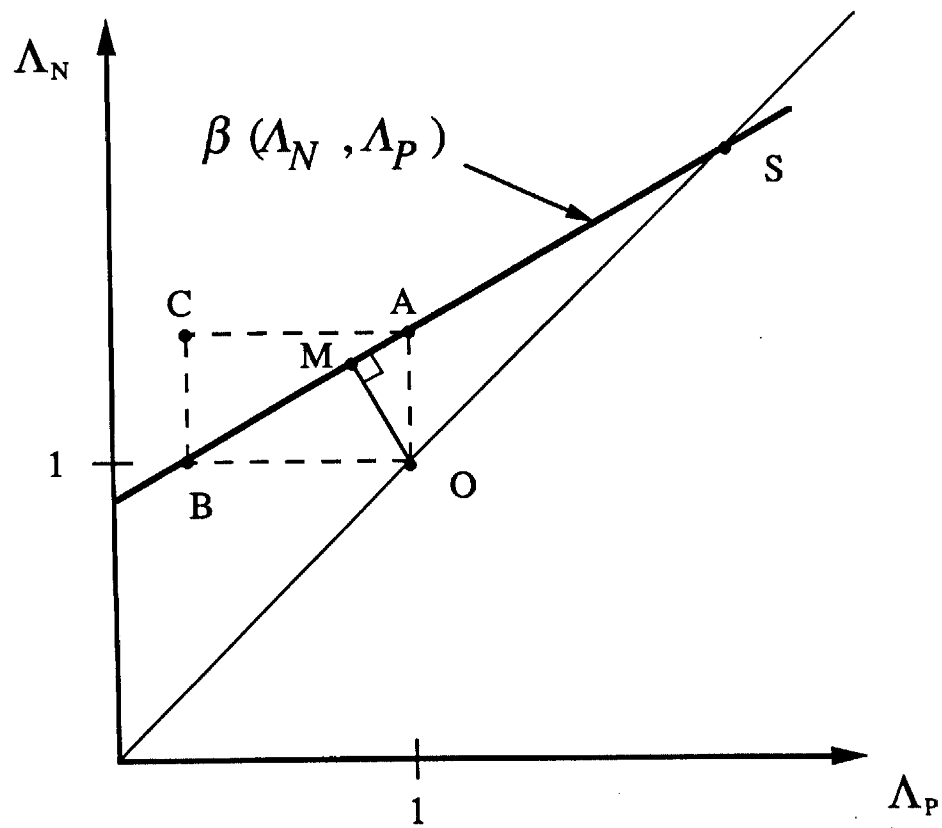


Figure 1: ASTROS Diagram



Point A is $\left(\Lambda_N = \beta^{-1/\mu_N}, \Lambda_P = 1\right)$

Point B is $\left(\Lambda_N = 1, \Lambda_P = \beta^{1/\mu_P}\right)$

Point C is $\left(\Lambda_N = \beta^{-1/\mu_N}, \Lambda_P = \beta^{1/\mu_P}\right)$

Point M is $\left(\Lambda_N = \beta_N^{-1/\mu_N}, \Lambda_P = \beta_P^{1/\mu_P}\right)$

Point S is $\left(\Lambda_N = \beta^{1/\mu} = \Lambda_P\right)$

Figure 2: Approximate Target Response Scaling Line

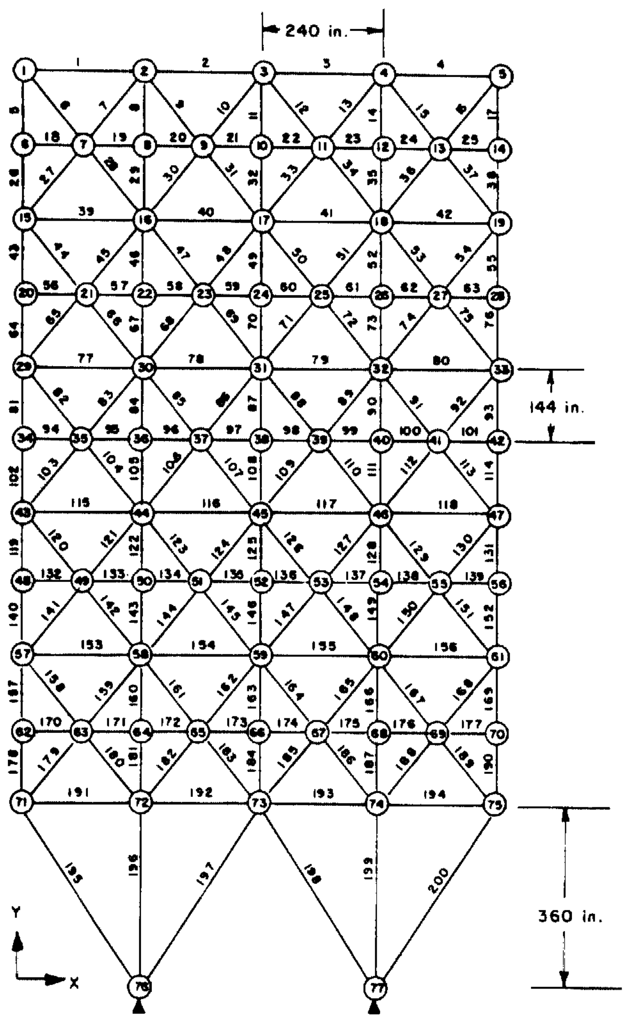


Figure 3: 200 Member Plane Frame Model

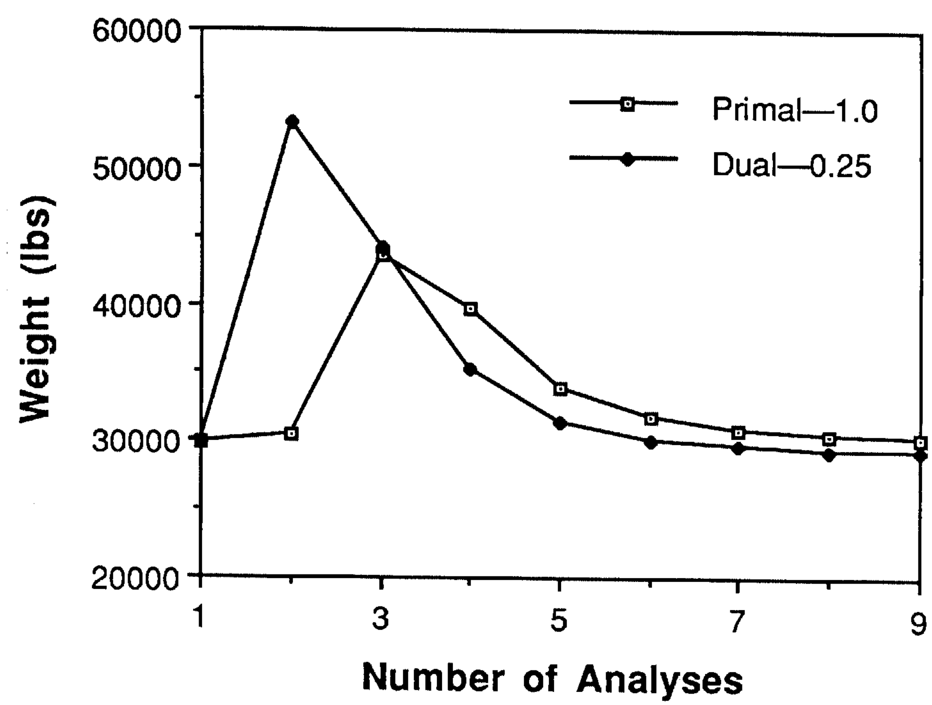


Figure 4: 200 Member Frame Iteration History

No. of Nodes	No. of Elements	No. of DOF's
88	39 Rods	294 Constrained
	55 Shear Panels	234 Unconstrained
	62 Quadrilateral Membrane	528 Total
	2 Triangular Membrane	
	158 Total	

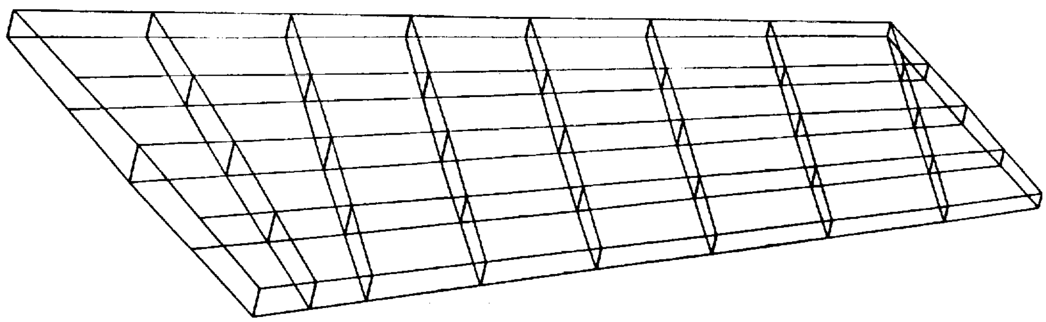


Figure 5: Intermediate Complexity Wing Model

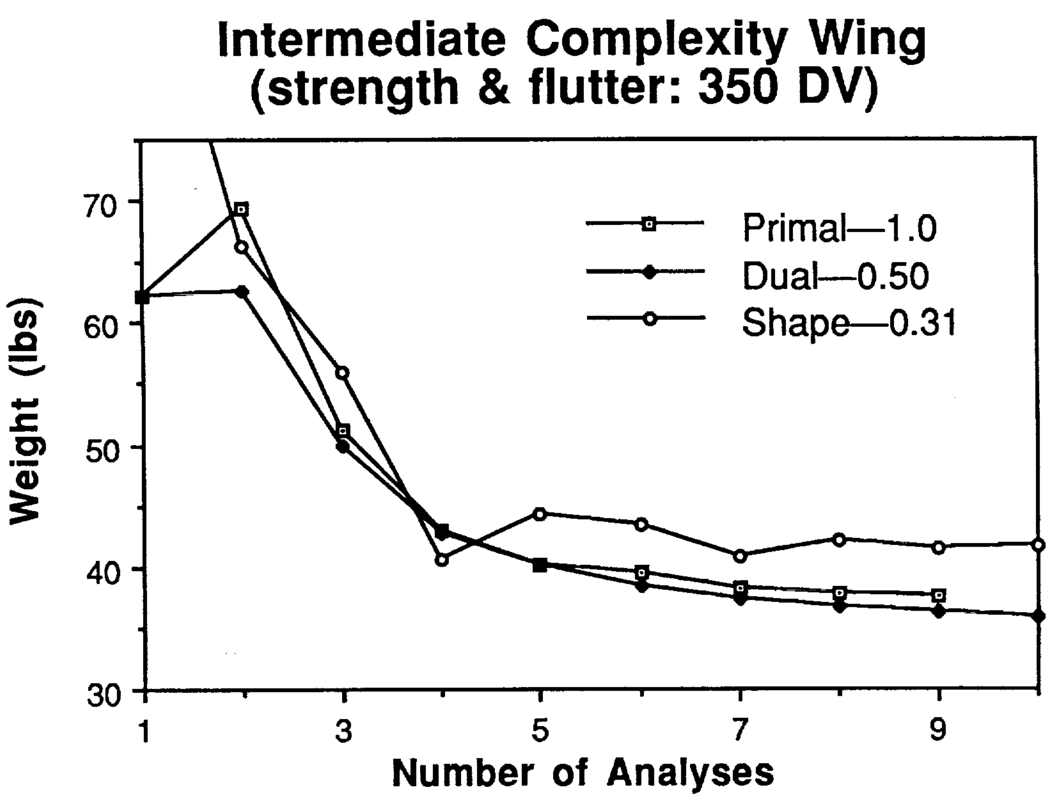


Figure 6: Intermediate Complexity Wing Iteration History

High Altitude Long Endurance Aircraft Wing

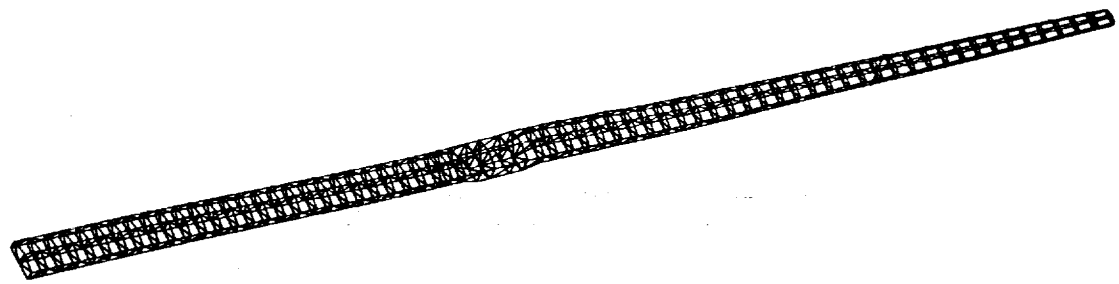


Figure 7: HALE Aircraft Wing Model

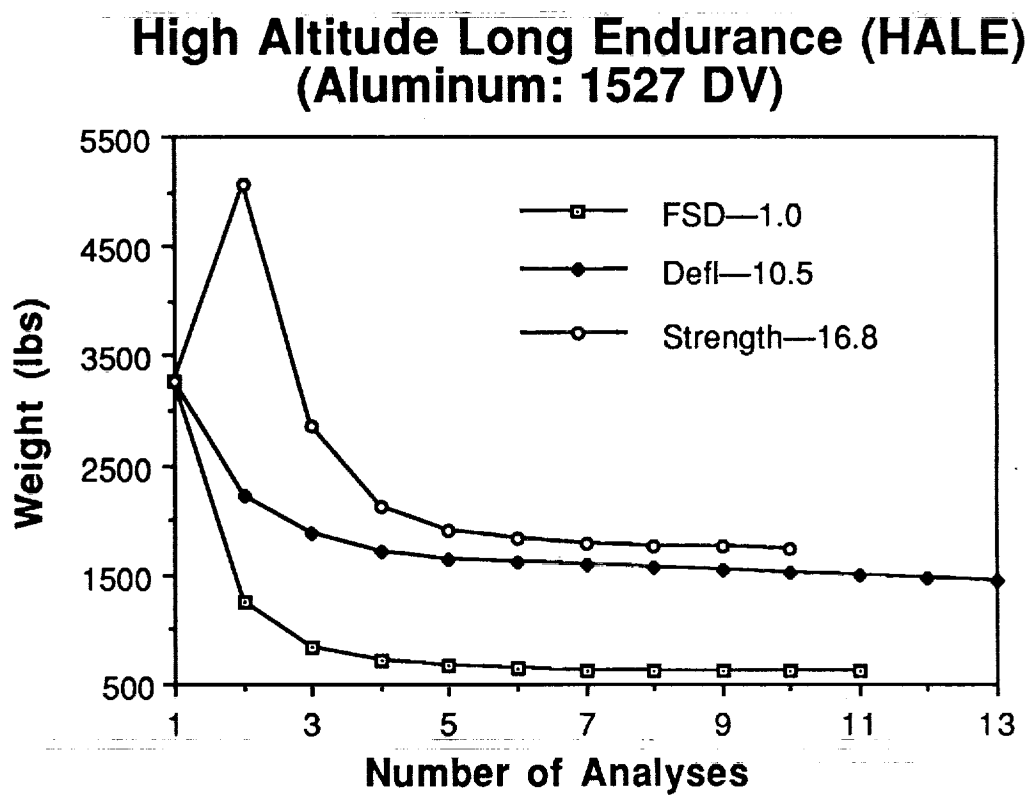


Figure 8: HALE Iteration History