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Classical Theory of Radiating Strings.

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E. Copeland,

*School of Mathematical and Physical Sciences,
University of Sussex,
Brighton BN1 9QH.*

D. Haws,

*NASA/Fermilab Astrophysics Center,
MS 209 Fermi National Accelerator Laboratory,
P. O. Box 500 Batavia, Illinois 60510.*

M. Hindmarsh.

*Department of Physics,
Newcastle University,
Newcastle-upon-Tyne NE1 7RU.*

Abstract

We calculate to first order in classical perturbation theory the divergent part of the self force of a radiating string coupled to gravity, an antisymmetric tensor and a dilaton in four dimensions. While this divergence can be absorbed into a renormalisation of the string tension, demanding that both it and the divergence in the energy momentum tensor vanish forces the string to have the couplings of compactified $N = 1$ $D = 10$ supergravity. In effect, supersymmetry cures the classical infinities.

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The classical treatment of radiating charges has a venerable history¹, and from it much useful information can be gained such as the Larmor formula for radiation damping. The analogous problem for a classical string also has important applications: for example the backreaction on a radiating cosmic string². Without backreaction, the metric in weak field perturbation theory develops singularities along null lines originating at cusps, points in spacetime where the string tangent vector vanishes and it moves at the speed of light³. It is reasonable to expect that radiation reaction must modify this somehow by slowing down the string, but as yet the question of what really happens at a cusp remains unanswered. Attempts have been made to include backreaction by calculating the decay rates of highly excited quantum strings⁴, but the connection with the classical problem remains obscure. In any case, a cosmic string is really a classical object, and the question should be resolvable within the framework of the classical theory. Some headway has already been made: Quashnock and Spergel² have investigated how the trajectory of a radiating loop changes with time. In this letter we examine the first order corrections to the equations of motion of the string paying particular attention to the divergent parts of the self force and the energy momentum tensor. To remain general, we consider the string coupled to gravity, an antisymmetric tensor field and to the dilaton. This enables us to make contact with the work of Dabholkar and Quashnock⁵ and Dabholkar and Harvey⁶.

Our theory is a classical bosonic string in 3+1 dimensions, which may be considered as a truncated version of a higher dimensional theory. We allow arbitrary couplings to "the" dilaton (which is a linear combination of the higher dimensional supergravity dilaton and that resulting from the compactification of the extra dimensions) and the anti-symmetric tensor (AST). We show that there is a logarithmic divergence in the equations of motion when backreaction is included, which can be absorbed into a renormalisation of the mass per unit length (this was previously

shown for the AST⁵ and for a gauge field⁷). When the couplings of the string to gravity (μ) and to the AST field (λ) satisfy $\mu^2 = \lambda^2 e^{(\beta-2\alpha)\bar{\Phi}}$, where α and β are the dilaton couplings (defined below) and $\bar{\Phi}$ is the dilaton expectation value, the divergence vanishes. This is intriguingly similar to the calculation of the self-energy of a straight superstring⁶, where logarithmic divergences conspired to cancel between all the fields. Generalizing these results to arbitrary couplings, we find that requiring the energy momentum tensor to vanish as well as the self force constrains α to equal 1. These cancellations are no accident: they occur for precisely those couplings for which the action is the bosonic part of a dimensionally reduced $N = 1$ $D = 10$ supergravity theory. Thus we might say that the classical divergences cancel because of supersymmetry.

With the conventions of Landau and Lifshitz¹, our classical action is the sum of a string action S_2 and the action for the massless four-dimensional fields S_4 :

$$S_2 = -\frac{\mu}{2} \int d^2\sigma \sqrt{\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} e^{\alpha\bar{\Phi}} - \frac{\lambda}{2} \int d^2\sigma \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu B_{\mu\nu} \quad (1)$$

$$S_4 = \frac{1}{16\pi G} \int d^4x \sqrt{g} [-R + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} e^{-\beta\bar{\Phi}}] \quad (2)$$

where $g = |\det(g_{\mu\nu})|$, $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the AST field strength and $X^\mu(\sigma)$ are the worldsheet coordinates. This is part of the bosonic sector of the conformally rescaled and dimensionally reduced $N = 1$ supergravity action⁸ with string sources when $\alpha = 1$ and $\beta = 2$.

The equations of motion for the string that follow from this action are

$$\mu(\partial_a(\sqrt{\gamma} \gamma^{ab} \partial_b X^\mu) + (\Gamma_{\nu\rho}^\mu + \alpha \delta_\rho^\mu \partial_\nu \Phi) \mathcal{L}^{\nu\rho}) = -\lambda e^{-\alpha\bar{\Phi}} g^{\mu\sigma} \partial_\rho B_{\sigma\nu} \mathcal{K}^{\nu\rho} \quad (3)$$

where $\gamma = |\det(\gamma_{ab})|$ and where $\mathcal{L}^{\mu\nu}$ and $\mathcal{K}^{\mu\nu}$ are given by

$$\begin{aligned} \mathcal{L}^{\mu\nu} &= \sqrt{\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \\ \mathcal{K}^{\mu\nu} &= \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu \end{aligned} \quad (4)$$

The equation of motion for the massless fields $g_{\mu\nu}$, $B_{\mu\nu}$ and Φ are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} \quad (5)$$

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\Phi) = -\frac{\beta}{12}e^{-\beta\Phi}H^2 - \alpha 8\pi G\mu \int d^2\sigma g_{\mu\nu}\mathcal{L}^{\mu\nu}e^{\alpha\Phi}\frac{1}{\sqrt{g}}\delta^{(4)}(x-X(\sigma)) \quad (6)$$

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}H^{\mu\nu\rho}) - \beta\partial_\mu\Phi H^{\mu\nu\rho} = -16\pi G\lambda \int d^2\sigma\mathcal{K}^{\nu\rho}e^{\beta\Phi}\frac{1}{\sqrt{g}}\delta^{(4)}(x-X(\sigma)) \quad (7)$$

The energy momentum tensor is the sum of a string piece

$$T_{\mu\nu}^{\text{string}} = \mu \int d^2\sigma \mathcal{L}^{\mu\nu} e^{\alpha\Phi} \frac{1}{\sqrt{g}} \delta^{(4)}(x-X(\sigma)) \quad (8)$$

and the field pieces

$$\begin{aligned} T_{\Phi}^{\mu\nu} &= \frac{1}{16\pi G}(\partial^\mu\Phi\partial^\nu\Phi - \frac{1}{2}g^{\mu\nu}(\partial\Phi)^2) \\ T_B^{\mu\nu} &= \frac{1}{32\pi G}e^{-\beta\Phi}(H_{\rho\sigma}^\mu H^{\nu\rho\sigma} - \frac{1}{6}g^{\mu\nu}H^2) \end{aligned} \quad (9)$$

We expand around a constant field background, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x)$, $\Phi(x) = \bar{\Phi} + \phi(x)$, $B_{\mu\nu}(x) = \bar{B}_{\mu\nu} + b_{\mu\nu}(x)$ to first order in the dimensionless couplings $G\mu$ and $G\lambda$ with an arbitrarily moving string as a source. Choosing the gravitational harmonic gauge $g^{\mu\nu}\Gamma_{\mu\nu}^\rho = 0$, and the antisymmetric tensor analogue of the Lorentz gauge $\partial_\mu B^{\mu\nu} = 0$, we obtain the weak field equations of motion

$$\begin{aligned} \partial^2 h_{\mu\nu} &= -16\pi G\mu \int d^2\sigma (\mathcal{L}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathcal{L})e^{\alpha\Phi}\delta^{(4)}(x-X(\sigma)) \\ \partial^2 b_{\mu\nu} &= -16\pi G\lambda \int d^2\sigma \mathcal{K}_{\mu\nu}e^{\beta\Phi}\delta^{(4)}(x-X(\sigma)) \\ \partial^2 \phi &= -\alpha 8\pi G\mu \int d^2\sigma \mathcal{L}e^{\alpha\Phi}\delta^{(4)}(x-X(\sigma)) \end{aligned} \quad (10)$$

\mathcal{L} is defined to be \mathcal{L}_μ^μ . These equations are of the form

$$\partial^2 A(x) = - \int d^2\sigma \Sigma(\sigma)\delta^{(4)}(x-X(\sigma)) \quad (11)$$

where the potential A represents the gravitational, antisymmetric tensor or dilation perturbations and Σ is $16\pi G\mu(\mathcal{L}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathcal{L})e^{\alpha\Phi}$, $16\pi G\lambda\mathcal{K}_{\mu\nu}e^{\beta\Phi}$ or $8\pi G\mu\alpha\mathcal{L}e^{\alpha\Phi}$

respectively. Equation (11) is easily solved using the retarded Green's function $G(x, y) = (2\pi)^{-1} \theta(x^0 - y^0) \delta((x - y)^2)$ (Jackson¹)

$$A(x) = - \int d^4 y d^2 \sigma \Sigma(\sigma) G(x, y) \delta^{(4)}(y - X(\sigma)) \quad (12)$$

Introducing $\Delta^\mu = x^\mu - X^\mu(\sigma)$, we may perform all but one of the integrations to obtain

$$A(x) = - \frac{1}{4\pi} \int d\sigma \left[\frac{\Sigma}{\Delta \cdot \dot{X}} \right]_{\tau=\tau_r} \quad (13)$$

where the integrand is to be evaluated at the retarded proper time, given by $\Delta^2(\tau, \sigma) |_{\tau=\tau_r} = 0$. Each of the quantities in (13) is logarithmically divergent as $x \rightarrow X(\sigma)$. However we will see that these divergences are not fatal. In order to calculate the self forces due to the various fields requires an expression for the derivative of $A(x)$. Generalizing a similar expression for the electron¹ we find

$$\partial_\rho A(x) = - \frac{1}{4\pi} \int d\sigma \left[\frac{1}{\dot{X} \cdot \Delta} \frac{\partial}{\partial \tau} \left(\frac{\Delta_\rho \Sigma}{\dot{X} \cdot \Delta} \right) \right]_{\tau=\tau_r} \quad (14)$$

The divergence is extracted by taking the field point on the string and expanding the integrand around it. It is convenient to fix the worldsheet coordinates by using the conformal gauge

$$\begin{aligned} \dot{X} \cdot X' &= 0 \\ \dot{X}^2 + X'^2 &= 0 \end{aligned} \quad (15)$$

To order $G\mu$ it is irrelevant whether we use $g_{\mu\nu}$ or $\eta_{\mu\nu}$ to contract the spacetime vectors. Changing the worldsheet variables in the integrand to primed quantities, taking $x = X^\mu(\sigma)$ and $\tau' = \tau_r$ (the retarded proper time) so that $\Delta\tau = \tau_r - \tau = -|\Delta\sigma| = -|\sigma' - \sigma|$, and using the gauge conditions we find that the divergent part of $\partial_\rho A(X(\sigma))$ which we denote by $\mathcal{D}_\rho(X)$ is

$$\mathcal{D}_\rho(X) = \frac{1}{4\pi} \int d\sigma' \frac{1}{|\Delta\sigma|} \frac{1}{E} \left[\frac{\Sigma}{2E} (X''_\rho - \ddot{X}_\rho) + \frac{\partial}{\partial \sigma} \left(\frac{\Sigma}{E} \right) X'_\rho - \frac{\partial}{\partial \tau} \left(\frac{\Sigma}{E} \right) \dot{X}_\rho \right] \quad (16)$$

where $E = \dot{X}^2$.

a) *Dilaton*: in this case $\Sigma = 16\pi G\mu\alpha e^{\alpha\bar{\Phi}} E$ so that

$$\mathcal{D}_\rho(X) = -2G\mu e^{\alpha\bar{\Phi}} \alpha \int d\sigma' \frac{1}{|\Delta\sigma|} \frac{1}{\dot{X}^2} (\ddot{X}_\rho - X''_\rho) \quad (17)$$

The divergent piece in the force per unit length f_{div}^μ exerted by the dilaton field on the string is, by equation (3) given by

$$f_{\phi,div}^\mu = -\alpha\mu\mathcal{D}_\nu\mathcal{L}^{\nu\mu} = 0 \quad (18)$$

by the gauge conditions.

b) *AST*: here Σ is a tensor quantity, $\Sigma^{\sigma\nu} = 16\pi G e^{\beta\bar{\Phi}} \lambda \epsilon^{ab} \partial_a X^\sigma \partial_b X^\nu$, and the divergent part of the self force $f_{B,div}^\mu$ is

$$\begin{aligned} f_{B,div}^\mu &= -\lambda e^{\alpha\bar{\Phi}} \mathcal{K}^{\nu\rho} \mathcal{D}_{\rho\sigma\nu} \eta^{\sigma\mu} \\ &= -4G\lambda^2 (\ddot{X}^\mu - X''^\mu) e^{(\beta-\alpha)\bar{\Phi}} \int \frac{d\sigma'}{|\Delta\sigma|} \end{aligned} \quad (19)$$

c) *Graviton*: for the graviton, in the conformal gauge, $\Sigma \equiv \Sigma^{\mu\nu} = 16\pi G\mu e^{\alpha\bar{\Phi}} (\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu - \eta^{\mu\nu} \dot{X}^2)$ and we find

$$f_{g,div}^\mu = +4G\mu^2 e^{\alpha\bar{\Phi}} (\ddot{X}^\mu - X''^\mu) \int \frac{d\sigma'}{|\Delta\sigma|} \quad (20)$$

Putting (18-20) together, we arrive at our first result: the classical equations of motion for a string including backreaction are in the conformal gauge

$$\mu \left(1 + 8e^{\alpha\bar{\Phi}} \left(\frac{G\lambda^2}{\mu} e^{(\beta-2\alpha)\bar{\Phi}} - G\mu \right) \ln\left(\frac{R}{\delta}\right) \right) (\ddot{X}^\mu - X''^\mu) = f^\mu(finite) \quad (21)$$

where R is an infrared cut-off provided by the curvature of the string and δ an ultraviolet cut-off which, for a cosmic string (see e.g. Vilenkin⁹), is its width. This is to be compared with the result for a charged point particle¹

$$m \left(1 - \frac{e^2}{4\pi m\delta} \right) \ddot{X}^\mu = \frac{2}{3} \frac{e^2}{4\pi} (\ddot{X}^\mu + \dot{X}^\mu (\ddot{X}^2)) \quad (22)$$

There is no simple local expression for the self force for a string because the self-interactions are non-local, and the string divergence is only logarithmic because it is an extended object.

We note that it is not in general correct to deduce the divergent part of (21) from the divergent part of the effective action, which is obtained by substituting back into S_2 the fields for which the string is a source. To first order,

$$\begin{aligned} S_2^{(1)} = & \mu(8\pi G\mu)e^{2\alpha\bar{\Phi}} \int d^2\sigma d^2\sigma' \mathcal{L}^{\mu\nu}(\sigma) G(\sigma, \sigma') \mathcal{L}_{\mu\nu}(\sigma') \\ & + \lambda(8\pi G\lambda)e^{\beta\bar{\Phi}} \int d^2\sigma d^2\sigma' \mathcal{K}^{\mu\nu}(\sigma) G(\sigma, \sigma') \mathcal{K}_{\mu\nu}(\sigma') \\ & + \mu(\alpha^2 - 1)(4\pi G\mu)e^{2\alpha\bar{\Phi}} \int d^2\sigma d^2\sigma' \mathcal{L}(\sigma) G(\sigma, \sigma') \mathcal{L}(\sigma') \end{aligned} \quad (23)$$

where $G(\sigma, \sigma') = (2\pi)^{-1} \theta(X^0(\sigma) - X^0(\sigma')) \delta((X(\sigma) - X(\sigma'))^2)$. Performing the integration over τ' the δ -function leaves behind a factor $(2|\Delta \cdot \dot{X}(\sigma')|)^{-1}$, which when expanded in powers of $\Delta\sigma = \sigma' - \sigma$ becomes $(2\sqrt{m}|\Delta\sigma|)^{-1}$, where $m_{ab} = \partial_a X \cdot \partial_b X$ is the induced metric on the world sheet. Thus

$$S_{2,div}^{(1)} = 4G\mu^2 e^{2\alpha\bar{\Phi}} \ln(R/\delta) \int d^2\sigma \frac{1}{\sqrt{m}} \left[\mathcal{L}^{\mu\nu} \mathcal{L}_{\mu\nu} + \frac{\lambda^2}{\mu^2} e^{(\beta-2\alpha)\bar{\Phi}} \mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} + \frac{1}{2}(\alpha^2 - 1) \mathcal{L}^2 \right] \quad (24)$$

Using the equation of motion for γ_{ab} it is easy to show that $\gamma_{ab} = \Omega^2 m_{ab}$ where Ω is an arbitrary conformal factor, but the variation of (24) with respect to X^μ vanishes. The reason for this discrepancy is clear: varying (23) with respect to X^μ produces terms involving derivatives of the Green's function, which are not included in (24). We now turn to $T^{\mu\nu}$, which diverges quadratically as the string is approached. This divergence can be found by extracting the highest (linear) divergences in the relevant versions of $\partial_\rho A$ and substituting into (9) and the weak field gravitational energy momentum pseudotensor. This has already been done for the straight superstring⁶, from which it is easy to show that in the case of arbitrary couplings the divergence is proportional to $\mu^2 e^{2\alpha\bar{\Phi}} (\alpha^2/2 - 1) + \lambda^2 e^{\beta\bar{\Phi}}/2$. Taken together with (21), the divergences

cancel in both the self force and the energy momentum tensor only if $\lambda^2 = \mu^2 e^{(2\alpha-\beta)\Phi}$ and $\alpha = 1$. When $\beta = 2$ the cancellation occurs independently of the value of the background field, and we have precisely the parameters in the classical action for a fundamental string coupled to the background fields $g_{\mu\nu}$, $B_{\mu\nu}$ and Φ in four dimensions^{10,6}. Interestingly enough, this cancellation occurs for the fundamental string in $d = 4$, where d is the number of “large” dimensions, irrespective of the compactified dimensions. The classical action in $D = d + n$ dimensions, where n is the dimension of the compact space is¹¹

$$\begin{aligned} S &= S_2 + S_D \\ S_2 &= -\frac{\mu}{2} \int d^2\sigma (\sqrt{\gamma} \mathcal{L}^{\mu\nu} g_{\mu\nu} + \mathcal{K}^{\mu\nu} B_{\mu\nu}) + \frac{1}{4\pi} \int d^2\sigma \sqrt{\gamma} R^{(2)} \Phi \\ S_D &= \frac{1}{16\pi G} \int d^D x \sqrt{\hat{g}} e^{-2\Phi} (-R^{(D)} - 4(\nabla\Phi)^2 + \frac{1}{12} H^2) \end{aligned}$$

So long as the curvature of the world sheet is much less than μ , the last term in S_2 can be ignored. We choose coordinates $x^A = (x^\mu, y^i)$ where $\mu = 0, \dots, d-1$ and $i = 1, \dots, n$. Rewriting the D-dimensional metric \hat{g}_{AB} as

$$\hat{g}_{AB}(x, y) dx^A dx^B = e^{(4\Phi - 2n\sigma)/(d-2)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{2\sigma} \tilde{g}_{ij} dy^i dy^j \quad (25)$$

with $\sigma = \sigma(x)$, we find that the dimensionally reduced action becomes, upon defining $\phi = \alpha(2\Phi - n\sigma)$ with $\alpha = \sqrt{2/(d-2)}$

$$\begin{aligned} S_2 &= -\frac{\mu}{2} \int d^2\sigma (\sqrt{\gamma} \mathcal{L}^{\mu\nu} g_{\mu\nu} e^{\alpha\phi} + \mathcal{K}^{\mu\nu} B_{\mu\nu}) \\ S_d &= \frac{V}{16\pi \hat{G}} \int d^d x \sqrt{g} (-R^{(d)} + \frac{1}{2}(\nabla\phi)^2 + n(\nabla\sigma)^2 + \frac{1}{12} e^{-2\alpha\phi} H^2 + U(\phi, \sigma)) \end{aligned}$$

where V is the volume of the internal space and U an effective potential whose form depends on the particular compactification¹¹. A massive dilaton will not however affect the divergence structure which is a short distance result. We have dropped the vectors and scalars that result from the compactifications of the massless D-dimensional

fields, for if the string moves in only the non-compact dimensions ($X^i(\sigma) = \text{const}$) it will not generate any of these fields. Thus our results do not depend on the original number of dimensions of the theory. Although they are classical results and not obviously connected to the quantum string theory, it is nevertheless intriguing that the divergences in $T^{\mu\nu}$ could cancel between fields of different spin when the graviton and antisymmetric tensor couplings are equal. It was previously shown that there was no renormalisation of the string tension to one loop in the quantum superstring theory and argued that supersymmetry was to blame⁶. Here we can see explicitly how the supersymmetry works; the theory is the bosonic part of an $N = 1$ supergravity theory only when $\mu = \lambda$ and $\alpha = \sqrt{2/(d-2)}$.

Our extraction of the divergent part of the self force (21) is of interest for cosmic string backreaction calculations. Quashnock and Spergel² developed a perturbation theory which evolved the string along a Nambu trajectory for one period, and then calculated the change in $\dot{X}^\mu + X'^\mu$ and $\dot{X}^\mu - X'^\mu$ from (21). No infinities appeared, which we can now see is because the divergence is proportional to $\ddot{X}^\mu - X''^\mu$, which always vanishes with this technique.

Finally, we discuss the relation between the divergences in the classical theory to those in the quantum theory. When a charged spinless point particle, whose classical divergence in the self-mass is linear, is quantized the divergence becomes worse – quadratic, in fact. This divergence can be cancelled by making the theory supersymmetric, so that the quadratic divergence from gauge boson bubbles on the scalar propagator is cancelled by fermion loops. Similarly, quantizing the bosonic string theory makes the divergence in the self-mass worse, and supersymmetry sorts the problem out. Classically, there is no divergence in the self energy as we have seen, provided $\mu = \lambda$ and $\alpha = \beta/2 = 1$, but upon quantization, which can only be done consistently for these values of the parameters (and with an extra 22 bosonic degrees of

freedom) it is found that the one loop correction to the self mass is again divergent¹². The divergence comes from parts of moduli space corresponding to parts of the torus becoming long thin tubes, down which only massless modes (and tachyons) can propagate. If we ignore the tachyon, the divergence is logarithmic and proportional to the dilaton one loop expectation value: the geometrical picture is that conformal invariance enables us to deform the thin handles down which the massless modes are propagating into a sphere attached by a stalk to a torus with no external legs. For the closed *superstring*, the dilaton expectation value vanishes at one loop, supersymmetry again removes the divergence, and the self-mass is finite. These arguments are, of course, far from rigorous, but they are intended only to highlight the intriguing relationships between quantum and classical string theory and the role that supersymmetry plays in both.

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