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Geometric Programming Prediction of Design Trends for OMV Protective Structures

R. A. Mog and J. R. Horn

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R. A. Mog
Science Applications International Corporation
Huntsville, Alabama

J. R. Horn
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

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NOTATION

a_i	system exponent for impact variable i
d	projectile diameter
K_1	impact predictor coefficient
$\overline{K}$	impact system constant
t_1	first facing sheet thickness
t_2	second facing sheet thickness
t_c	honeycomb core thickness
V	projectile impact velocity
W	system mass per unit area
δ_i	geometric programming dual variable
θ	projectile impact angle
ν	geometric programming dual objective function
ρ	projectile mass-density
ρ_1	first facing sheet mass-density
ρ_2	second facing sheet mass-density
ρ_c	honeycomb core mass-density

TECHNICAL MEMORANDUM

GEOMETRIC PROGRAMMING PREDICTION OF DESIGN TRENDS FOR OMV PROTECTIVE STRUCTURES

I. INTRODUCTION

The orbital maneuvering vehicle (OMV) design activity presents many challenges to the design engineer. The OMV will be used to transfer Earth-orbiting satellites to different orbits and will then be returned to Earth aboard the space shuttle orbiter. The design of an optimized protective shield which can prevent orbital debris particles from impacting and damaging the OMV could be a major design and development effort. Thousands of large orbiting man-made particles are being tracked each year by the space science community; many more smaller untrackable particles are known to exist from evidence of impact damage on returned satellites. The OMV, with exposed high-pressure fuel bottles, must be adequately protected for its average orbital lifetime of 7 days to prevent possible penetration and catastrophic rupture of the bottles.

As for any launched space vehicle, weight is a primary structural concern. The geometric programming optimization technique has been employed to optimize the metallic honeycomb structure, which is currently required as a stiffening panel on the OMV configuration. This technique determines the facing sheet and core areal densities needed to meet a protection requirement at the minimum structural weight.

This report covers the first step in using geometric programming to optimize the honeycomb panel configuration. By parametrically relating the material design variables in model forms indicated from past experience, general trends can be predicted for the honeycomb panel as an optimized protective structure. This information can then be used as a stepping stone for further work in this area by providing researchers with clearer insight into the relationships of the design parameters.

II. PROBLEM FORMULATION

The formulation of the protective structures design optimization problem begins with the specification of an objective function in the form of engineering models and the statement of applicable constraints in terms of the design variables. For the minimization of protective structures weight, it has been shown that an objective function in the form of structural mass per unit area is sufficiently representative, provided the spacecraft configuration to be protected is not dominated by high curvature surfaces [1,2,3]. This requirement is met by OMV. For the honeycomb configuration shown in figure 1, the system mass per unit area is given by

$$W = \rho_1 t_1 + \rho_2 t_2 + \rho_c t_c \quad . \tag{1}$$

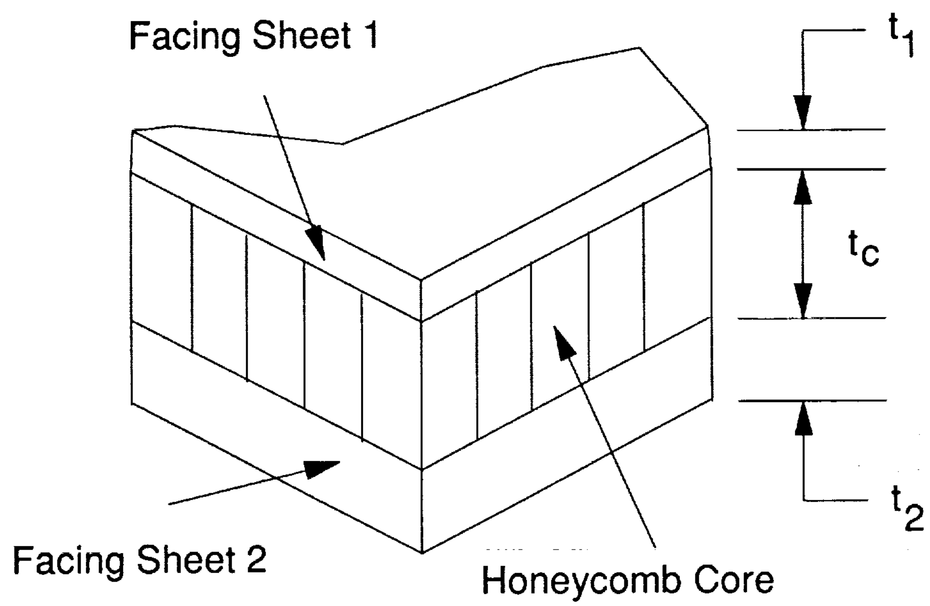

Figure 1. Notation for honeycomb structure configuration.

The wall thickness required to resist penetration/spallation is a function of the impact parameters, structural material properties, and structural configuration [1,3]. For the purposes of this development, we assume a single term posynomial (or monomial) relationship given by

$$t_2 = K_1 \rho_1^{a_1} t_1^{a_2} \rho_c^{a_3} t_c^{a_4} d^{a_5} \rho^{a_6} V^{a_7} (\tan(\theta))^{a_8} \rho_2^{a_{10}} \quad . \tag{2}$$

We will see later that this assumption is not particularly restrictive and can be greatly generalized with equivalent results. Five different problem formulations are considered by substituting equation (2) into equation (1). These formulations involve different combinations of independent variables. Case (i) considers T_1 and t_c as independent variables. Case (ii) also includes the material densities of the facing sheets and core. Case (iii) is the same as case (ii) but without the second facing sheet material density as an independent variable. Case (iv) is the same as case (iii) with some dimensional analysis considerations to reduce the number of required estimated parameters. Finally, case (v) is the same as case (iv) with the second facing sheet material density independent.

III. STRUCTURAL OPTIMIZATION USING GEOMETRIC PROGRAMMING

The five cases are solved by maximizing the corresponding dual problem subject to the derived linear constraints. The dual objective function takes the product form as specified by the arithmetic-geometric inequality [4-7]. The linear constraints include normality as well as orthogonality constraints. The normality constraint merely specifies that the dual variables sum to 1. This is a required condition for the application of the arithmetic-geometric inequality. The orthogonality conditions specify for each independent variable that the product of the appropriate dual variable with the independent variable's exponent summed over all of the terms in the primal posynomial be equal to 0. The main reason for transforming the problem to a dual form is to take advantage of the linear constraints in a low degree of difficulty problem. Additionally, the dual problem results in analytic solutions for zero degree of difficulty problems as given here. Finally, geometric programming provides globally optimal solutions for problems in posynomial form [4–7]. For **case (i)**, the dual problem is given by

$$\max \nu(\delta) = \left(\frac{\rho_1}{\delta_1}\right)^{\delta_1} \left(\frac{K_1 \rho_2^{1+a_{10}} \rho_c^{a_3} d^{a_5} \rho^{a_6} V^{a_7} (\tan(\theta))^{a_8}}{\delta_2}\right)^{\delta_2} \left(\frac{\rho_c}{\delta_3}\right)^{\delta_3} \quad , \tag{3}$$

subject to

$$A_1 \vec{\delta} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad , \tag{4}$$

$$A_1 = \begin{pmatrix} 1 & a_2 & 0 \\ 0 & a_4 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad . \tag{5}$$

The solution is given by

$$\vec{\delta} = \begin{pmatrix} \dfrac{a_2}{a_2 + a_4 - 1} \\ \dfrac{-1}{a_2 + a_4 - 1} \\ \dfrac{a_4}{a_2 + a_4 - 1} \end{pmatrix} \quad . \tag{6}$$

Since the dual variables must all be positive, we have

$$a_2 \leq 0 \qquad a_4 \leq 0 \quad . \tag{7}$$

Objective function solution and conversion to the primal variables will be deferred until case (v). For **case (ii)**, the linear constraints are given by

$$A_2 \vec{\delta} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad , \tag{8}$$

where

$$A_2 = \begin{pmatrix} 1 & a_2 & 0 \\ 0 & a_4 & 1 \\ 1 & a_1 & 0 \\ 0 & 1+a_{10} & 0 \\ 0 & a_3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad . \tag{9}$$

This system is inconsistent unless

$$a_{10} = -1 \quad . \tag{10}$$

If condition (10) did not hold, then each of the dual variables would be identically zero. This would contradict the normality condition that the dual variables must sum to 1. If condition (10) holds then

$$a_1 = a_2 \qquad a_3 = a_4 \quad , \tag{11}$$

and (6) again provides the solution for the dual variables.

Case (iii) results in the same solution as case (ii) without requirement (10). In **case (iv)**, the structural mass per unit areas are normalized by the projectile diameter density product to assure dimensionless variables and reduce the amount of regression required. The functional relationship is given by

$$t_2 = K_1\left(\frac{\rho_1 t_1}{\rho d}\right)^{a_2}\left(\frac{\rho_c t_c}{\rho d}\right)^{a_4} V^{a_7}(\tan(\theta))^{a_8}\rho_2^{a_{10}} \quad , \tag{12}$$

with the dual problem given by

$$\max v(\delta) = \left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{K_1\rho_2^{1+a_{10}}V^{a_7}(\tan(\theta))^{a_8}}{(\rho d)^{a_2+a_4}\delta_2}\right)^{\delta_2}\left(\frac{1}{\delta_3}\right)^{\delta_3} \quad , \tag{13}$$

subject to

$$A_4\vec{\delta} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad , \tag{14}$$

$$A_4 = \begin{pmatrix} 1 & a_2 & 0 \\ 0 & a_4 & 1 \\ 1 & a_2 & 0 \\ 0 & a_4 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad . \tag{15}$$

The dual variable solution is again given by (6). **Case (v)** includes second facing sheet density as an independent variable. The results are identical to case (iv) with requirement (10) included. The minimum system mass per unit area for the structure is found by substituting (6) into (13) and is given by

$$W_0 = \left(\frac{a_2 + a_4 - 1}{a_2}\right)^{\frac{a_2}{a_2+a_4-1}} \left(\frac{\overline{K}(a_2 + a_4 - 1)}{-1}\right)^{\frac{-1}{a_2+a_4-1}} \left(\frac{a_2 + a_4 - 1}{a_4}\right)^{\frac{a_4}{a_2+a_4-1}} \quad , \tag{16}$$

where

$$\overline{K} = \frac{K_1 V^{a_7}(\tan(\theta))^{a_8}}{(\rho d)^{a_2+a_4}} \quad . \tag{17}$$

Additionally, the optimal mass per unit areas for the first facing sheet, the second facing sheet, and the honeycomb core are found by multiplying the minimum system mass per unit area by the corresponding dual variable. These values are given by

$$\rho_{1_0} t_{1_0} = \left(\frac{a_2 + a_4 - 1}{a_2}\right)^{\frac{1-a_4}{a_2+a_4-1}} \left(\frac{\overline{K}(a_2 + a_4 - 1)}{-1}\right)^{\frac{-1}{a_2+a_4-1}} \left(\frac{a_2 + a_4 - 1}{a_4}\right)^{\frac{a_4}{a_2+a_4-1}} \quad , \tag{18}$$

$$\rho_{2_0} t_{2_0} = \left(\frac{a_2 + a_4 - 1}{a_2} \right)^{\frac{a_2}{a_2 + a_4 - 1}} (K)^{\frac{-1}{a_2 + a_4 - 1}} \left(\frac{a_2 + a_4 - 1}{-1} \right)^{\frac{-(a_2 + a_4)}{a_2 + a_4 - 1}} \left(\frac{a_2 + a_4 - 1}{a_4} \right)^{\frac{a_4}{a_2 + a_4 - 1}} , \tag{19}$$

$$\rho_{c_0} t_{c_0} = \left(\frac{a_2 + a_4 - 1}{a_2} \right)^{\frac{a_2}{a_2 + a_4 - 1}} \left(\frac{K(a_2 + a_4 - 1)}{-1} \right)^{\frac{-1}{a_2 + a_4 - 1}} \left(\frac{a_2 + a_4 - 1}{a_4} \right)^{\frac{1 - a_2}{a_2 + a_4 - 1}} . \tag{20}$$

IV. PARAMETRIC DESIGN TRENDS

This section gives parametric trends for solutions (16) to (20) in terms of the impact system constant, a_1 $(=a_2)$ and a_3 $(=a_4)$. Figure 2 shows the optimal mass per unit areas as functions of the impact system constant for $a_2 = -0.5$ and $a_4 = -0.5$. The first sheet and honeycomb core lines are equal and bounded above by the second sheet mass per unit area.

Figure 3 shows the decreasing relationship between minimum system mass per unit area and a_4, the honeycomb core mass per unit area exponent. Several curves are shown, corresponding to different values of a_1 $(=a_2)$. The case $a_1 = 0$ corresponds to a hypothetical structural configuration without a first sheet material. As the honeycomb core exponent increases, the core is less effective in terms of penetration resistance, and the structural configuration increasingly becomes a bumper/wall situation. The crossings of the $a_1 = -0.75$ and $a_1 = -1.0$ lines represent a relative tradeoff in the utility of the honeycomb core and the penalty imposed by its weight.

Figure 4 (fig. 6) shows the nonmonotonic relationship between optimal first (second) sheet mass per unit area and a_4 for various values of a_2. Increases in a_4 imply a decreasing role in the honeycomb core, which requires increases in the first and second sheet mass per unit areas, to a point. Beyond that point (corresponding to $a_4 = -0.25$), the honeycomb structure becomes increasingly inefficient relative to a bumper/wall configuration. At this point, decreases in first and second sheet mass per unit areas are in order.

Figure 5 shows the sharp decreases in optimal honeycomb core mass per unit area as a function of a_4. Note that the anomalous curve $a_1 = 0$ is bounded by $a_1 = -0.75$ and $a_1 = -1.0$.

Figures 7, 8, and 9 show the relationships of optimal ratios for the first, second, and honeycomb core materials as a function of a_4 for various values of a_1. This ratio is defined as the optimal mass per unit area of the material divided by the minimum system mass per unit area and is given by the dual variable values given by equation (6). Note that for the first and second sheets, this ratio is monotonically increasing, while for the honeycomb core, the ratio is monotonically decreasing.

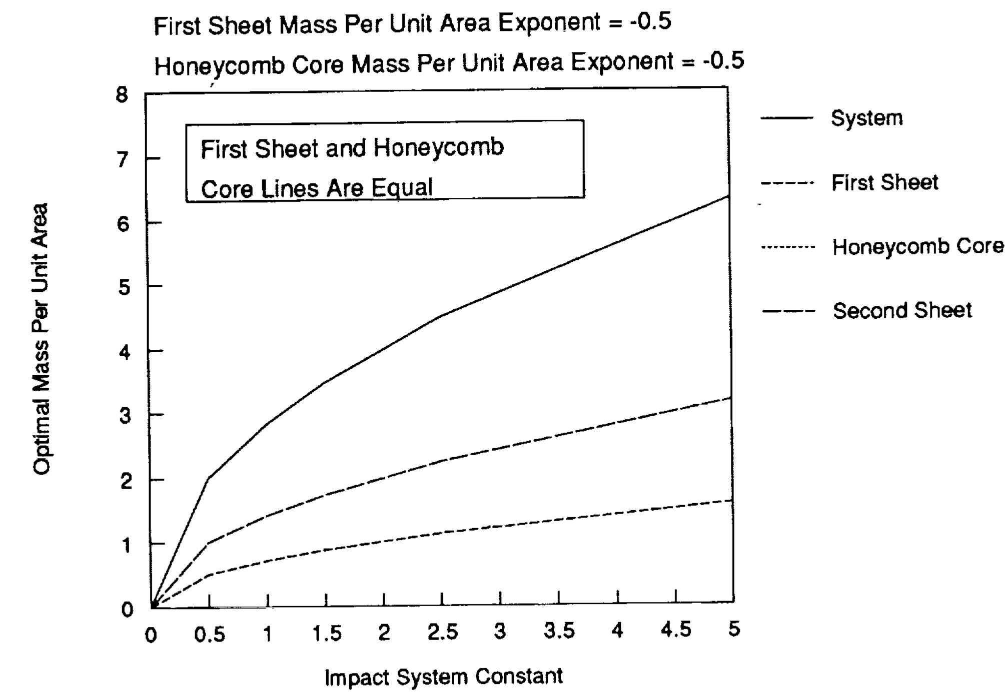

Figure 2. Optimal mass per unit areas versus impact system constant ($\overline{K}$).

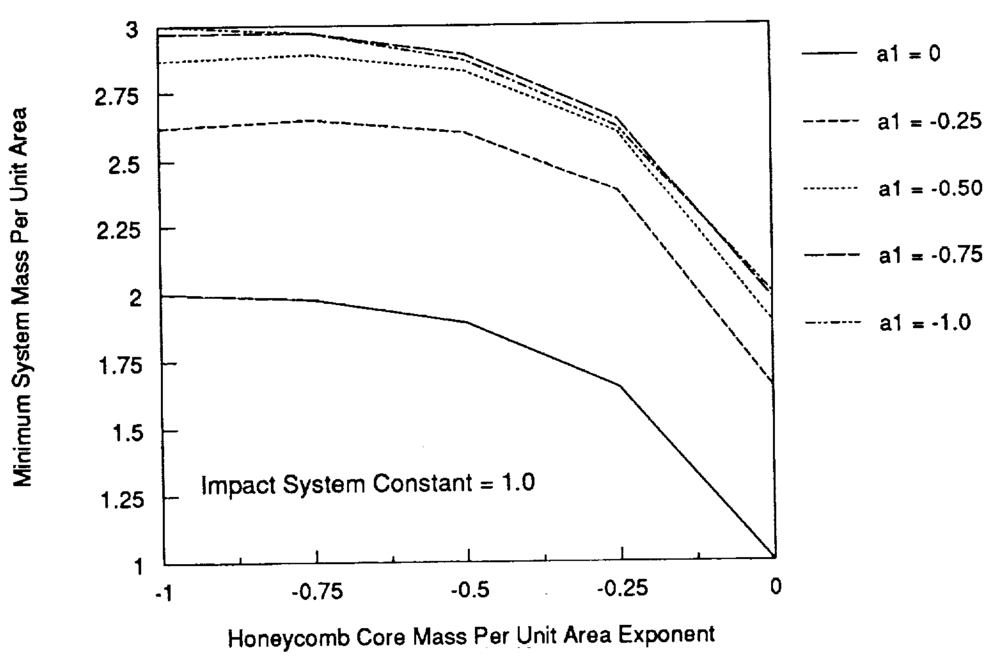

Figure 3. Minimum system mass per unit area versus honeycomb core mass per unit area exponent (a_4).

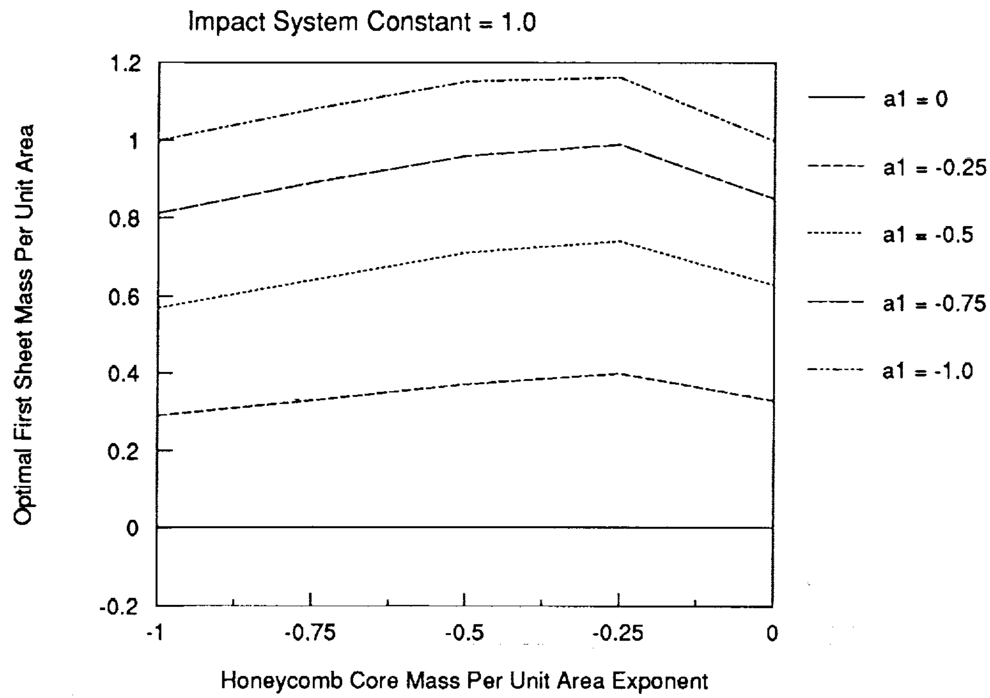

Figure 4. Optimal first sheet mass per unit area versus honeycomb core mass per unit area exponent (a_4).

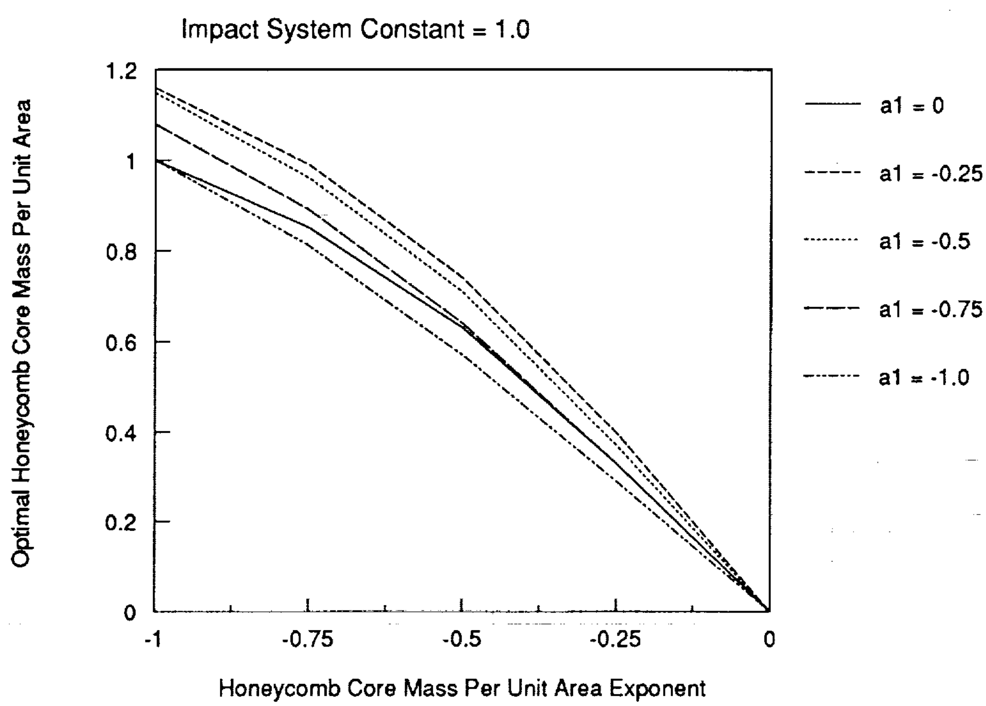

Figure 5. Optimal honeycomb core mass per unit area versus honeycomb core mass per unit area exponent (a_4).

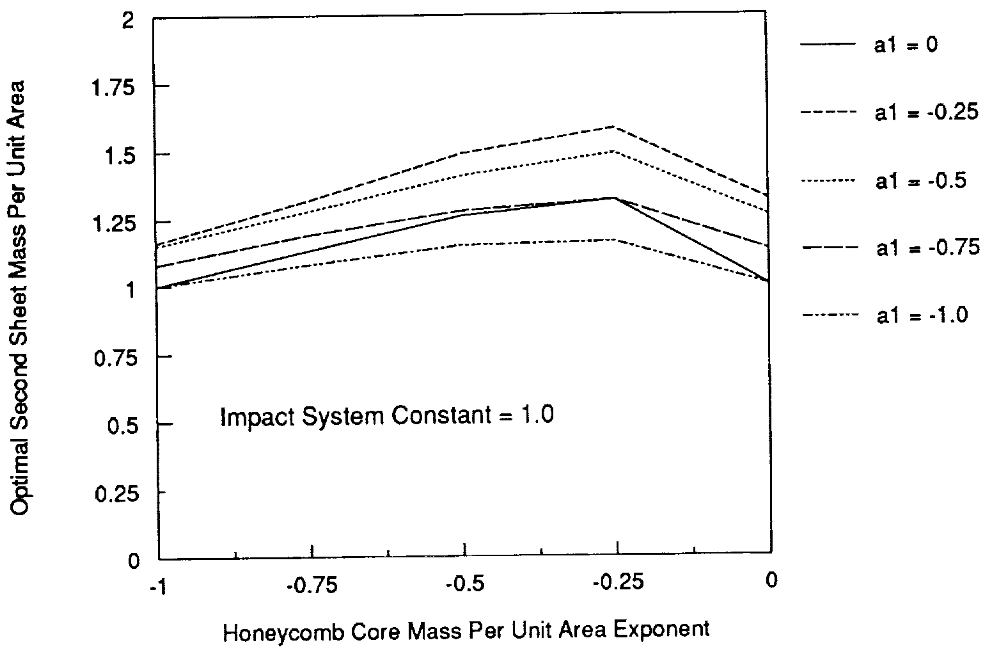

Figure 6. Optimal second sheet mass per unit area versus honeycomb core mass per unit area exponent (a_4).

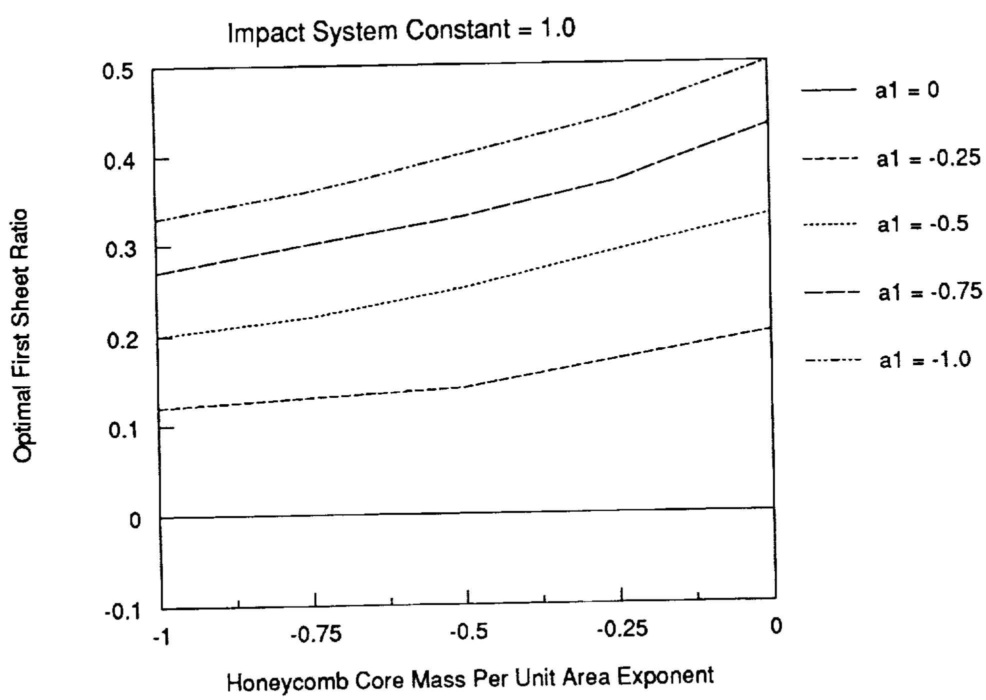

Figure 7. Optimal first sheet ratio versus honeycomb core mass per unit area exponent (a_4).

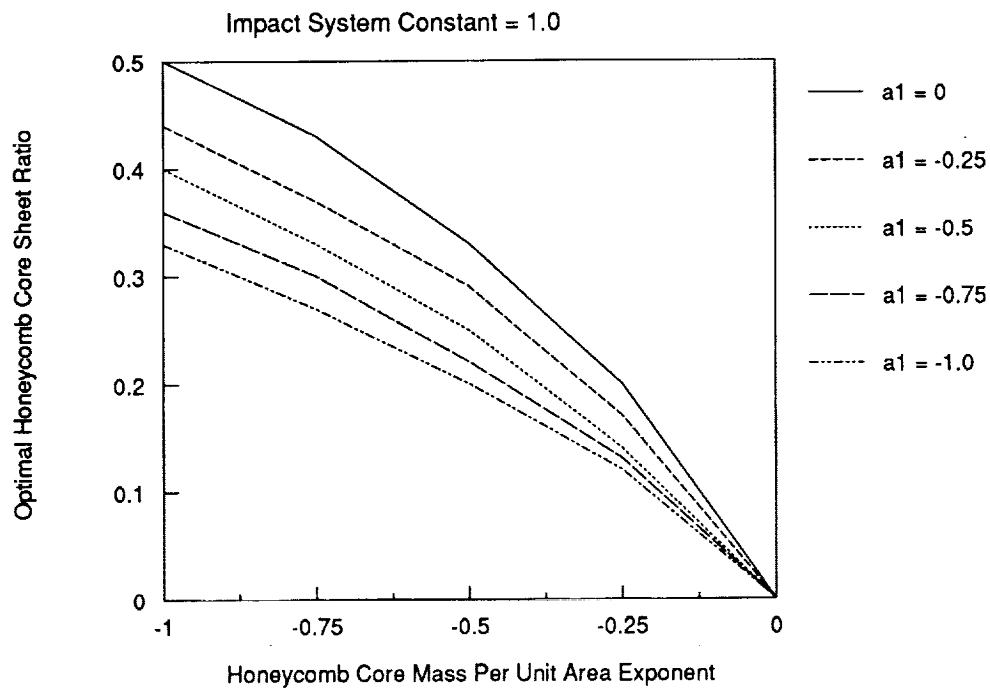

Figure 8. Optimal honeycomb core ratio versus honeycomb mass per unit area exponent (a_4).

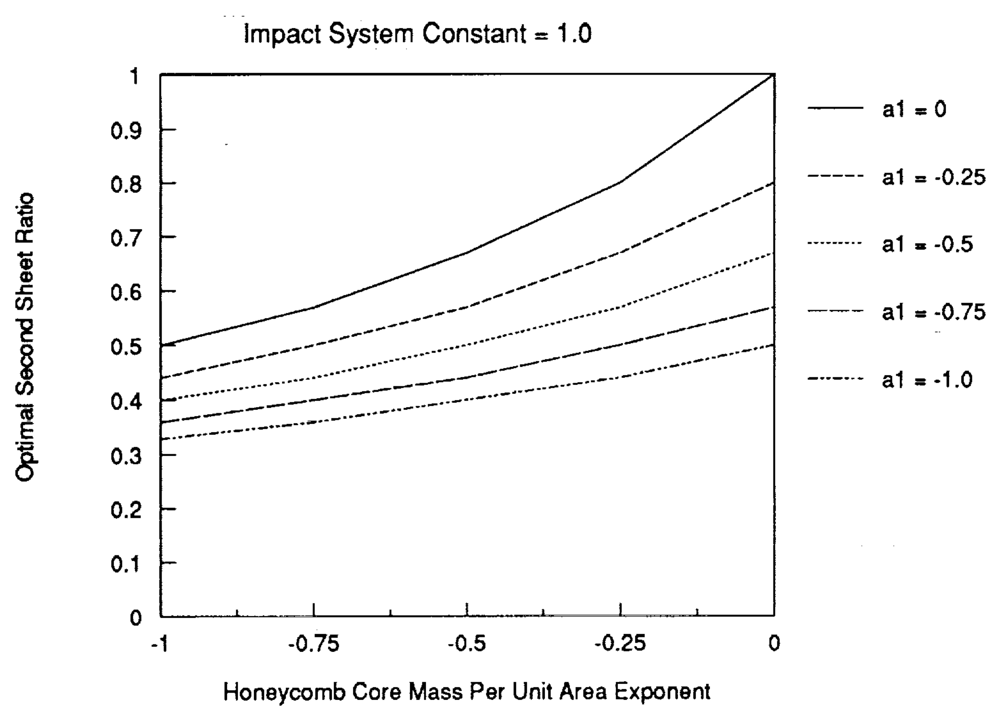

Figure 9. Optimal second sheet ratio versus honeycomb core mass per unit area exponent (a_4).

V. CONCLUSIONS

Using the geometric programming optimization technique, the following conclusions can be drawn for a honeycomb-type panel:

1. Optimal ratios are monotonically increasing for the first and second facing sheets and monotonically decreasing for the honeycomb core.

2. Optimal mass per unit areas are unimodal for the first and second facing sheets and monotonically decreasing for the honeycomb core.

3. The numerator of the impact system constant is not restricted to equation (17). Any form for the numerator will result in identical results for the parametric trends given in figures 1 to 8 and equations (16), (18), (19), and (20).

4. The minimum system mass per unit area is an essentially monotonically decreasing function of honeycomb core mass per unit area exponent.

5. The optimal mass per unit area of the facing sheets, honeycomb core, and system are monotonically increasing functions of the impact system constant.

In addition to these conclusions, one more point should be made. Geometrical considerations, particularly involving honeycomb cell size and shape, have been purposely omitted to reduce the number of estimated parameters required for model development. Certainly, equation (17) can be generalized to include these parameters, provided the impact test database supports this development. An interesting tradeoff involving cell size, penetration resistance, and cell density could result.

The conclusions from this study indicate that the geometric programming method can aid the designer of a honeycomb panel for optimal protection from particle impacts while minimizing weight. Additionally, an important conclusion is that the geometric programming method of optimization can be helpful in many fields of design engineering where optimal solutions may involve tens or hundreds of independent variables and design parameters. These design problems may be solved with geometric programming more efficiently than with other methods, and result in globally optimal, rather than locally optimal, solutions.

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*Science Applications International Corp., Huntsville, Alabama.

16. Abstract

The global optimization trends of protective honeycomb structural designs for spacecraft subject to hypervelocity meteoroid and space debris impacts are presented. This nonlinear problem is first formulated for weight minimization of the orbital maneuvering vehicle (OMV) using a generic monomial predictor. Five problem formulations are considered, each dependent on the selection of independent design variables. Each case is optimized by considering the dual geometric programming problem. The dual variables are solved for in terms of the generic estimated exponents of the monomial predictor. The primal variables are then solved for by conversion. Finally, parametric design trends are developed for ranges of the estimated regression parameters. Results specify nonmonotonic relationships for the optimal first and second sheet mass per unit areas in terms of the estimated exponents.

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