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ROBUST FIXED ORDER DYNAMIC COMPENSATION FOR LARGE SPACE STRUCTURE CONTROL

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ABSTRACT

This paper presents a simple formulation for designing fixed order dynamic compensators which are robust to both uncertainty at the plant input and structured uncertainty in the plant dynamics. The emphasis is on designing low order compensators for systems of high order. The formulation is done in an output feedback setting which exploits an observer canonical form to represent the compensator dynamics. The formulation also precludes the use of direct feedback of the plant output. The main contribution lies in defining a method for penalizing the states of the plant and of the compensator, and for choosing the distribution on initial conditions so that the loop transfer matrix approximates that of a full state design. To improve robustness to parameter uncertainty, the formulation avoids the introduction of sensitivity states, which has led to complex formulations in earlier studies where only structured uncertainty has been considered.

INTRODUCTION

Linear Quadratic Regulator (LQR) design methods are easy to use and have guaranteed stability margins at the plant input. Unfortunately, this requires full state feedback for implementation. With Loop Transfer Recovery (LTR) techniques, the loop characteristics of an LQR design for square, minimum phase plants can be asymptotically realized using output feedback with a full order observer.[1] This design methodology can be used to improve the robustness of observer based controller designs to unstructured uncertainty. However, for control of large order plants, this approach may result in controllers that are computationally expensive to implement. Moreover, although there is good robustness to unstructured uncertainty at the plant input, the design may remain sensitive to structured uncertainty in the plant parameters.

Optimal output feedback of fixed order dynamic compensators[2] has received limited attention due to numerous difficulties associated with this approach. Initially, the proposed compensator representation was overparameterized, which means it lacked a predefined structure. To

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overcome this obstacle, several canonical structures have been proposed which result in a minimal parameterization.[3,4] This study utilizes the observer canonical form since it yields a convenient form when the design is treated as a constant gain output feedback problem.

Another major objection to fixed order dynamic compensation is that there are no guarantees on the stability margins. This drawback was eliminated by a new methodology for designing fixed order compensators.[5] This technique approximates the LQR/LTR method, by appropriate selection of the plant and compensator state weighting matrices. Much like the full order observer design, a two step process is used. First, full state gains are computed for desirable loop properties, followed by the approximate LTR compensator design. The fundamental assumption of this approach is that if the closed loop return signals of the two designs are equal, then the loop transfer functions (with the loop broken at the plant input) must be equivalent. Unlike the LQR/LTR design, there is no requirement that the system be minimum phase or square.

A popular method of parameter sensitivity reduction consists of including a quadratic trajectory sensitivity term in the linear regulator formulation. In Ref. 6, the approximate LTR methodology for low order compensators was extended to include sensitivity reduction for real plant parameter variations. The sensitivity reduction is accomplished by a simple modification of the quadratic performance index. Unlike earlier studies in this area,[7,8,9] this formulation does not require increasing the dimension of the problem to include the sensitivity states. In addition, the parameter sensitivity reduction is achieved with minimal sacrifice in the loop transfer characteristics.

An outline of this paper is as follows. First, the approximate LTR methodology of Ref. 5 is reviewed. Then, the sensitivity reduction formulation is presented for the specific case of a scalar uncertainty in the state equation. The generalizations of the trajectory sensitivity approach, given in Ref. 6, are summarized afterwards. Several future extensions of this research are also discussed.

CONTROLLER DESIGN FORMULATIONS

Dynamic Compensation

Consider a linear system of the form

$$\dot{x}_p = A_p x_p + B_p u_p \qquad x_p \in \Re^n,\ u_p \in \Re^m \tag{1}$$

$$y_p = C_p x_p + D_p u_p \qquad y_p \in \Re^p \tag{2}$$

where y_p represents the measured outputs. A dynamic compensator can be parameterized in the following observer canonical form:[4]

$$\dot{z} = P^o z + u \qquad z_o \in \Re^{nc} \tag{3}$$

$$u = P_z u_p - N y_p \qquad u_o \in \Re^{nc} \tag{4}$$

$$u_p = - H^o z \tag{5}$$

where nc is the order of the compensator and P_z and N are the matrices containing the design parameters. The matrices P^o and H^o are specified by the choice of observability indices, and their structures are given in Ref. 4.

An equivalent, constant gain output feedback problem is obtained using the augmented system dynamics.

$$\dot{x} = Ax + Bu_c \tag{6}$$

$$y = Cx \tag{7}$$

$$u_c = -Gy \tag{8}$$

where $x^T = \{x_p^T, z^T\}$, $y^T = \{y_p^T, -u_p^T\}$ and

$$A = \begin{bmatrix} A_p & -B_p H^o \\ 0 & P^o \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} C_p & -D_p H^o \\ 0 & H^o \end{bmatrix} \qquad G = \begin{bmatrix} N & P_z \end{bmatrix} \tag{10}$$

The performance index in this case is

$$J = E_{x_o}\left\{\int_0^{\infty} \left[x^T Q x + u_c^T R u_c\right] dt\right\} \tag{11}$$

It is shown in Ref. 5 that the loop transfer properties of a full state feedback design are approximately recovered at the plant input by choosing the following weighting matrices in (11).

$$Q = \begin{bmatrix} K^{*T}K^{*} & -K^{*T}H^{o} \\ -H^{oT}K^{*} & H^{oT}H^{o} \end{bmatrix} \qquad R = \rho B^{T}B = \rho I_{nc} \tag{12}$$

$$E\{x_o x_o^T\} = \begin{bmatrix} B_p B_p^T & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

where K^* is the full state feedback gain matrix, and repeating the design for decreasing values of ρ. Although the full state loop transfer properties are approximately recovered as ρ is reduced, the controller design may be sensitive to parameter uncertainty.

Uncertain State Equation

We first consider the system (1,2) with a scalar uncertainty parameter α in the state equation.

$$\dot{x}_p = A_p(\alpha)x_p + B_p(\alpha)u_p \tag{14}$$

In the case of dynamic compensation, (5) becomes

$$\dot{x} = A(\alpha)x + Bu_c \tag{15}$$

where

$$A(\alpha) = \begin{bmatrix} A_p(\alpha) & -B_p(\alpha)H^o \\ 0 & P^o \end{bmatrix} \tag{16}$$

The trajectory sensitivity dynamics are obtained by differentiating (13) with respect to α.

$$\dot{\sigma} = A_\alpha x + (\overline{A} - BGC)\sigma; \quad \sigma(0) = 0 \tag{17}$$

where $\sigma = \partial x/\partial\alpha|_{\alpha_0}$, $A_\alpha = \partial A(\alpha)/\partial\alpha|_{\alpha_0}$, $\bar{A} = A(\alpha_0)$ and α_0 denotes the nominal value for α. The standard approach would consider minimizing the following performance index.

$$J = E_{X_0}\left\{\int_0^\infty \left[x^T Q x + u_c^T R u_c + \sigma^T S \sigma\right] dt\right\} \tag{18}$$

where the weighting matrix S is used to penalize sensitivity to parameter uncertainty. However, this increases the dimension of the problem to 2(n+nc). To avoid this drawback, we adopt the following viewpoint. Note that $A_\alpha x$ acts as a forcing function in (15), and that $\sigma(0) = 0$. Also, note that since G stabilized the nominal system, the the dynamics of $\sigma(t)$ are also stable. Thus, $||\sigma||$ can be reduced by penalizing $||A_\alpha x||$. This suggests that the following index of performance be used to introduce a penalty on sensitivity to parameter uncertainty

$$J = E_{X_0}\left\{\int_0^\infty \left[x^T (Q + \eta A_\alpha^T A_\alpha) x + u_c^T R u_c\right] dt\right\} \tag{19}$$

Thus, a second design parameter, η, is introduced which can be used to penalize sensitivity to parameter variations, without increasing the order of the dynamic system. When Q, R and X_0 are chosen in accordance with (12) and (13), then increasing η permits a design trade off between desirable loop transfer properties at the plant input and parameter sensitivity reduction. Equations (7), (8), (15) and (19), with $A(\alpha) = \bar{A}$, constitute a static optimal output feedback problem, whose necessary conditions for optimality are well known.[10]

This approach to parameter sensitivity reduction can be generalized to include an uncertain output equations. With uncertainty in the state and output equation, the trajectory sensitivity dynamics become

$$\dot{\sigma} = (A_\alpha - BGC_\alpha)x + (\bar{A} - BG\bar{C})\sigma; \quad \sigma(0) = 0 \tag{20}$$

where the input matrix B is specified by the observer canonical structure in (9). To minimize $\sigma(t)$, the logical extension is to penalize $||(A_\alpha - BGC_\alpha)x||$ in the performance index. Since this penalty depends explicitly on the gain matrix G, the standard necessary conditions for optimal output feedback no longer apply. In Ref. 6, these new necessary conditions are given as well as a generalization of this methodology to include a vector parameters. Also, the design approach is illustrated by a high bandwidth controller for a flexible satellite.

EXTENSIONS

There are several directions in which the current research on low order compensation is heading. First, the approximate LTR methodology used at the plant input has been extended to recovery of loop properties at the plant output. This design technique uses both the controller and the observer canonical forms and the duality which exists between these structures. Similar to the dual LTR formulation of Ref. 1, a full order observer is first designed for desirable loop properties at the dual system input. With the appropriate quadratic state and control penalties, the compensator design approximately recovers these loop characteristics at the plant output. This formulation can easily be extended to include a structured uncertainty penalty similar to the trajectory sensitivity dynamics used in this paper.

The next extension of this work is to develop a design methodology which will allow simultaneous approximate LTR at both the plant input and the plant output. This will allow the extension to H-Infinity design. Specifically, using the design approach in Ref. 11, the H-Infinity controller becomes a constant gain full state feedback design. This can be viewed as a design approach that simultaneously achieves loop shaping with the loop broken at both the plant input and the plant output. Thus, the loop characteristics of a H-Infinity design can be realized with a low order dynamic compensator if simultaneous LTR can be achieved.

SUMMARY

A method has been developed for designing fixed order dynamic compensators which are robust to both unstructured uncertainty at the plant input and structured uncertainty in the plant dynamics. The design approach specifies weighting matrices which allow the loop transfer properties to approximate that of a full state design. The robustness to real structured parameter variations is accomplished by a modification of the quadratic performance index. The approach avoids the introduction of the sensitivity states. Hence, it does not increase the dimension of the problem to acheive the robustness to real plant parameter variations. Extension to H-Infinity design are currently under development.

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