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Evolution of light domain walls interacting with dark matter I

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Abstract

In this paper we discuss the evolution of domain walls generated in the early Universe considering, unlike the previous studies, an interaction between the walls and a major gaseous component of the dark matter. The walls are supposed able to reflect the particles elastically and with a reflection coefficient of unity. We discuss a toy Lagrangian that could give rise to such a phenomenon. In the simple model studied we obtain highly non-relativistic and slowly varying speeds for the domain walls ($\sim 10^{-2}(1+z)^{-1}$) and negligible distortions of the microwave background. In addition, these topological defects may provide a mechanism of forming the large scale structure of the Universe, by creating fluctuations in the dark matter $\delta\rho/\rho \sim O(1)$ on a scale comparable with the distance the walls move from the formation (in our model $d < 20h^{-1}$ Mpc). The characteristic scale of the wall separation can be easily chosen to be of the order of 100 Mpc instead of being restricted to the horizon scale, as usually obtained.

1. Introduction

The cosmological consequences of primordial phase transitions associated with scalar fields have been the subject of many studies in recent years. The topological defects created in the transitions, such as domain walls, strings and monopoles are potentially of great interest for Cosmology, since they could supply seeds for the formation of the large scale structure of the Universe. Specifically, domain walls are sheet-like regions of false vacuum in-between domains having different and disconnected vacuum ground states of the scalar field. The simplest and the most studied model involves a real scalar field with a quartic potential and a negative sign for the mass term; after the phase transition the field rolls down to one of the two zero temperature minima for the potential; this leaves a domain structure on scales bigger than the correlation length of the field, resembling closely what happens in an Ising model [1]. When originally introduced, the phase transitions considered were on the GUT scale [2]. The trouble is that domain walls on the GUT scale rapidly become the dominant form of matter in the Universe and produce much too big distortions in the present microwave background.

Recently, interest in domain walls has been raised again considering late phase transitions (at $z \sim 100$) that would give rise to so called "soft" domain walls [3]. These walls may never be massive enough as to distort the microwave background but may a priori be a dominant gravitational component of the present Universe, triggering the formation of galaxies and changing the expansion rate. These possibilities have been excluded by a numerical study [4] of the evolution of the field itself through the phase transition and after, as the walls appear and evolve by their surface tension . The domain walls soon reach relativistic speeds and the average scale of the system becomes comparable to the horizon scale, making these walls unusable for the formation of the large scale structures we see [5,6]. Very similar results have been obtained [7] by considering directly the evolution of the walls after the phase transition . In that calculation the approximation taken is that the wall thickness is much smaller than the radius of curvature of the wall surface.

The problems mentioned arise due to the lack of energy dissipation in the models considered; the mass-energy stored in the walls gets efficiently converted into their kinetic energy, rapidly raising them to relativistic speeds. We therefore consider the effect of introducing in the equation of motion of the walls a friction term that is a function of the wall speed relative to the background matter and to its density. The idea of studying the consequences of friction on domain walls can be traced back to refs.1,8,9, but it was never fully developed because it was introduced in the context of GUT scale phase transitions, in which case including friction would even worsen the problems pointed out previously. In this paper we will consider much lower energy scales, of the same order of those obtained in ref.3. It will be shown that indeed there exists an interesting range of the wall energy density for which the average "inter-wall "distance is of the order of 100 Mpc today, and that these domain walls are compatible with the limits on the anisotropy of the microwave background.

The paper is organized in the following way: in section 2 we derive the equation of motion of an element of domain wall without any friction term other than the usual due to the universal expansion; in section 3 we concentrate our attention on the friction pressure arising when walls move through a homogeneous gas reflecting all incident particles elastically; in section 4 we introduce the results of section 3 into the equation of motion previously calculated; in section 5 we discuss what kind of particle Lagrangian may lead to the premises of this paper and the consequences of our model on the microwave background.

2. The equation of motion in the absence of friction

In order to approach the problem we will assume, first of all, that we are dealing with domain walls late enough after the phase transition so that the thin wall approximation can be considered roughly valid [7]. We are therefore interested in the motion of sharp interfaces under their surface tension. The shape of the network, which is related to the details of the model chosen, will turn out to be unimportant in the discussion. The important assumption made here is that we are dealing with two or more degenerate values for the vacuum ground state, so that the driving force for the motion of the walls is only due to their surface tension.

Many approaches may be considered to get the local equation of motion of the walls; the most direct of these is just to start with the well known equation of motion for real scalar fields [4,10],

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} - \frac{1}{a^2}\nabla^2\Phi = -\frac{\partial V}{\partial \Phi} \tag{1}$$

where a is the scale factor, given by $a = t^{2/3}$ if $\Omega = 1$ (we express t in $2H_o^{-1}/3$ units, H_o being the present value of Hubble constant; $a = 1$ today). Eq. (1) is expressed in comoving coordinates and universal time. After the phase transition there are regions of different vacua separated by kinks (which are classical solutions of eq.(1)). Throughout the following calculations we will assume that these kinks are moving non-relativistically; this will turn out to be a sensible choice (see section 4).

In general we can define as the surface of a kink the 2-d space on which $\partial V/\partial\Phi = 0$; at each point of the surface we can label as x the axis normal to it. If $x_o(t)$ is the intercept of the surface with the x axis and the principal radii of curvature at the point are much bigger than the wall thickness Δ we can represent the kink by a function $\Phi(x - x_o(t))$. The calculation is easily performed when we recall that

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}(\vec{\nabla}\cdot\hat{x})$$

where x is the unit vector perpendicular to the kink surface. The divergence $\vec{\nabla}\cdot\hat{x}$ is, in 3-d, the sum of the curvatures along the principal axes of the surface at the point considered [11]. In this way from eq. (1) we get

$$\left(-\ddot{x}_o\frac{\partial\Phi}{\partial x} + \dot{x}_o^2\frac{\partial^2\Phi}{\partial x^2}\right) - 3\frac{\dot{a}}{a}\frac{\partial\Phi}{\partial x}\dot{x}_o - \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial\Phi}{\partial x}(\vec{\nabla}\cdot\hat{x}) = -\frac{\partial V}{\partial\Phi} \tag{2}$$

Evaluating (2) at $x = x_o$ we see that $\partial\Phi/\partial x$ becomes very big when $\Delta \to 0$ ($\partial\Phi/\partial x \sim \Delta^{-1}$), while the $\partial^2\Phi/\partial x^2$ term is very small (it would be exactly $\partial^2\Phi/\partial x^2 = 0$ if the wall were straight). In the thin wall approximation we therefore get to the final expression

$$\ddot{x}_o + 3\frac{\dot{a}}{a}\dot{x}_o = -\frac{1}{a^2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{3}$$

where r_1 and r_2 are the principal comoving radii of curvature at the chosen point of the wall network.

If there were no universal expansion (set $a = 1$ constant) then eq.(3) would look like $\ddot{x}_o = -(1/r_1 + 1/r_2)$; if σ is the mass-energy density of the walls, then $P_T = \sigma(1/r_1 + 1/r_2)$ would simply be the pressure due to the surface tension, exactly the same form that one obtains in condensed matter. This also reminds us that eq.(3) is just Newton's second law divided by σ.

3. The friction term

We now derive the pressure exerted on the walls moving with speed $v \ll 1$ through some homogeneous medium interacting with it. We are going to study only the case in which the medium remains homogeneous throughout all the period of evolution considered. This can be considered valid, for example, if perfectly reflecting walls move so little that they are not able to reshuffle the bulk of the matter, i.e. if they move of a small fraction of the distance between each other, so that no particle interacts with two different walls in a cosmological time (see also the discussion at page 7). In all the following we are restricting ourself to this simple case.

We begin by writing down the general expression for the friction force acting on the domain walls as they move non-relativistically through a homogeneous gas. The particles coupled with the walls will be taken to be weakly interacting (WIMP's). We will also suppose that the walls are able to reflect elastically all the incident particles, regardless of their energy at the impact. This condition could be relaxed, as we will discuss at the end of this section.

For a non-relativistic gas we can write that the pressure exerted by the gas on the wall is given by (see Appendix):

$$\begin{aligned} P_f &= -2mn \int_y^{\infty} B^{-2\alpha} (y - y_x)^2 f(|y_x|)\, dy_x + \\ &\quad 2mn \int_{-\infty}^{y} B^{-2\alpha} (y - y_x)^2 f(|y_x|)\, dy_x \end{aligned} \tag{4}$$

where $B \equiv m/T$, $y = B^{\alpha} v$ and $y_x = B^{\alpha} v_x$.

Let's consider the limits in which $y \ll 1$ and $y \gg 1$. In the former case the thermal speed of the particles is much greater than the speed of the domain wall, since the average thermal momentum of the particles is $\bar{p} \sim T$; in the latter case the wall is moving through particles effectively at rest and the volume spanned remains depleted of the gas. The case $y \gg 1$ will turn out to be the most interesting in our discussion.

For $y \ll 1$, changing the variables inside the integrals ($y_1 = y - y_x$) and expanding $f(|y_x|)$ in power series around y_1, we get

$$P_f = -4mnB^{-2\alpha} y \int_0^{\infty} y_1^2 f'(y_1)\, dy_1 = 4mn(T/m)^{\alpha} vF \tag{5}$$

where $F \equiv -\int_0^\infty y_1^2 f'(y_1)\, dy_1$ is a constant of order unity. For $y \gg 1$ instead we obtain

$$P_f = 2mn\, v^2 \tag{6}$$

since $f(y_x) \sim 0$ for $y_x \gg 1$ and therefore we can substitute $(y - y_x)^2$ by y^2 in the integrals. Such a result is not surprising if we recall that in this case $\Delta p \sim 2mv$ and that the number of particles hitting the unit area per unit time is nv.

The case in which the gas is relativistic is even easier, since the number of particles hitting the wall per unit time is simply given by n ($c = 1$) on both sides of the surface. Taking $v \ll 1$ we get

$$P_f = n\left[\int_0^\infty p_x\,(1+v)\,f(|p_x|)\,dp_x - \int_0^\infty p_x\,(1-v)\,f(|p_x|)\,dp_x\right] = 2nv\bar{p}_x \tag{7}$$

so that $P_f \sim v\,T^4$ which is the limit discussed in the review [1].

We can also give an evaluation of the average thermal speed of the particles interacting with the walls e.g. for light neutrinos and for gas having the Boltzmann distribution, supposing that the particles decouple at a certain z_d (note that throughout the paper $z+1 = a^{-1}$). The momentum of the particles shifts with the expansion of the Universe so that $\overline{p}_d/\gamma m\overline{v}(z) = (z_d+1)/(z+1)$, where $\gamma^{-1} = \sqrt{1-\overline{v}^2}$; if at decoupling $T_d \gg m$ then $\overline{p}_d \sim T_d$; if $T_d \ll m$ then $m\overline{v}_d^2 \sim T_d$, so that $\overline{p}_d \sim (mT)^{1/2}$. Assuming the particles to be non-relativistic today we get

$$\bar{v}(z) \sim \left(\frac{T_d}{m}\right)^{1/2} \frac{z+1}{z_d+1} \tag{8}$$

for $T_d \ll m$ and

$$\bar{v}(z) \sim \left(\frac{T_d}{m}\right) \frac{z+1}{z_d+1} \tag{9}$$

for $T_d \gg m$. If we assume neutrinos of mass $m = 10$ eV and $T_d \sim 1$ Mev we get $\overline{v}(z) \sim 10^{-5}$ at $z = 0$. This result will turn out to be useful in the following discussion.

In closing this section we go back briefly to our initial assumptions. Although we are interested here in studying the consequences of a reflection coefficient close to unity, there could be cases in which one has to deal with an energy dependent partial transmission of the incident particles through the walls. This would lead to a class of solutions in which the walls may decouple from the matter after a certain stage, when their speed with respect to the matter becomes bigger than a critical value. These possibilities are at present under investigation and go beyond the goals of this paper.

4. Domain walls and friction: a simple case

In this section we introduce the previously calculated friction term into the equation of motion of the domain walls.

Define $R = ax$ and $v = a\dot{x}_o$. We can rewrite eq.(3) in terms of physical coordinates and peculiar speed v and add up the friction term we have been talking about in the previous section. As we are going to show soon the most interesting case to study is the $y \gg 1$, when the domain walls move fast with respect to the thermal motion of the gas. If the matter interacting is a major component of the dark matter then it follows that the gas is highly non relativistic and $y \gg 1$ during most of the evolution of the network, as shown in a check of self-consistency at the end of this section.

We define $\rho_m \equiv mn$ so that $\rho_m = \rho_{mo}/a^3$, where ρ_{mo} is the mass density of the matter interacting with the walls today. Inserting eq.(6) into eq.(3) we get [12]

$$\dot{v} + 2\frac{\dot{a}}{a}v + \frac{k_1}{a^3}v^2 = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{10}$$

where $k_1 = 2\rho_{mo}/\sigma$. The constant σ is the energy density of the walls. Let's now suppose that $\dot{v} \ll k_1 v^2/a^3$ and $2\dot{a}/a \ll k_1 v/a^3$. This also will turn out to be self-consistent. We therefore remain with the important equation:

$$v^2/a^3 = -K\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \tag{11}$$

where we define $K \equiv k_1^{-1}$.

Eq.(11) can be finally written in the more useful comoving coordinates as

$$\dot{x}_o^2 = -K\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \tag{12}$$

We are interested in how much the walls move in average from the original configuration, and therefore we average the curvature over the surface S of the network contained in an arbitrarily large volume $V \gg \bar{r}^3$. The quantity $\bar{r}$ is the mean interwall distance in comoving coordinates, defined as a point by point average of the distance to the next neighbouring wall integrated [13] over S. We therefore obtain

$$\frac{1}{\hat{r}} \equiv \frac{1}{2S}\int\left(\frac{1}{r_1}+\frac{1}{r_2}\right)dS \tag{13}$$

In general $\hat{r} = \beta\bar{r}$ (with $\beta \sim O(1)$), so that we finally get to

$$(\overline{\dot{x}_o^2})^{1/2} = \left(\frac{K}{\beta\bar{r}}\right)^{1/2} \tag{14}$$

where $K = \sigma/2\rho_{mo}$.

Since we are studying the case in which $\bar{r}$ changes little from the phase transition to the present time, we can consider eq.(14) as an estimate of the average comoving speed $\dot{r}$, which remains roughly constant. Our goal would be to determine σ given $\bar{r}$, but before doing so we should slightly modify eq.(14) taking into account the following correction. The friction term we utilize in eq.(10) is based on the assumption that only one reflection occurs to each particle. In comoving coordinates the speed of a free particle goes like $\dot{r}_p \sim a^{-2}$. In the case $y \gg 1$ after being reflected particles have a speed double than that of the wall, but this decreases due to the universal expansion and soon they get scattered again. A priori this fact could modify the form of the law of motion of the walls, but this is not the case, as we will see in a moment.

Let's give a numerical estimate. Consider a wall moving at constant comoving speed $\dot{r}$ (we will check the validity of the assumption at the end of the calculation). At a certain time t_i a particle at rest is reflected and its speed is henceforth given by $\dot{r}_p(t) = 2\dot{r}(t_i/t)^{4/3}$ (since $a = t^{2/3}$ if $\Omega = 1$). The maximum comoving distance x_{max} from the wall is reached at the time t_{max}, when the wall and the particle have equal speed. This yields $\dot{r}_p(t_{max}) = \dot{r} \rightarrow t_{max} = 1.7t_i \rightarrow x_{max} = 0.2\dot{r}t_{max} = 0.2\dot{r}a^{3/2}$, which is roughly as far as any scattered particle can get from the kink.

All the matter in the volume swept by the wall from its formation is contained within a distance x_{max} in front of the kink, while the total distance traveled by the wall is $\overline{\Delta r} \sim \dot{r}t_{max} = \dot{r}a^{3/2}$. Since $\overline{\Delta r}/x_{max} \simeq const.$, we take the density of the matter in front of the wall to be roughly constant; in this way we get $\rho_{front} \sim 6\rho_m$ at any time. This means that the initial assumption of constant comoving speed Eq.(14) is, at least approximately, self-consistent, substituting K by $K' \equiv K/6$,

$$(\overline{\dot{x}_o^2})^{1/2} = \left(\frac{\sigma}{12\rho_{mo}\beta\bar{r}}\right)^{1/2} \tag{15}$$

A more accurate calculation would require a numerical simulation that takes into account also gravitational effects.

To proceed we need now an estimate of K'. If we associate the walls with the peaks of the distribution of galaxies observed in the survey [6], which suggests the domain walls may be related to the clustering process, then the scale of our network will be $\bar{r} = \bar{R}_o = 120\, h^{-1}$ Mpc today. In this way we obtain $K' = \sigma/12\rho_{mo} = 6\beta \cdot 10^{-2}\overline{\Delta r}^2$; this can also be written as

$$\frac{\Omega_{wo}}{\Omega_{mo}} \sim 12\beta\overline{\Delta r}^2 \tag{16}$$

since $\Omega_{wo}/\Omega_{mo} \sim \sigma/\rho_o\bar{R}_o$ by geometry. If we want the walls to produce a density fluctuation on a scale of the order, for example, of $20\, h^{-1}$ Mpc in a major component of the dark matter (assuming $\Omega_{mo} = 1$), setting $\overline{\Delta r} = 10^{-2} = 20\, h^{-1}$ Mpc yields

$\Omega_{wo} \sim 1.2\beta\,10^{-3}$(equivalently, $\sigma \sim 1.2\beta MeV^3$); this constitutes an upper bound on Ω_{wo} in our model and shows once again that the domain walls never get to dominate the energy density of the Universe.

We can now easily see that the self-consistency conditions on eq.(12) are ensured. In fact, we know that

$$(\overline{v^2})^{1/2} \sim a 10^{-2} \tag{17}$$

which says that our initial assumption $v \ll 1$ is satisfied by a big margin. It also means that at $z > 30$ the speed of the walls is smaller than the average thermal speed of particles like neutrinos with mass $m > 10$ eV (see estimate given by eq.(9) and compare it with eq.(17)). In general, when this occurs the friction term we used eq.(6) has to be replaced by eq.(5). At early times one should write $(\rho_{mo}/\sigma a^3)\,(T_o/am)\,v \sim 1/a\bar{r} \rightarrow v \sim 20\,a^3 = 20\,(z+1)^{-3}$ (where $T_o \equiv T_{today}$) but, since the evolution of the configuration takes place for the most part at $z < 5$, such a change wouldn't affect our previous conclusions.

Let's continue our self-consistency check, going back to our original eq.(10). We know that in a Universe with critical density $\Omega_{mo} = 1$ and $a = t^{2/3}$, so that $\dot{v}/v = 2/3t$ at each point of the network; we therefore get

$$\dot{v} = \frac{2}{3}\frac{v}{t} \ll 12\left(\frac{\rho_{mo}\bar{R}_o}{\sigma}\right)\frac{v^2}{R_o t^2} = \frac{10^4}{6\beta \cdot 10^{-2}}\frac{v^2}{t^2} \rightarrow v \gg 4 \cdot 10^{-6}\beta t$$

and

$$2\frac{\dot{a}}{a}v \ll \frac{1.5 \cdot 10^5 v^2}{\beta t^2} \rightarrow v \gg 8 \cdot 10^{-6}\beta t$$

As anticipated at the beginning of this section, at any time considered we meet the conditions for friction dominated motion (analogous considerations apply also at $z > 30$ in the $m = 10$ eV neutrino case).

5. Discussion

We have shown that, if domain walls coming from some late phase transition are able to perfectly reflect the particles of gas of a component of the dark matter, then the domain wall network is bound to expand with the scale factor, provided that $\Omega_{wo}/\Omega_{mo} \leq 1.2\beta 10^{-3}$. The coupling between the scalar field Φ and the particles in question (call the associated field Ψ) may assume the very simple form of a mass term dependent on the spatial coordinates. For the sake of discussion take Ψ to be fermions. A toy Lagrangian for Ψ could be written as $\mathcal{L}(\psi) = \bar{\Psi}\ \partial\!\!\!/\Psi + (m + f(\Phi))\bar{\Psi}\Psi$ (a Lagrangian of this form is obtained e.g. in ref.3 where $f(\Phi)$ is a real function of the field Φ that gives the domain structure and it is assumed to get higher values within the kinks than outside. Due to the presence of the kinks, the mass of the particles changes when they get close to the soliton. In the non-relativistic case this situation is equivalent to obtaining a Schroedinger equation for free particles with a potential $V = f(\Phi(\vec{r}))$. We can say that $V(\vec{r})$ is a perfect barrier if the reflection coefficient on both sides of the kink is unity.

Now we turn to consider a possible estimate of the wall thickness, that has been, up to this point, a free parameter. If the domain walls maintain their position from the formation (in the comoving coordinate system), we are actually bound to consider second order phase transitions not earlier than $z_f = \bar{R}_o/R_H(z_f) = \bar{R}_o/3t_f = 2 \cdot 10^{-2} z_f^{3/2} \rightarrow z_f \leq 2500$ (R_H is the horizon scale at z_f) due to simple causality considerations. As a consequence there is a lower bound to the thickness Δ of the domain walls; since the interwall distance is $\bar{R}_f \sim \Delta \sim \bar{R}_o z_f^{-1}$ at formation, $\Delta \geq 3.5 \cdot 10^{-5}$ in our units, which is $\Delta \geq 7 \cdot 10^{-2} h^{-1}$ Mpc. Such a distance is far greater than the wavelength usually associated to any dark matter particle candidate (e.g. for neutrinos $\lambda_{thermal} < 10^4 eV^{-1}$ at any z). We infer that $V(r)$ can be considered a classical barrier of height E_{max} such that for $E < E_{max}$ the reflection coefficient is unity and for $E > E_{max}$ it is zero. In this paper we only consider the case $E_{max} \rightarrow \infty$.

A couple of issues still remain to be solved. The walls carry a gravitational field that shifts the frequency of the microwave background radiation a slight amount when this passes through the potential. Such a problem has been treated in refs.[3,14].

The infinitesimal shift of the average photon energy T while the photon is moving for a dt time through the gravitational influence of a wall is given by $dT \sim \delta T + T\delta V$, where V is the gravitational potential of the wall. V is roughly given by $V \sim G\sigma R$ at a distance R from the kink surface, within a cut off value $\sim \bar{R}/2$; $\bar{R}$ is the average interwall distance in physical coordinates at the time considered [15]. The value of V varies in time due to the evolution of the network, so that $\delta V = (\partial V/\partial t)\delta t + \vec{\nabla} V \cdot \delta\vec{R}$(where $|\delta\vec{R}| = \delta t$). We want to calculate the total shift in the temperature of the photons as they pass through the gravitational potential of a single wall, i.e. within the cut off distance of V. If we take roughly $\partial V/\partial t \sim G\sigma\dot{\bar{R}}$ in a region of order $\bar{R}$ in size(this is clearly an over-estimate), when we integrate the above expression for dT to find the total shift of the temperature we get a term $\delta T/T = \alpha G\sigma\bar{R}^2/t$ in addition to the usual term due to the expansion; α is a fudge factor of order unity and t is the age of the Universe at the epoch considered. The biggest distortion can be reached at the present epoch: $\delta T/T \sim 10^{-8}$.

Another effect may be considered. The fluctuation in the matter density due to the sweeping action of the wall gives rise to a gravitational influence limited to the region of thickness $d \sim 20h^{-1}Mpc$ in which $\delta\rho_m/\rho_m \neq 0$. The minimum value of the gravitational potential just due to this distribution of matter is $V_m \sim G\rho_m d^2$. Using the same arguments as above we can calculate the distortion due to the matter in $\delta T/T|_m \sim \beta G\rho_{mo}d^3/t$ (β is fudge factor of order unity). Again the biggest $\delta T/T$ is reached today: $\delta T/T \sim 10^{-7}[d/20h^{-1}Mpc]^3$.

All other effects, including gravitational distortion at the last photon scattering surface (if $z_f \geq 1000 \to \delta T/T|_{LSS} \sim G\sigma\bar{R}_o a_{LSS}$, with $a_{LSS} = 10^{-3}$) and effects originated at the phase transition, are comparatively much smaller.

The values obtained refer to the distortion originated from a single wall. Even supposing that the phase transition takes place before the photon decoupling there are only $N \sim R_H/\bar{R}_o \sim 3/6 \cdot 10^{-2} = 50$ walls between us and the last surface of scattering. An evaluation of the $\delta T/T|_m$ due to the matter swept from the walls, which is the biggest distortion, can be obtained multiplying the single wall distortion by $\sqrt{N}$ and gives $\delta T/T|_m \sim 10^{-6}$. For the effects directly related to the domain walls our values of $\delta T/T$ are, for the same σ, one order of magnitude lower than that calculated in the previous papers $\delta T/T \sim 10\alpha G\sigma\bar{R}^2/t \sim 10^{-7}$; this derives from an interwall separation an order of magnitude smaller.

The gravitational interaction of the domain walls with matter is secondary with respect to the sweeping action. In fact, taking for the sake of discussion the favorable case of straight infinite walls, the peculiar speed gained by the particles due to the gravitational influence would be, after a cosmological time [14], of the order $v_m \sim 2\pi G\sigma t \sim 10^{-4.5}$.

In concluding the discussion we point out that one can also consider late first order phase transitions in order to achieve our big values for the average interwall distance, even while starting with a much smaller comoving correlation length at the critical temperature. In this way one can remove the lower bound on Δ obtained in this section. Such an analysis is left for future investigation.

6. Conclusion

This paper wants to offer a framework for future work. We have made the following assumptions:

- A network of domain walls is established in the primordial Universe through a second order phase transition.
- The walls interact with an important gaseous component of the present energy density of the Universe, reflecting elastically all incoming particles regardless of their kinetic energy.
- The configuration is bound to expand with the background comoving coordinates (up to higher order corrections).

We reach the following conclusions:

- There is a wide range of values for σ, surface density of the domain walls, such that $\bar{R}_o$, the average inter-wall distance today is of the order of the large scale structure observed for the galaxies.
- The mechanism that generates the fluctuations in the distribution of the dark matter could be related also to the particle Lagrangian, and not just gravitational.
- This suggests that the large scale structure could indeed form in intimate connection with the presence of the domain walls, although studying the evolution of the fluctuations and the long distance gravitational effects (see the discussion on the Great Attractor in ref.14 obtained goes beyond the present work.
- Domain walls never come to dominate the energy density of the Universe.

- Walls with σ of Mev order and such a small interwall separation ($\bar{R}_o \sim 100$ Mpc) are not able to distort the microwave background. Also the effects related to the matter density fluctuations are small.

Some of the assumptions made to obtain our results may be relaxed, giving rise to the different scenarios we earlier mentioned. Particularly intriguing is the possibility of the wall decoupling mentioned in section 3 [16]: domain walls may give rise to a spectrum of density perturbations and at some point decouple and start growing in the way described in the previous work [4]. This paper represents just a first attempt to approach the late phase transition issue from an angle that could solve some of the problems other investigations have found. This paper is meant to stimulate interest in such non-standard scenarios.

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Appendix

Consider an infinite wall moving along the x axis of a chosen coordinate system, with speed $v > 0$. At one side of the wall particles with speed v_x get reflected gaining momentum $\Delta p = 2m(v - v_x)$ (particles with $v_x > v$ will not interact with the wall).

The momentum distribution of the particles is defined on a $3+3$ dimensional phase space; neverthless we are interested in the statistical distribution of the momenta only in the x direction, and we therefore integrate out all other degrees of freedom. In this way we can write, in a very general way, a statistical distribution $f((m/T)^{\alpha}|v_x|)$ defined so that $B^{\alpha}\int_{-\infty}^{\infty} f(B^{\alpha}|v_x|)\,dv_x = 1$ (where $B \equiv m/T$) [17]. The coefficient α depends on the actual original distribution we are considering. For a Boltzmann distribution $\alpha = 1/2$ while for light neutrinos ($m \ll 1$ Mev) $\alpha = 1$.

There are $dN = B^{\alpha} n\,(v - v_x) f(B^{\alpha}|v_x|)\,dv_x$ interactions per each second and per unit area with momentum exchange Δp (n is the number density of the particles). On the other side of the wall Δp has opposite sign, so that we can write that the pressure exerted by the gas on the wall is given by:

$$\begin{aligned} P_f &= -2mn\int_{y}^{\infty} B^{-2\alpha}\,(y-y_x)^2\, f(|y_x|)\,dy_x + \\ & \quad 2mn\int_{-\infty}^{y} B^{-2\alpha}\,(y-y_x)^2\, f(|y_x|)\,dy_x \end{aligned}$$

where $y = B^{\alpha}v$ and $y_x = B^{\alpha}v_x$.The first integral refers to particles having speed $v_x > v > 0$ and hitting the wall from the back, while the second refers to particles hitting the wall from the front.

We now derive, as an example, the form that $f(B^{\alpha}v)$ assumes in the case of light neutrinos ($m \ll 1MeV$). We start up with the statistical distribution of neutrinos in thermal equilibrium:

$$n = \frac{g}{(2\pi)^3}\int \frac{d^3p}{exp\left(\frac{\sqrt{p^2+m^2}}{T}\right)+1}$$

at $T > T_d$. At $T < T_d$ this becomes

$$n = \frac{g}{(2\pi)^3}\int \frac{d^3p}{exp\sqrt{\frac{p^2}{T^2}+\frac{m^2}{T_d^2}}+1}$$

where $T \equiv T_d a/a_d$; since $m/T_d \ll 1$ we get

$$n = \frac{g}{(2\pi)^3} \int \frac{d^3p}{exp(p/T) + 1}$$

at all times.

The probability of finding a particle in an interval $p_x, p_x + dp_x$ of the x component of the momentum is then

$$g(p_x) = \frac{g}{n(2\pi)^3} \int_0^\infty \frac{2\pi p_\perp dp_\perp}{exp\left(\frac{\sqrt{p_x^2 + p_\perp^2}}{T}\right) + 1}$$

where $p_\perp$ is any component of the momentum perpendicular to p_x. Changing variables we get

$$f(y_x) = \frac{1}{4\zeta(3)} \int_0^\infty \frac{y_\perp dy_\perp}{exp\sqrt{y_x^2 + y_\perp^2} + 1} \tag{18}$$

which is an implicit function of $y_x \equiv mv_x/T$ $(y_\perp \equiv p_\perp/T)$. A similar calculation can be performed for a Boltzmann distribution.

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12. We can do this since both (3) and (6) are valid locally.

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17. We are assuming the statistical distribution to be thermal, in a broad sense. In this definition we would, e.g., include light ($m < 1MeV$) neutrinos after their decoupling.