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MODELING AND CONTROL OF FLEXIBLE SPACE STATIONS (SLEW MANEUVERS)

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ABSTRACT

Large orbiting space structures are expected to experience mechanical vibrations arising from several disturbing forces such as those induced by shuttle takeoff or docking and crew movements. In this paper, we consider the problem of modeling and control of large space structures subject to these and other disturbing forces. The system consists of a (rigid) massive body, which may play the role of experimental modules located at the center of the space station and flexible configurations, consisting of several beams, forming the space structure. A complete dynamic model of the system has been developed using Hamilton's principle. This model consists of radial equations describing the translational motion of the central body, rotational equations describing the attitude motions of the body and several beam equations governing the vibration of the flexible members (platform) including appropriate boundary conditions. In summary the dynamics of the space structure is governed by a complex system of interconnected partial and ordinary differential equations.

Using Lyapunov's approach the asymptotic stability of the space structure is investigated. For asymptotic stability of the rest state (nominal trajectory) we have suggested feedback controls. In our investigation stability of the slewing maneuvers is also considered.

Several numerical results are presented for illustration of the impact of coupling and the effectiveness of the stabilizing controls. This study is expected to provide some insight into the complexity of modeling, analysis and stabilization of actual space structures.

1.INTRODUCTION

Structural vibrations in future large space systems(Fig.1) such as space antennas, space platforms, space stations, or deployed flexible payloads attached to the space shuttle orbiter, and their interaction with the other members of the system have become a major concern in design of reliable systems satisfying stringent stability requirements. During the past few years, considerable attention have been focused on the development of mathematical model and stabilizing controls for such systems[1-13].

The most natural model for flexible space structures is given by a hybrid system of equations which is a combination of ordinary differential equations for rigid parts and partial differential differential equations for flexible members. Hybrid models for some simple structures have been considered in [5-11]. Recently Lim[12] developed a complete dynamic model, which includes the orbital dynamics, attitude dynamics and equations for vibrations of flexible members and all the relevant boundary conditions, for flexible space structures. Here, in this paper, based on the dynamic model we develop a control scheme to suppress the vibration induced by slew maneuvers in space stations.

The paper is organized as follows. In the next section we present the equations of motion for large spacecraft derived in [12]. In section 3, based on Lyapunov's approach, asymptotic stability of slew maneuvers for the system using feedback control is considered. For illustration, in section 4, we present some numerical results demonstrating the effectiveness of the stabilizing controls for vibration suppression associated with slew maneuvers. Finally, the concluding remarks are presented.

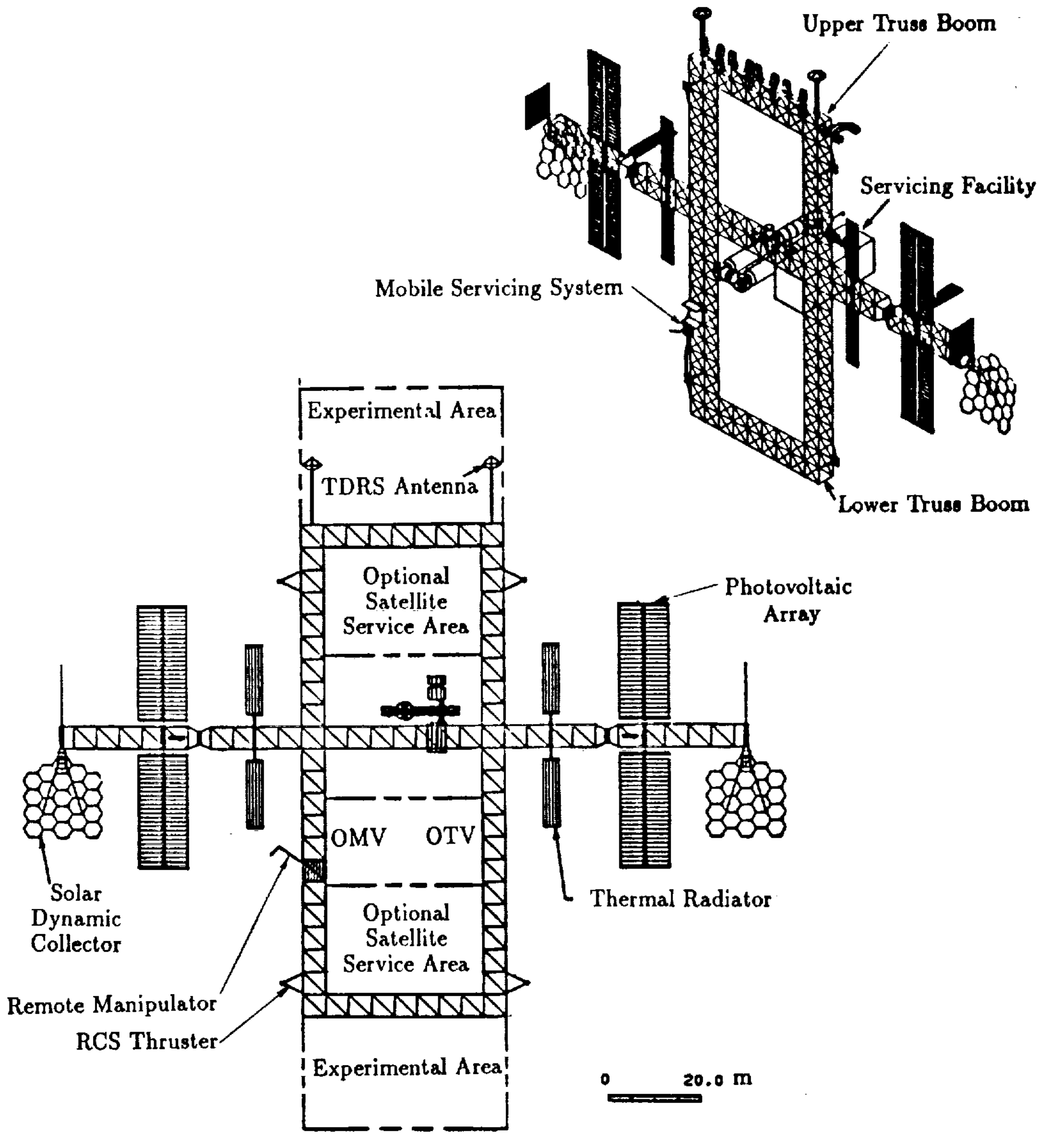


Fig.1 The U.S. Space Station.

2 DYNAMIC MODEL OF FLEXIBLE SPACE STATIONS

2.1 Introduction

We describe the dynamics of the space station by
Three Types of Motion:

- Rigid body translation perturbing the orbit.
- Rigid body rotation perturbing the orientation of the system.
- Vibration of the elastic members causing elastic deformation of the system.

To derive the dynamics we use the following coordinate systems:

2.2 Reference Coordinate Systems

- Body Coordinate System S_B: (i_b, j_b, k_b)
- Orbital Reference Coordinate System S_r: (i_r, j_r, k_r)
- Inertial Coordinate System S_I: (i_I, j_I, k_I)

Angular Velocities

$w_b = (w_x, w_y, w_z)'$: angular velocity of body frame w.r.t. the S_r.

$w_r = (w_X, w_Y, w_Z)'$:angular velocity of S_r w.r.t. S_I.

$w \equiv w_b + w_r = (w_1, w_2, w_3)'$:angular velocity of S_B w.r.t. S_I.

$$Q_i \equiv \begin{cases} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \text{for } i=1,2,3,4, \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \text{for } i=5,6. \end{cases} \tag{1}$$

$$r_i = R + \bar{r}_i, \quad \text{with } \bar{r}_i \equiv (\bar{x}_i, \bar{y}_i, \bar{z}_i)' = D_{i_0} + D_i, \tag{2}$$

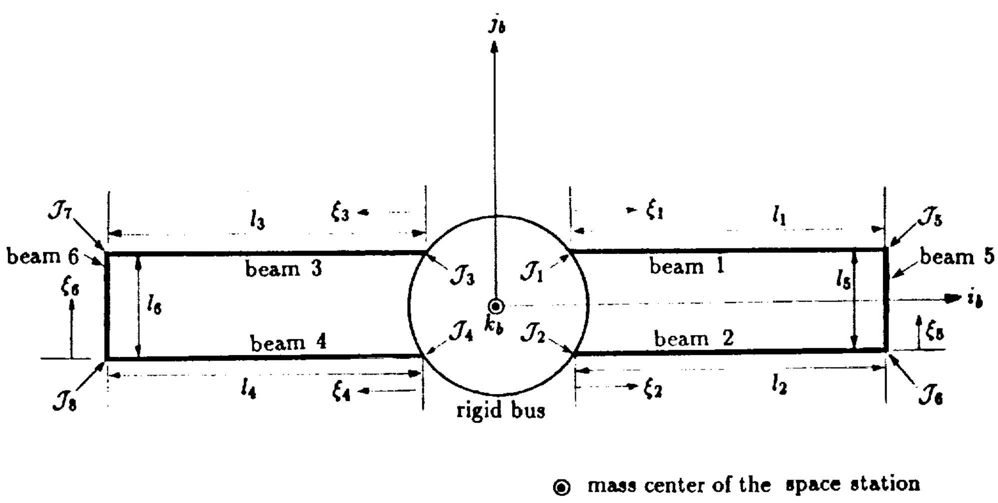
where $R \equiv (X,Y,Z)'$, $D_{i_0} \equiv (x_{i_0}, y_{i_0}, z_{i_0})'$, and $D_i \equiv Q_i\ (x_i, y_i, z_i)'$, i=1,2,...,6.

$\Omega_i \equiv (0, l_i)$, i=1,2,...,6 for l_i(the length of the beam i).

$$D_{1_o} = (s_{1_x} + \xi_1, s_{1_y}, 0)', \quad D_{2_o} = (s_{2_x} + \xi_2, s_{2_y}, 0)', \tag{3}$$

$$D_{3_o} = (-(s_{3_x} + \xi_3), s_{3_y}, 0)', \quad D_{4_o} = (-(s_{4_x} + \xi_4), s_{4_y}, 0)', \tag{4}$$

$$D_{5_o} = (s_{5_x}, -s_{5_y} + \xi_5, 0)', \quad D_{6_o} = (s_{6_x}, -s_{6_y} + \xi_6, 0)', \tag{5}$$



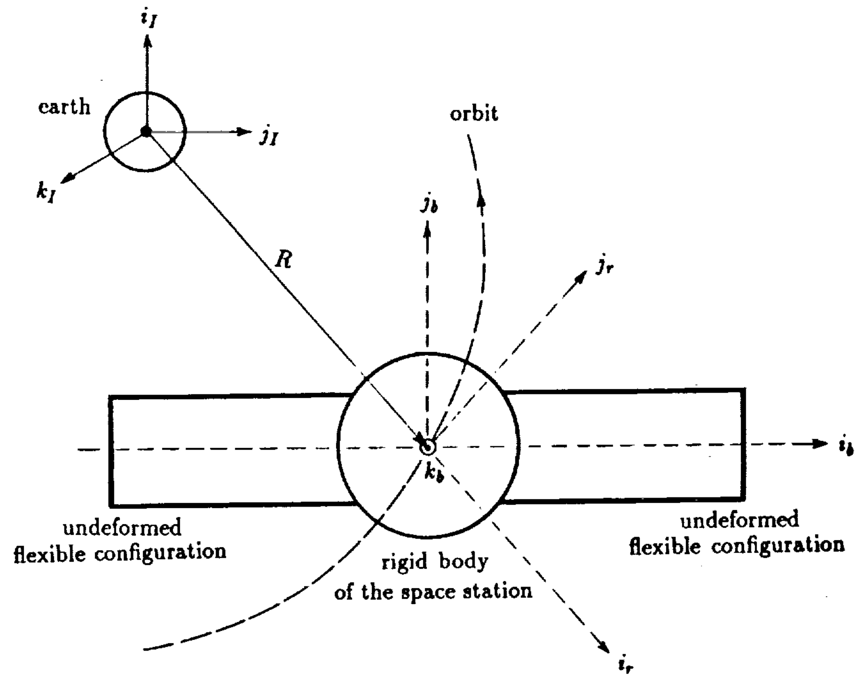


Fig.2 The space station and reference coordinate systems.

2.3 Inertia Tensor

The beam inertia tensor is given by

$$I_b \equiv \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix} \tag{6}$$

where

$$I_{xx} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i(\bar{y}_i^2 + \bar{z}_i^2) d\xi_i, \tag{7}$$

$$I_{yy} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i(\bar{x}_i^2 + \bar{z}_i^2) d\xi_i, \tag{8}$$

$$I_{zz} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i(\bar{x}_i^2 + \bar{y}_i^2) d\xi_i, \tag{9}$$

$$I_{xy} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \bar{x}_i \bar{y}_i d\xi_i, \tag{10}$$

$$I_{xz} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \bar{x}_i \bar{z}_i d\xi_i, \tag{11}$$

$$I_{yz} = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \bar{y}_i \bar{z}_i d\xi_i. \tag{12}$$

2.4 Dynamic Model of the Space Station

Let the Lagrangian of the whole system be denoted by L and external work by W. Then using the extended Hamilton's principle we obtain the relation[12,13]:

$$\begin{aligned} &\delta \int_{t_1}^{t_2} (L + W) dt \\ &\quad \equiv - \int_{t_1}^{t_2} \left\{ (\delta R) \circ \mathcal{I}_1 - (\delta \theta) \circ \mathcal{I}_2 + \sum_{i=1}^{6} \int_{\Omega_i} (\delta D_i) \circ \mathcal{I}_3 d\xi_i + \mathcal{I}_4 \right\} dt \\ &\quad \equiv 0, \end{aligned} \tag{13}$$

where

$$\mathcal{I}_1 = m_s \frac{d^2 R}{dt^2} + \frac{d}{dt} \sum_{i=1}^{6} \int_{\Omega_i} \left(\frac{dR}{dt} + \omega \times \bar{r}_i + \dot{D}_i \right) dm_i + \frac{G m_e m_T R}{|R|^3} - F_s, \tag{14}$$

$$\mathcal{I}_2 = T - \frac{d}{dt} \left\{ (I_s \circ \omega) + \sum_{i=1}^{6} \int_{\Omega_i} \bar{r}_i \times \frac{dr_i}{dt} dm_i \right\} + \sum_{i=1}^{6} \int_{\Omega_i} \left(\omega \times \bar{r}_i + \dot{D}_i \right) \times \left(\frac{dr_i}{dt} \right) dm_i, \tag{15}$$

$$\mathcal{I}_3 = \rho_i \frac{d^2 r_i}{dt^2} + \frac{\partial^2}{\partial \xi_i{}^2} \left(EI_i \frac{\partial^2 D_i}{\partial \xi_i{}^2} \right) - \bar{F}_{b_i}, \ i = 1, 2, ..., 6, \tag{16}$$

$$\mathcal{I}_4 = \sum_{i=1}^{6} \left\{ \left[EI_i \left(\frac{\partial^2 D_i}{\partial \xi_i{}^2} \right) \circ \frac{\partial}{\partial \xi_i} (\delta D_i) \right]_{\partial \Omega_i} - \left[\frac{\partial}{\partial \xi_i} \left(EI_i \frac{\partial^2 D_i}{\partial \xi_i{}^2} \right) \circ (\delta D_i) \right]_{\partial \Omega_i} \right\}, i = 1, 2, ..., 6. \tag{17}$$

Let

$$EI_i \equiv \begin{pmatrix} EI_{ix} & 0 & 0 \\ 0 & EI_{iy} & 0 \\ 0 & 0 & EI_{iz} \end{pmatrix}, \quad i = 1, 2, ..., 6. \tag{18}$$

$$\bar{F}_{b_i} \equiv \begin{cases} (0, F_{i_y}, F_{i_z})', & \text{for } i\text{=1,2,3,4}, \\ (F_{i_x}, 0, F_{i_z})', & \text{for } i\text{=5,6}. \end{cases} \tag{19}$$

For boundary conditions we introduce the following assumptions:

Assumptions for the Joints and Beams:

(i) The structures are in xy plane in undisturbed state,

(ii) The beams 1,2,3,4 are clamped at the joints $\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4$,

(iii) Displacements of the beams 1,2,...,6 are small,

(iv) The beams 5,6 are rigidly jointed with the other four beams as shown in Fig.2.

For convenience of presentation we use the following notations:

$$\alpha_i \equiv \dot{\omega} \times \bar{r}_i + 2\omega \times \dot{D}_i + \omega \times (\omega \times \bar{r}_i), \; i = 1, 2, ..., 6.$$

$$F_{b_i} \equiv \begin{cases} (F_{i_y}, F_{i_z})', & \text{for } i\text{=1,2,3,4}, \\ (F_{i_x}, F_{i_z})', & \text{for } i\text{=5,6}. \end{cases} \tag{20}$$

$$\tilde{Q}_i \equiv \begin{cases} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \text{for i=1,2,3,4}, \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \text{for i=5,6}, \end{cases} \tag{21}$$

$$\text{mass of beams} : m_b = \sum_{i=1}^{6} \int_{\Omega_i} dm_i, \quad \text{mass of the body} = m_s \tag{22}$$

$$\text{mass density of the beam} : \rho_i \equiv \rho_i(\xi_i), \quad m_\tau \equiv m_s + m_b \tag{23}$$

$$I_b \circ \omega = \sum_{i=1}^{6} \int \bar{r}_i \times (\omega \times \bar{r}_i) dm_i \tag{24}$$

$$I_\tau \equiv I_s + I_b, \quad I_s : \text{inertia of the body} \quad I_b : \text{inertia of the beams}. \tag{25}$$

Since the variations δR, $\delta\theta$ and δD_i are arbitrary in (13), $\mathcal{I}_i, i = 1, 2, 3$ are all zero provided $\mathcal{I}_4$=0 from the boundary conditions. Hence we obtain from this fact the following equations of motion[12,13]:

$\mathcal{I}_1$=0, that is,

$$m_\tau \frac{d^2R}{dt^2} + \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \Big\{ (\dot{\omega} \times \bar{r}_i) + \omega \times (\omega \times \bar{r}_i) + \ddot{D}_i + 2\omega \times \dot{D}_i \Big\} d\xi_i + \frac{G m_e m_\tau R}{|R|^3} = F_s, \tag{26}$$

$\mathcal{I}_2$=0, that is,

$$I_\tau \circ \dot{\omega} + \omega \times (I_\tau \circ \omega) + \dot{I}_b \circ \omega + \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \Big(\bar{r}_i \times \frac{d^2R}{dt^2} \Big) d\xi_i$$
$$+ \sum_{i=1}^{6} \int_{\Omega_i} \rho_i (\bar{r}_i \times \ddot{D}_i) d\xi_i + \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \Big\{ \omega \times (\bar{r}_i \times \dot{D}_i) \Big\} d\xi_i = T, \tag{27}$$

and for $\xi_i \in \Omega_i,\ \xi_j \in \Omega_j;\ i$=1,2,3,4, j=5,6, $\mathcal{I}_3$=0, that is,

$$\rho_i \frac{\partial^2}{\partial t^2}\begin{pmatrix} y_i \\ z_i \end{pmatrix} + \rho_i \tilde{Q}_i\left(\alpha_i + \frac{d^2R}{dt^2}\right) + \frac{\partial^2}{\partial \xi_i{}^2}\left(\begin{pmatrix} EI_{iy} & 0 \\ 0 & EI_{iz} \end{pmatrix}\frac{\partial^2}{\partial \xi_i{}^2}\begin{pmatrix} y_i \\ z_i \end{pmatrix}\right) = F_{b_i}, \tag{28}$$

$$\rho_j \frac{\partial^2}{\partial t^2}\begin{pmatrix} x_j \\ z_j \end{pmatrix} + \rho_j \tilde{Q}_j\left(\alpha_j + \frac{d^2R}{dt^2}\right) + \frac{\partial^2}{\partial \xi_j^2}\left(\begin{pmatrix} EI_{jx} & 0 \\ 0 & EI_{jz} \end{pmatrix}\frac{\partial^2}{\partial \xi_j^2}\begin{pmatrix} x_j \\ z_j \end{pmatrix}\right) = F_{b_j}. \tag{29}$$

Let C_B^I denote the coordinate transformation matrix from body to inertial frame and $(a_i, a_j, a_k)' \equiv (C_B^I)'$ $\frac{d^2}{dt^2}R$. Then the equations of motion given in (26-29) can be written in explicit form for computer simulation as follows.

BODY DYNAMICS(TRANSLATION)

$$m_\tau \frac{d^2}{dt^2}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + C_B^I \begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{pmatrix} + \begin{pmatrix} h_4(t) \\ h_5(t) \\ h_6(t) \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}, \tag{30}$$

where

$$f_1(t) = \sum_{i=1}^{6}\int_{\Omega_i}\rho_i\Big\{(\bar{z}_i\dot{\omega}_2 - \bar{y}_i\dot{\omega}_3) + 2(\omega_2\frac{\partial z_i}{\partial t} - \omega_3\frac{\partial y_i}{\partial t}) + \omega_1\omega_2\bar{y}_i + w_1 w_3\bar{z}_i - (w_2^2 + w_3^2)\bar{x}_i + \frac{\partial^2 x_i}{\partial t^2}\Big\}d\xi_i \tag{31}$$

$$f_2(t) = \sum_{i=1}^{6}\int_{\Omega_i}\rho_i\Big\{\bar{x}_i\dot{w}_3 - \bar{z}_i\dot{w}_1 + 2(w_3\frac{\partial x_i}{\partial t} - w_1\frac{\partial z_i}{\partial t}) + w_2 w_3\bar{z}_i + w_2 w_1\bar{x}_i - (w_3^2 + w_1^2)\bar{y}_i + \frac{\partial^2 y_i}{\partial t^2}\Big\}d\xi_i, \tag{32}$$

$$f_3(t) = \sum_{i=1}^{6}\int_{\Omega_i}\rho_i\Big\{\bar{y}_i\dot{w}_1 - \bar{x}_i\dot{w}_2 + 2(w_1\frac{\partial y_i}{\partial t} - w_2\frac{\partial x_i}{\partial t}) + w_3 w_1\bar{x}_i + w_3 w_2\bar{y}_i - (w_1^2 + w_2^2)\bar{z}_i + \frac{\partial^2 z_i}{\partial t^2}\Big\}d\xi_i, \tag{33}$$

$$h_4(t) = \frac{Gm_e m_\tau}{|R|^3}X, \tag{34}$$

$$h_5(t) = \frac{Gm_e m_\tau}{|R|^3}Y, \tag{35}$$

$$h_6(t) = \frac{Gm_e m_\tau}{|R|^3}Z, \tag{36}$$

BODY DYNAMICS(ROTATION)

$$I_\tau \begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix} + \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} I_\tau \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} + \begin{pmatrix} f_4 + f_7 \\ f_5 + f_8 \\ f_6 + f_9 \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}, \tag{37}$$

where

$$f_4(t) = \sum_{i=1}^{6}\int_{\Omega_i}\rho_i\Big\{\bar{y}_i\frac{\partial^2 z_i}{\partial t^2} - \bar{z}_i\frac{\partial^2 y_i}{\partial t^2} + 2w_1\bar{y}_i\frac{\partial y_i}{\partial t} + 2w_1\bar{z}_i\frac{\partial z_i}{\partial t} - 2w_2\bar{y}_i\frac{\partial x_i}{\partial t} - 2w_3\bar{z}_i\frac{\partial x_i}{\partial t}\Big\}d\xi_i, \tag{38}$$

$$f_5(t) = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \left\{ \bar{z}_i \frac{\partial^2 x_i}{\partial t^2} - \bar{x}_i \frac{\partial^2 z_i}{\partial t^2} + 2\omega_2 \bar{z}_i \frac{\partial z_i}{\partial t} + 2w_2 \bar{x}_i \frac{\partial x_i}{\partial t} - 2w_3 \bar{z}_i \frac{\partial y_i}{\partial t} - 2w_1 \bar{x}_i \frac{\partial y_i}{\partial t} \right\} d\xi_i, \tag{39}$$

$$f_6(t) = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \left\{ \bar{x}_i \frac{\partial^2 y_i}{\partial t^2} - \bar{y}_i \frac{\partial^2 x_i}{\partial t^2} + 2w_3 \bar{x}_i \frac{\partial x_i}{\partial t} + 2w_3 \bar{y}_i \frac{\partial y_i}{\partial t} - 2w_1 \bar{x}_i \frac{\partial z_i}{\partial t} - 2w_2 \bar{y}_i \frac{\partial z_i}{\partial t} \right\} d\xi_i, \tag{40}$$

$$f_7(t) = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \left\{ \bar{y}_i a_k - \bar{z}_i a_j \right\} d\xi_i, \tag{41}$$

$$f_8(t) = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \left\{ \bar{z}_i a_i - \bar{x}_i a_k \right\} d\xi_i, \tag{42}$$

$$f_9(t) = \sum_{i=1}^{6} \int_{\Omega_i} \rho_i \left\{ \bar{x}_i a_j - \bar{y}_i a_i \right\} d\xi_i, \tag{43}$$

BEAM DYNAMICS(BEAM VIBRATION)

$$\begin{aligned}
&\rho_i \frac{\partial^2}{\partial t^2} \begin{pmatrix} y_i \\ z_i \end{pmatrix} + \rho_i \begin{pmatrix} 0 & -2\omega_1 \\ 2\omega_1 & 0 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} y_i \\ z_i \end{pmatrix} + \frac{\partial^2}{\partial {\xi_i}^2} \left(\begin{pmatrix} EI_{iy} & 0 \\ 0 & EI_{iz} \end{pmatrix} \frac{\partial^2}{\partial {\xi_i}^2} \begin{pmatrix} y_i \\ z_i \end{pmatrix} \right) \\
&\quad + \rho_i \begin{pmatrix} -\omega_1^2 - \omega_3^2 & -\dot{\omega}_1 + \omega_2\omega_3 \\ \dot{\omega}_1 + \omega_2\omega_3 & -\omega_1^2 - \omega_2^2 \end{pmatrix} \begin{pmatrix} \bar{y}_i \\ \bar{z}_i \end{pmatrix} + \rho_i \bar{x}_i \begin{pmatrix} \dot{\omega}_3 + \omega_1\omega_2 \\ -\dot{\omega}_2 + \omega_1\omega_3 \end{pmatrix} \\
&\quad + \rho_i \tilde{Q}_i C_I^B \frac{d^2 R}{dt^2} = \begin{pmatrix} F_{i_y} \\ F_{i_z} \end{pmatrix}, \qquad \text{for } \xi_i \in \Omega_i,\ i = 1,2,3,4,
\end{aligned} \tag{44}$$

$$\begin{aligned}
&\rho_j \frac{\partial^2}{\partial t^2} \begin{pmatrix} x_j \\ z_j \end{pmatrix} + \rho_j \begin{pmatrix} 0 & 2\omega_2 \\ -2\omega_2 & 0 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} x_j \\ z_j \end{pmatrix} + \frac{\partial^2}{\partial \xi_j^2} \left(\begin{pmatrix} EI_{jx} & 0 \\ 0 & EI_{jz} \end{pmatrix} \frac{\partial^2}{\partial \xi_j^2} \begin{pmatrix} x_j \\ z_j \end{pmatrix} \right) \\
&\quad + \rho_j \begin{pmatrix} -w_2^2 - w_3^2 & \dot{w}_2 + w_3w_1 \\ -\dot{w}_2 + w_3w_1 & -w_1^2 - w_2^2 \end{pmatrix} \begin{pmatrix} \bar{x}_j \\ \bar{z}_j \end{pmatrix} + \rho_j \bar{y}_j \begin{pmatrix} -\dot{w}_3 + w_2w_1 \\ \dot{w}_1 + w_2w_3 \end{pmatrix} \\
&\quad + \rho_j \tilde{Q}_j C_I^B \frac{d^2 R}{dt^2} = \begin{pmatrix} F_{j_x} \\ F_{j_z} \end{pmatrix}, \qquad \text{for } \xi_j \in \Omega_j,\ j = 5,6.
\end{aligned} \tag{45}$$

BOUNDARY CONDITIONS FOR BEAM DYNAMICS

Since the beams(1-4) are clamped to the central body and the end beams(5,6) are rigidly jointed to the first four beams as shown in Fig.2, the following boundary conditions must hold.

Clamped at $\xi_i = 0$, i=1,2,3,4 for the joints $\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4$:

$$y_i(0,t) = 0, \qquad z_i(0,t) = 0, \tag{46}$$

$$\frac{\partial y_i}{\partial \xi_i}(0,t) = 0, \qquad \frac{\partial z_i}{\partial \xi_i}(0,t) = 0, \tag{47}$$

Continuity of displacements at the joints $\mathcal{J}_5, \mathcal{J}_6, \mathcal{J}_7, \mathcal{J}_8$:

$$x_5(l_5,t) = 0, \;\; z_1(l_1,t) = z_5(l_5,t), \;\; at\ \mathcal{J}_5, \tag{48}$$

$$x_5(0,t) = 0, \;\; z_2(l_2,t) = z_5(0,t), \;\; at\ \mathcal{J}_6, \tag{49}$$

$$x_6(l_6,t) = 0, \;\; z_3(l_3,t) = z_6(l_6,t), \;\; at\ \mathcal{J}_7, \tag{50}$$

$$x_6(0,t) = 0, \;\; z_4(l_4,t) = z_6(0,t), \;\; at\ \mathcal{J}_8, \tag{51}$$

Continuity of slopes at the joints $\mathcal{J}_5, \mathcal{J}_6, \mathcal{J}_7, \mathcal{J}_8$:

$$\frac{\partial y_1}{\partial \xi_1}(l_1,t) = -\frac{\partial x_5}{\partial \xi_5}(l_5,t), \text{ at } \mathcal{J}_5, \tag{52}$$

$$\frac{\partial y_2}{\partial \xi_2}(l_2,t) = -\frac{\partial x_5}{\partial \xi_5}(0,t), \text{ at } \mathcal{J}_6, \tag{53}$$

$$\frac{\partial y_3}{\partial \xi_3}(l_3,t) = \frac{\partial x_6}{\partial \xi_6}(l_6,t), \text{ at } \mathcal{J}_7, \tag{54}$$

$$\frac{\partial y_4}{\partial \xi_4}(l_4,t) = \frac{\partial x_6}{\partial \xi_6}(0,t), \text{ at } \mathcal{J}_8, \tag{55}$$

Equilibrium equations:

$$\text{at } \mathcal{J}_5, \quad EI_{5z}\frac{\partial^2 z_5}{\partial \xi_5^2}(l_5,t) = 0, \quad EI_{1z}\frac{\partial^2 z_1}{\partial \xi_1^2}(l_1,t) = 0, \tag{56}$$

$$\text{at } \mathcal{J}_6, \quad EI_{5z}\frac{\partial^2 z_5}{\partial \xi_5^2}(0,t) = 0, \quad EI_{2z}\frac{\partial^2 z_2}{\partial \xi_2^2}(l_2,t) = 0, \tag{57}$$

$$\text{at } \mathcal{J}_7, \quad EI_{6z}\frac{\partial^2 z_6}{\partial \xi_6^2}(l_6,t) = 0, \quad EI_{3z}\frac{\partial^2 z_3}{\partial \xi_3^2}(l_3,t) = 0, \tag{58}$$

$$\text{at } \mathcal{J}_8, \quad EI_{6z}\frac{\partial^2 z_6}{\partial \xi_6^2}(0,t) = 0, \quad EI_{4z}\frac{\partial^2 z_4}{\partial \xi_4^2}(l_4,t) = 0, \tag{59}$$

$$EI_{1y}\frac{\partial^3 y_1}{\partial \xi_1^3}(l_1,t) = 0, \qquad EI_{3y}\frac{\partial^3 y_3}{\partial \xi_3^3}(l_3,t) = 0, \tag{60}$$

$$EI_{2y}\frac{\partial^3 y_2}{\partial \xi_2^3}(l_2,t) = 0, \qquad EI_{4y}\frac{\partial^3 y_4}{\partial \xi_4^3}(l_4,t) = 0, \tag{61}$$

$$EI_{1z}\frac{\partial^3 z_1}{\partial \xi_1^3}(l_1,t) + EI_{5z}\frac{\partial^3 z_5}{\partial \xi_5^3}(l_5,t) = 0, \tag{62}$$

$$EI_{3z}\frac{\partial^3 z_3}{\partial \xi_3^3}(l_3,t) + EI_{6z}\frac{\partial^3 z_6}{\partial \xi_6^3}(l_6,t) = 0, \tag{63}$$

$$EI_{2z}\frac{\partial^3 z_2}{\partial \xi_2^3}(l_2,t) - EI_{5z}\frac{\partial^3 z_5}{\partial \xi_5^3}(0,t) = 0, \tag{64}$$

$$EI_{4z}\frac{\partial^3 z_4}{\partial \xi_4^3}(l_4,t) - EI_{6z}\frac{\partial^3 z_6}{\partial \xi_6^3}(0,t) = 0, \tag{65}$$

$$EI_{1y}\frac{\partial^2 y_1}{\partial \xi_1^2}(l_1,t) - EI_{5x}\frac{\partial^2 x_5}{\partial \xi_5^2}(l_5,t) = 0, \tag{66}$$

$$EI_{3y}\frac{\partial^2 y_3}{\partial \xi_3^2}(l_3,t) + EI_{6x}\frac{\partial^2 x_6}{\partial \xi_6^2}(l_6,t) = 0, \tag{67}$$

$$EI_{2y}\frac{\partial^2 y_2}{\partial \xi_2^2}(l_2,t) + EI_{5x}\frac{\partial^2 x_5}{\partial \xi_5^2}(0,t) = 0, \tag{68}$$

$$EI_{4y}\frac{\partial^2 y_4}{\partial \xi_4^2}(l_4,t) - EI_{6x}\frac{\partial^2 x_6}{\partial \xi_6^2}(0,t) = 0. \tag{69}$$

3. STABILIZATION OF SPACE STATIONS

3.1 The Reference Trajectory

Let R_0 denote the reference trajectory(nominal trajectory) governed by the equation

$$\frac{d^2 R_0}{dt^2} + \frac{G m_e R_0}{|R_0|^3} = 0 \,, \tag{70}$$

with the initial conditions:

$$\frac{dR_0}{dt} \times \left(R_0 \times \frac{dR_0}{dt} \right) = \frac{G m_e R_0}{|R_0|}, \qquad \text{at } t = 0. \tag{71}$$

Denoting the excursion of the radial vector R around the nominal trajectory by $\tilde{R} \equiv R - R_0 = (\tilde{X}, \tilde{Y}, \tilde{Z})'$ we obtain from eqs.(26) and (70) the perturbed radial dynamics:

$$m_T \frac{d^2 \tilde{R}}{dt^2} + \sum_{i=1}^{6} \frac{d}{dt} \int_{\Omega_i} \rho_i \left(\omega \times \bar{r}_i + \dot{D}_i \right) d\xi_i + \frac{G m_e m_T \tilde{R}}{|R_0|^3} = F_s. \tag{72}$$

3.2 The Rest State

For stability of the system subject to external disturbances we consider the rest state:

$$w_1 = 0, \quad w_2 = 0, \quad w_3 = 0, \tag{73}$$

$$\tilde{X}(t) = \tilde{Y}(t) = \tilde{Z}(t) = 0, \quad \tilde{v}_1(t) = \tilde{v}_2(t) = \tilde{v}_3(t) = 0, \tag{74}$$

and for $k = 5, 6;\ j = 1, 2, 3, 4;\ i = 1, 2, ..., 6,$

$$x_k(\xi_k, t) = y_j(\xi_j, t) = z_i(\xi_i, t) = 0, \quad \frac{\partial x_k}{\partial t}(\xi_k, t) = \frac{\partial y_j}{\partial t}(\xi_j, t) = \frac{\partial z_i}{\partial t}(\xi_i, t) = 0, \tag{75}$$

where $\tilde{v} \equiv \frac{d\tilde{R}}{dt} \equiv (\tilde{v}_1, \tilde{v}_2, \tilde{v}_3)'$.

Theorem 1 **(Distributed Control)**

Consider the perturbed system described by the equations (30-69) around the reference trajectory. Suppose that the controls applied to the system are given by the feedback law:

$$T = (T_x - k_1 \omega_1,\ T_y - k_2 \omega_2,\ T_z - k_3 \omega_3)', \quad k_1, k_2, k_3 > 0, \tag{76}$$

$$F_{b_i} = Q_i \left(\tilde{A}_{i_x} - d_{i_1} \frac{\partial x_i}{\partial t}, \tilde{A}_{i_y} - d_{i_2} \frac{\partial y_i}{\partial t},\ \tilde{A}_{i_z} - d_{i_3} \frac{\partial z_i}{\partial t} \right)', \quad d_{i_1}, d_{i_2}, d_{i_3} > 0,\ i = 1, 2, ..., 6, \tag{77}$$

$$F_s = (-e_1 \tilde{v}_1,\ -e_2 \tilde{v}_2,\ -e_3 \tilde{v}_3)', \qquad e_1, e_2, e_3 > 0, \tag{78}$$

where $(\tilde{A}_{i_x}, \tilde{A}_{i_y}, \tilde{A}_{i_z})' \equiv -\rho_i \frac{G m_e}{|R_0|^3} R_0$ and $(T_x, T_y, T_z)' \equiv -\frac{G m_e}{|R_0|^3} (\sum_{i=1}^{6} \int_{\Omega_i} \rho_i \bar{r}_i d\xi_i) \times R_0$. Then the system is asymptotically stable (in the sense of Lyapunov) with respect to the rest state (73-75).

Proof see the refs.[12,13].

Remark

Defining the state variable x appropriately and incorporating the boundary conditions in the differential operator one can rewrite the system equation as an ordinary differential equation in a Banach space (probably Orlicz space) as follows

$$\dot{x} = Ax + F(x, \dot{x}), \tag{79}$$

where A can be proved to be the infinitesimal generator of a C_0- semigroup in the Banach space and F is a strongly nonlinear operator having polinomial growth. This is a descriptor system in an infinite dimensional space and very little is known about these systems concerning the existence and uniqueness of nonlinear semigroups.

4 NUMERICAL RESULTS

For numerical simulation we assume that (i) the bus inertia tensor is diagonal. (ii) the flexible members consist of three beams (i.e., beams 1,2 and 5 in Fig.4) and each beam is uniform.

The following data and parameters were used.

System Parameters:

I_s=diag(I_1, I_2, I_3) = diag(31750, 5000, 33450) $slug\ \ ft^2$

the length of the beams: $l_1 = l_2 = 187.7\ ft$, $l_5 = 66\ ft$,

Flexural rigidity: $EI_{5x} = EI_{5z} = 10^5 lb\ ft^2$, $EI_{iy} = EI_{iz} = 3565.5\ lb\ ft^2$, for i=1,2,5,

Mass density: $\rho_i = 8.25 \times 10^{-2}\ slug/ft$,

$D_{1_0} = (3+ \xi_1, 33, 0)'$, $D_{2_0} = (3+ \xi_2, -33, 0)'$, $D_{5_0} = (190.7, -33+ \xi_5, 0)' ft$, $m_s = 300\ slug$.

Initial Conditions:

ω_1 (0)=0.03, $\omega_2(0)$ =0.02, $\omega_3(0)$ = 0.01 rad/sec and for $\xi_i \in [0, l_i]$ and i=1,2, 5, $\frac{\partial x_i}{\partial t}(\xi_i, 0) = \frac{\partial y_i}{\partial t}(\xi_i, 0) = \frac{\partial z_i}{\partial t}(\xi_i, 0) = 0$, $x_i(\xi_i, 0) = y_i(\xi_i, 0) = z_i(\xi_i, 0) = \phi_o(\xi_i)$, where ϕ_o satisfies the boundary conditions (equations 3.97-3.120).

Control Gains:

k_1 =30000, k_2 =10000, k_3 =100 for $t \in [0, 2.5]$ and k_3 =50000 for $t \in [2.5, 10]$; $d_{i_1} = d_{i_2} = d_{i_3} = 0.05$ for i= 1,2,5; e_1=9000, e_2=3000, e_3=6000. $F_1(t) = F_2(t) = F_3(t) = 0$.

Slew maneuver using Bang -Pause -Bang control with the parameters:

T_{s_1}=10035, t_{s_1}=0.5sec, t_{s_2}=2.0sec, t_{s_3}=2.5sec.

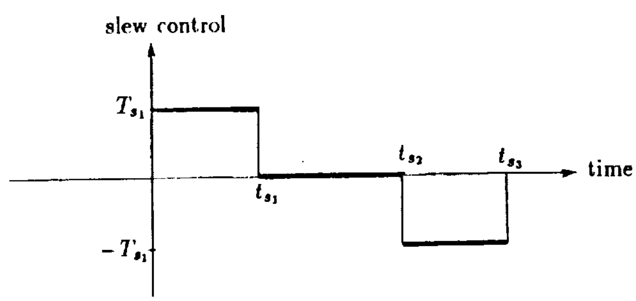


Fig.3 Bang-Pause-Bang Control.

Detailed numerical results showing stabilization of the various state trajectories were obtained from the simulation and presented in Figs. 4-23. Figs.4 and 5 represent the slew angle and the angular rate of the spacecraft corresponding to the slew control (Fig.3). We note from the Fig.4 that slew maneuver was successfully achieved by the slew control commands. As one can see from Eqs.(30-45), the body translation, body rotation and vibration of beams are strongly coupled. This implies that any perturbation in one part of the system will induce disturbances in the other members of the system. Hence the slew maneuver induced significant perturbations leading to beam vibrations and body rotation. This is clearly observed from the responses without controls. From Figs.4-23 it is clear that without stabilizing controls beam vibrations, body oscillations and radial excursions persist or grow. However, with application of the proposed feedback controls, oscillations induced by the slew maneuver are effectively eliminated throughout the entire system.

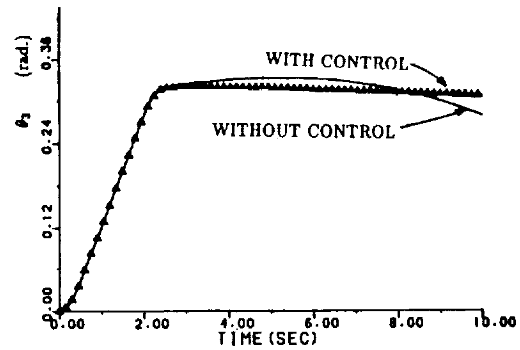


Fig. 4 Slew angle θ_3 in k_b direction.

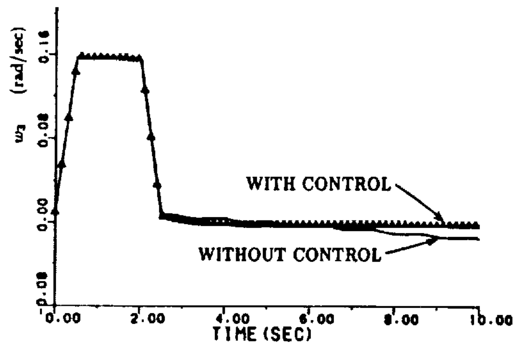


Fig. 5 Bus angular velocity w_3.

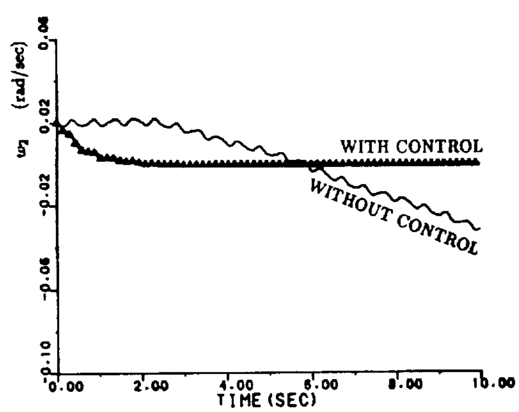


Fig. 6 Bus angular velocity w_2.

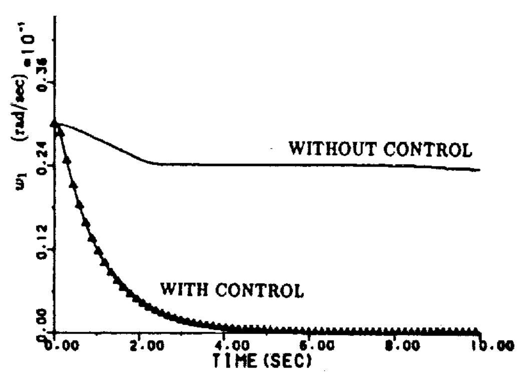


Fig. 7 Bus angular velocity w_1.

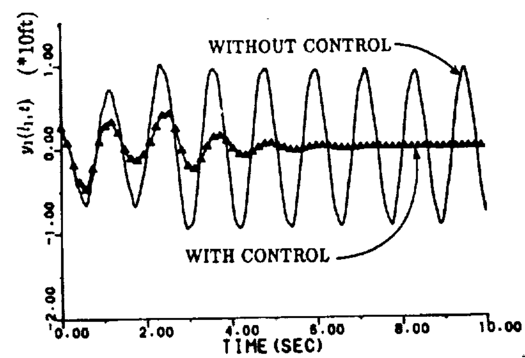


Fig. 8 Beam displacement $y_1(l_1, t)$.

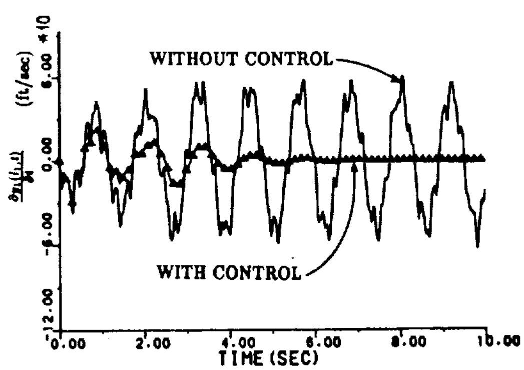


Fig. 9 Beam velocity $\frac{\partial y_1(l_1,t)}{\partial t}$.

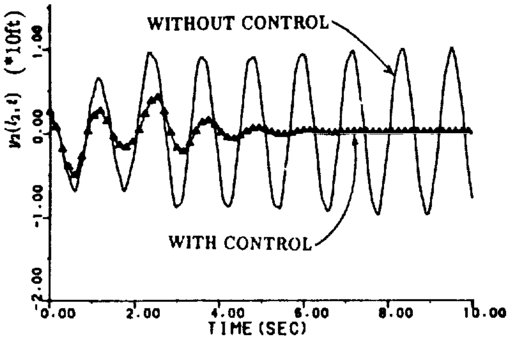


Fig. 10 Beam displacement $y_2(l_2,t)$.

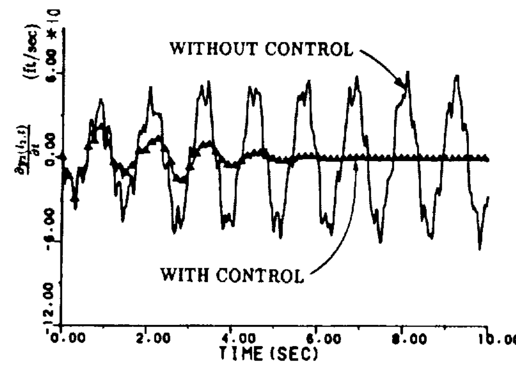


Fig 11 Beam velocity $\frac{\partial y_2(l_2,t)}{\partial t}$.

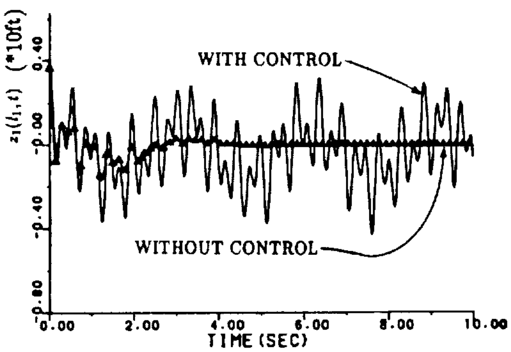


Fig. 12 Beam displacement $z_1(l_1,t)$.

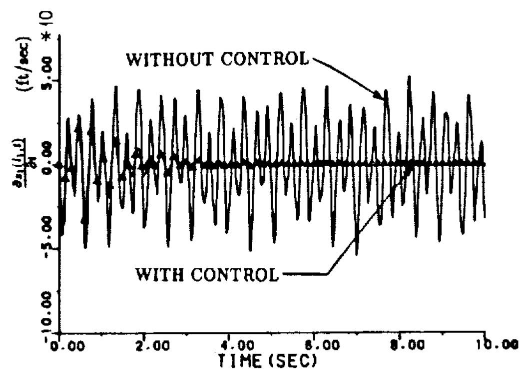


Fig. 13 Beam velocity $\frac{\partial z_1(l_1,t)}{\partial t}$.

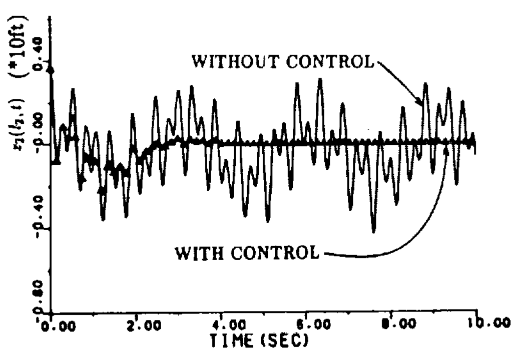


Fig. 14 Beam displacement $z_2(l_2,t)$.

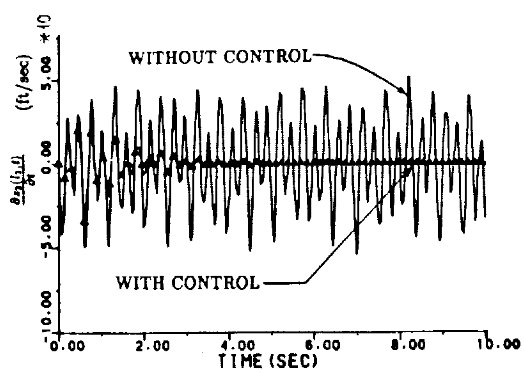


Fig. 15 Beam velocity $\frac{\partial z_2(l_2,t)}{\partial t}$.

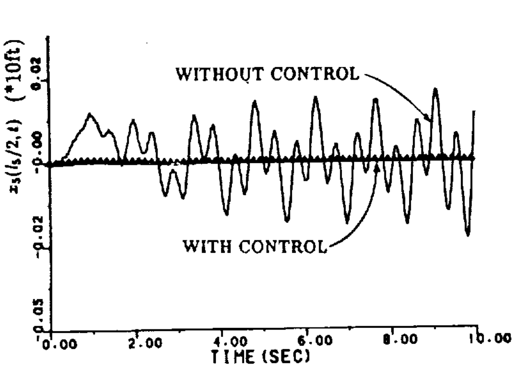


Fig. 16 Beam displacement $z_5(l_5/2,t)$.

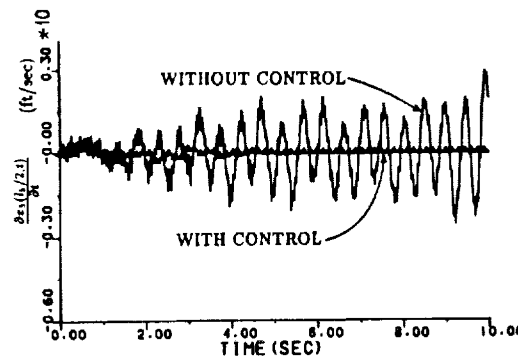


Fig. 17 Beam velocity $\frac{\partial z_5(l_5/2,t)}{\partial t}$.

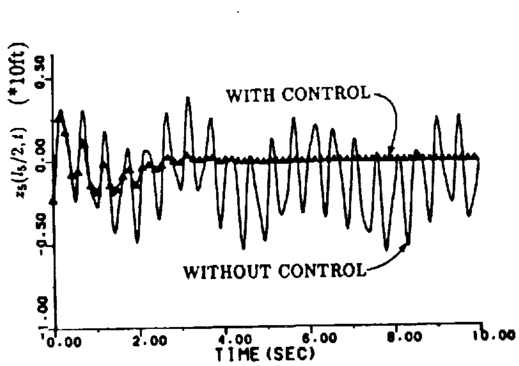


Fig. 18 Beam displacement $z_5(l_5/2,t)$.

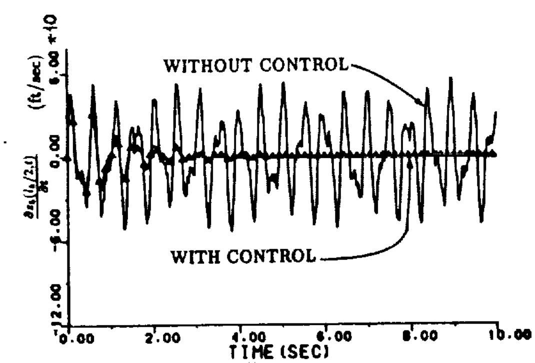


Fig. 19 Beam velocity $\frac{\partial z_5(l_5/2,t)}{\partial t}$.

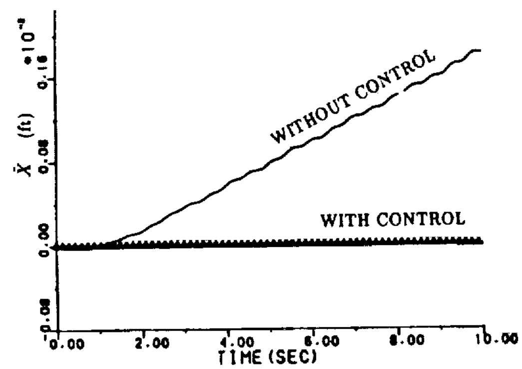


Fig. 20 Radial perturbation $\tilde{X}$.

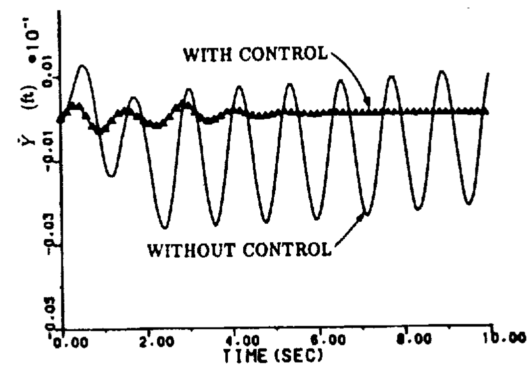


Fig. 21 Radial perturbation $\tilde{Y}$.

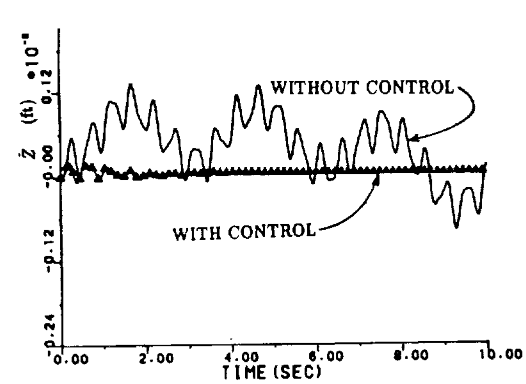


Fig. 22 Radial perturbation $\tilde{Z}$.

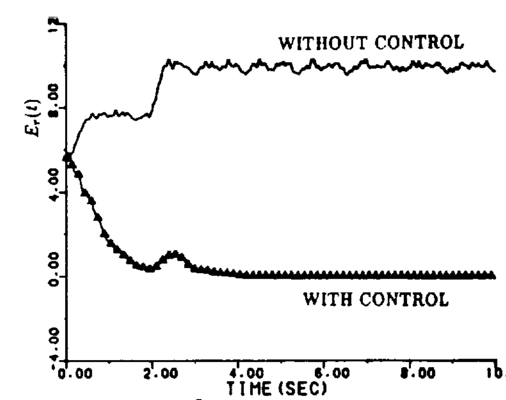


Fig. 23 Relative beam energy.

5. CONCLUSIONS

In this paper, suppression of vibration induced by slew maneuver in flexible space stations has been considered. Based on the dynamic model developed by the authors[13], asymptotic stability of the system subject to perturbation is investigated. The rotational perturbing forces were applied to the system and their corresponding stabilizations have been demonstrated. From the numerical results it is clearly observed that (i)if the disturbances following external perturbing forces persist, then in the absence of proper controls, these small motions may build up leading to instability of the entire system, (ii)during the slew maneuver, vibration in the beams and oscillations in the angular velocities of the body are induced and it has been shown that the stabilizing control can effectively eliminate the oscillations and stabilize the entire system.

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