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N90-24649

COUPLED BE/FE/BE APPROACH FOR SCATTERING FROM FLUID-FILLED STRUCTURES

by

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ABSTRACT

NASHUA is a coupled finite element/boundary element capability built around NASTRAN for calculating the low frequency far-field acoustic pressure field radiated or scattered by an arbitrary, submerged, three-dimensional, elastic structure subjected to either internal time-harmonic mechanical loads or external time-harmonic incident loadings. This paper describes the formulation and use of NASHUA for solving such structural acoustics problems when the structure is fluid-filled. NASTRAN is used to generate the structural finite element model and to perform most of the required matrix operations. Both fluid domains are modeled using the boundary element capabililty in NASHUA, whose matrix formulation (and the associated NASTRAN DMAP) for evacuated structures can be used with suitable interpretation of the matrix definitions. After computing surface pressures and normal velocities, far-field pressures are evaluated using an asymptotic form of the Helmholtz exterior integral equation. The proposed numerical approach is validated by comparing the acoustic field scattered from a submerged fluid-filled spherical thin shell to that obtained with a series solution, which is also derived in this paper.

INTRODUCTION

Two basic problems in computational structural acoustics are (1) the calculation of the acoustic pressure field radiated by a general submerged three-dimensional elastic structure subjected to internal time-harmonic loads, and (2) the calculation of the acoustic pressure scattered by such a structure subjected to an incident time-harmonic wavetrain. The most common, as well as the most accurate, approach for solving these problems at low frequencies is to couple a finite element model of the structure with a boundary element model of the surrounding fluid. This is the approach taken by NASHUA, which is a boundary element program built around NASTRAN, a widely-used finite element computer program for structural dynamics.

Several previous papers (Ref. 1-4) described the basic formulation and development for acoustic radiation and scattering from evacuated structures. Here we describe the formulation and use of NASHUA for modeling submerged structures which are fluid-filled. Internal fluid can occur because the structure is free-flooded or contains fluid-filled tanks. It is possible to use existing NASTRAN capability to model the interior fluid with finite elements (Ref. 5-7), but three-dimensional models with large numbers of fluid degrees of freedom might result. An attractive alternative to the fluid finite element model is to represent the contained fluid using a boundary element approach. In principle, any computer program capable of generating the appropriate boundary element matrices for an exterior fluid is also capable of generating such matrices for the complementary region (the interior region). NASTRAN's versatility in user-controlled matrix operations (DMAP) makes the implementation of such an approach straightforward.

THEORETICAL APPROACH

The basic theoretical development for NASHUA's radiation and scattering approach for evacuated structures has been presented in detail previously (Ref. 1-4). For completeness, this paper summarizes that approach and describes the changes necessary to model the interior fluid with boundary elements in the same procedure. There is no requirement that the interior and exterior fluids be the same.

The Surface Solution for Evacuated Structures

Consider any submerged three-dimensional, evacuated elastic structure subjected to either internal time-harmonic loads or an external time-harmonic incident wavetrain. If the structure is modeled with finite elements using NASTRAN, the resulting matrix equation of motion can be written as

$$Zv = F - GAp, \tag{1}$$

where matrix Z (of dimension s x s) is the structural impedance, vector v (s x r) is the complex velocity amplitude for all structural DOF (wet and dry) using the coordinate systems selected by the user, vector F (s x r) is the complex amplitude of the mechanical forces applied to the structure, matrix G (s x f) is the rectangular transformation of direction cosines to transform a vector of outward normal forces at the wet points to a vector of forces at all points in the coordinate systems selected by the user, matrix A (f x f) is the diagonal area matrix for the wet surface, and vector p (f x r) is the complex amplitude of total pressures (incident + scattered) applied at the wet grid points. In this equation, the time dependence $\exp(i\omega t)$ has been suppressed. In the above dimensions, s denotes the total number of independent structural DOF (wet and dry), f denotes the number of fluid DOF (wet points), and r denotes the number of load cases. In general, the surface areas, the normals, and the transformation matrix G are obtained in NASHUA from the NASTRAN calculation of the load vector resulting from an outwardly directly static unit pressure load on the structure's wet surface.

In Eq. 1, the structural impedance matrix Z, which converts velocity to force, is given by

$$Z = (-\omega^2 M + i\omega B + K)/(i\omega), \tag{2}$$

where M, B, and K are the structural mass, viscous damping, and stiffness matrices, respectively, and ω is the circular frequency of excitation. For structures with a nonzero loss factor, K is complex. In addition, K can include the differential stiffness effects of hydrostatic pressure, if any (Ref. 3). A standard NASTRAN finite element model of the structure supplies the matrices K, M, and B.

For the exterior fluid domain, the total fluid pressure p satisfies the Helmholtz differential equation

$$\nabla^2 p + k^2 p = 0, \tag{3}$$

where $k = \omega/c$ is the acoustic wave number, and c is the fluid sound speed. Equivalently, for the exterior fluid, p is the solution of the Helmholtz integral equations (Ref. 8)

$$\int_S p(\mathbf{x}) \frac{\partial D(r)}{\partial n} dS - \int_S q(\mathbf{x}) D(r) dS = \begin{cases} p(\mathbf{x}')/2 - p_I, & \mathbf{x}' \text{ on } S, \\ p(\mathbf{x}') - p_I, & \mathbf{x}' \text{ in } E, \\ -p_I, & \mathbf{x}' \text{ in } I, \end{cases} \tag{4}$$

where S, E, and I denote the surface, exterior, and interior domains, respectively, p_I is the incident free-field pressure (if any), r is the distance from **x** to **x'** (Fig. 1), D is the free-space Green's function

$$D(r) = \frac{e^{-ikr}}{4\pi r}, \tag{5}$$

$$q = \frac{\partial p}{\partial n} = -i\omega\rho v_n, \tag{6}$$

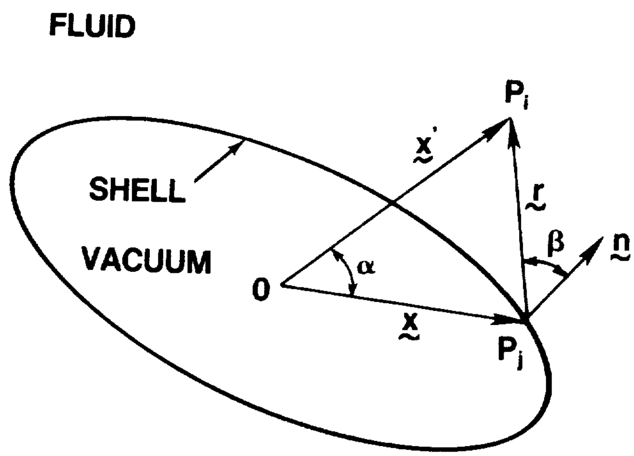

Fig. 1. Notation for Helmholtz Integral Equation

ρ is the fluid mass density, and v_n is the outward normal velocity on S. As shown in Fig. 1, **x** in Eq. 4 is the position vector for a typical point P_j on the surface S, **x'** is the position vector for the point P_i on the surface or in the exterior field, the vector **r** = **x'** - **x**, and **n** is the unit outward normal at P_j. We denote the lengths of the vectors **x**, **x'**, and **r** by x, x', and r, respectively. The normal derivative of the Green's function D is (Ref. 1)

$$\frac{\partial D(r)}{\partial n} = \frac{e^{-ikr}}{4\pi r}\left(ik + \frac{1}{r}\right) \cos\beta, \tag{7}$$

where β is the angle between the normal **n** and the vector **r**, as shown in Fig. 1.

All three integral equations in Eq. 4 are needed for exterior fluids. The surface equation provides the fluid impedance at the fluid-structure interface. Since the surface equation exhibits non-uniqueness at certain discrete characteristic frequencies (Ref. 9), the interior equation is used to provide additional constraint equations which ensure the required uniqueness. The exterior equation is used to compute the exterior pressure field once the surface solution (which includes the fluid pressure and its gradient) is known.

The substitution of Eqs. 6 and 7 into the surface equation (4) yields

$$\frac{p(\mathbf{x}')}{2} - \int_S p(\mathbf{x}) \frac{e^{-ikr}}{4\pi r} (ik + \frac{1}{r}) \cos\beta \, dS = i\omega\rho \int_S v_n(\mathbf{x}) \frac{e^{-ikr}}{4\pi r} dS + p_I, \quad \mathbf{x}' \text{ on } S. \quad (8)$$

This integral equation relates the total pressure p and normal velocity v_n on S. If the integrals in Eq. 8 are discretized for numerical computation (Ref. 1), we obtain the matrix equation (for the exterior fluid)

$$Ep = Cv_n + p_I, \quad (9)$$

where vector p (of dimension f x r) is the vector of complex amplitudes of the total pressure on the structure's wet surface, matrices E and C (both f x f) are fully-populated, complex, nonsymmetric, and frequency-dependent, and vector p_I (f x r) is the complex amplitude of the incident pressure vector. The number of unknowns in this system is f, the number of wet points on the fluid-structure interface.

The normal velocities v_n in Eq. 9 are related to the total velocities v by the same rectangular transformation matrix G:

$$v_n = G^T v, \quad (10)$$

where T denotes the matrix transpose. If velocities v and v_n are eliminated from Eqs. 1, 9, and 10, the resulting equation for the coupled fluid-structure system is

$$(E + CG^T Z^{-1} GA)\, p = CG^T Z^{-1} F + p_I. \quad (11)$$

This equation is solved for the total surface pressures p, since the rest of the equation depends only on the geometry, the material properties, and the frequency. Since the two right-hand side terms in Eq. 11 correspond to mechanical and incident loadings, only one of the two terms would ordinarily be present for a given case. The details of the incident pressure vector p_I for scattering problems were presented previously (Ref. 2) and will not be repeated here.

The velocity vector v for all structural DOF is recovered by solving Eq. 1 for v:

$$v = Z^{-1}F - Z^{-1}GAp. \quad (12)$$

The surface normal velocity vector v_n is recovered by substituting this solution for v into Eq. 10.

Modeling Interior Fluid

The theoretical development presented in the preceding section can be modified slightly to account also for an interior fluid. The wave equation, Eq. 3, applies also to interior fluids. Although all three integral equations in Eq. 4 are generally needed for exterior fluids, only the surface equation is needed to represent the surface impedance of interior fluids. Eq. 4a also applies to interior fluids if the incident pressure p_I is set to zero, and the normal vector **n** is negated. That is, the surface integral equation applies to both exterior and interior fluids so long as the unit normal is always directed from the structure into the fluid. One other consideration, perhaps unique to NASHUA, is that wet surface curvatures (which are needed in the calculation of the "self" terms in matrix E) are negative at some interior points (Ref. 1).

A matrix equation similar to Eq. (9) is therefore obtained for the interior fluid except that the incident pressure p_I is zero. The fluid matrices E and C are different for exterior and interior domains (even if the separating surface S has infinitesimal thickness) because the normals are of opposite sign.

Two-Fluid Formulation

Denote the exterior fluid as Fluid 1 and the interior fluid as Fluid 2, and use the subscripts 1 and 2 to refer to these two domains. Also define new pressure and normal velocity unknowns p and v_n which include the solutions for both fluid domains:

$$p = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}, \qquad v_n = \begin{Bmatrix} v_{n1} \\ v_{n2} \end{Bmatrix}. \tag{13}$$

Since there is no direct fluid coupling between the interior and exterior fluids, and the incident pressure vanishes in the interior domain, Eqs. 1, 9, 10, and 11 apply also to the two-fluid problem if the new definitions in Eq. 13 are used, and the matrices A, G, E, C, and p_I are re-defined as

$$A = \begin{bmatrix} A_1 & \\ & A_2 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \; G_2 \end{bmatrix}, \quad E = \begin{bmatrix} E_1 & \\ & E_2 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & \\ & C_2 \end{bmatrix}, \quad p_I = \begin{Bmatrix} p_{I1} \\ 0 \end{Bmatrix}. \tag{14}$$

The principle benefit of formulating the two-fluid problem in this way is that the required modifications to extend the procedure to three or more independent fluid domains is now clear.

The Far-Field Calculation

With the solution for the total pressures and velocities on the surface, the exterior Helmholtz integral equation, Eq. 4b, can be integrated to obtain the radiated (or scattered) pressure at any desired location $\mathbf{x}'$ in the exterior field. We first substitute Eqs. 5 - 7 into Eq. 4b to obtain

$$p(\mathbf{x}') = \int_S [i\omega\rho v_n(\mathbf{x}) + (ik + \frac{1}{r})p(\mathbf{x}) \cos\beta] \frac{e^{-ikr}}{4\pi r}\, dS, \quad \mathbf{x}' \text{ in E.} \tag{15}$$

In applications, however, the field pressures generally of interest are in the far-field, so we use instead the asymptotic form of Eq. 15 (Ref. 1):

$$p(\mathbf{x}') = \frac{ike^{-ikx'}}{4\pi x'} \int_S [\rho c v_n(\mathbf{x}) + p(\mathbf{x}) \cos\beta] e^{ikx \cos\alpha}\, dS, \quad \mathbf{x}' \text{ in E}, \; x' \gg d, \tag{16}$$

where d is a characteristic dimension of the structure, and α is the angle between the vectors $\mathbf{x}$ and $\mathbf{x}'$ (Fig. 1). For far-field points, $\cos\beta$ is computed using the asymptotic approximation

$$\cos\beta \rightarrow \mathbf{n}\cdot\frac{\mathbf{x}'}{x'} . \tag{17}$$

For both Eqs. 15 and 16, numerical quadrature is used.

OVERVIEW OF SOLUTION PROCEDURE

The NASHUA solution procedure uses NASTRAN to generate the matrices K, M, B, and F and to generate sufficient geometry information so that the matrices E, C, G, A, and p_I can be computed by a separate program called SURF. Then, NASTRAN DMAP is used to form the matrices appearing in Eq. 11, which is solved for the total pressures p (in both fluid domains) using the block solver OCSOLV (Ref. 10). Next, NASTRAN DMAP is used to recover the surface normal velocities v_n and the vector v of velocities at all structural DOF (NASTRAN's "g-set"). This step completes the surface solution. Then, with the total pressures and velocities on the (exterior) surface, the asymptotic (far-field) form of the Helmholtz exterior integral equation is integrated in program FAROUT to compute the far-field radiated pressures. Various tables and graphical displays are generated.

The overall setup of the solution procedure is organized into four steps. In Step 1, a separate NASTRAN structural model is prepared and run for each unique set of symmetry constraints and each fluid region. Since, for general three-dimensional analysis, up to three planes of reflective symmetry are allowed, there would be one, two, four, or eight such runs for each fluid region. Since the purpose of this step is to generate a file containing geometry information and a checkpoint file for subsequent use in the other steps, the only difference between the two runs associated with a given symmetry case is the specification of the outwardly directed unit pressure load which defines the wet surface for a given fluid region.

For each symmetry case and drive frequency, several programs are run sequentially to form Step 2. For each fluid region, the SURF program reads the geometry file generated by NASTRAN in Step 1 and, using the Helmholtz surface and interior integral equations, generates the fluid matrices E_1, E_2, C_1, and C_2, the area matrices A_1 and A_2, the structure-fluid transformation matrices G_1 and G_2, the incident pressure vector p_{I1}, and a geometry file to be used later by the far-field integration program FAROUT in Step 3. In addition, a partitioning vector is generated to facilitate the merging and partitioning of the various matrices associated with the two fluid domains.

The two SURF jobs in Step 2 are followed by a NASTRAN job which takes the structural matrices K, M, B, and F from Step 1 and the matrices generated by the SURF jobs and forms the matrices in Eq. 11, where the definitions in Eq. 14 apply. Eq. 11 is then solved for the total surface pressure vector p by program OCSOLV, which is a general out-of-core block solver designed specifically for large, full, complex, nonsymmetric systems of linear, algebraic equations. NASTRAN is then re-entered in Step 2 with p so that the velocities v and v_n can be recovered using DMAP operations. The surface pressures, normal velocities, and full g-set displacements are then reformatted, sorted, and merged into a single file (for each symmetry case) using program MERGE. Recall that there are one, two, four, or eight possible symmetry cases.

Steps 1 and 2 are repeated for each symmetry case. After all symmetry cases have been completed and merged, program FAROUT (Step 3) combines the symmetry cases and integrates over the surface. The far-field pressure solution is obtained by integrating the surface pressures and velocities using the asymptotic (far-field) form of the exterior Helmholtz integral equation, Eq. 16. Output from FAROUT consists of both tables and files suitable for various types of plotting.

The remaining steps in the NASHUA procedure are for graphical display. Deformed structural plots of the frequency response are obtained by restarting NASTRAN (Step 4) with the checkpoint file from Step 1 and a results file from FAROUT. In addition, animated plots can be generated on the Evans & Sutherland PS-330 graphics terminal using the CANDI program written for the DEC/VAX computer by R.R. Lipman of DTRC (Ref. 11). X-Y plots of various quantities (both surface and far-field) versus frequency may be obtained using IPLOT or other interactive plotting programs (Ref. 12). Polar plots of the far-field sound pressure levels in each of the three principal coordinate planes can also be generated using the interactive graphics program FAFPLOT (Ref. 1).

DMAP ALTERS

Several DMAP alters are used in the overall NASHUA procedure to implement the precedure described in preceding section. For Step 1, the following alter is used:

```
ALTER       1 $ NASHUA STEP 1, COSMIC 1988 RF8 (REVISED 12/7/89)
ALTER       2,2 $ DELETE PRECHK
ALTER       21,21 $ REPLACE GP3
GP3         GEOM3,EQEXIN,GEOM2/SLT,GPTT/S,N,NOGRAV/NEVER=1 $
ALTER       117,117 $ REPLACE FRRD
SSG1        SLT,BGPDT,CSTM,SIL,EST,MPT,GPTT,EDT,MGG,CASECC,DIT,/
            PG,,,,/LUSET/NSKIP $ PG
SSG2        USET,GM,YS,KFS,GO,DM,PG/QR,PO,PS,PL $ PL
OUTPUT2     AXIC,BGPDT,EQEXIN,USET,PG $
OUTPUT2     PL,CSTM,ECT,, $
OUTPUT2     ,,,,//-9 $
PARAMR      //*EQ*//C,Y,HSP=0./0.////NOHSP $
COND        LBL4D,NOHSP $ SKIP DIFF. STIFF. IF NO HYDRO. P
PARAMR      //*COMPLEX//C,Y,HSP=0./0./HSPC $ HSP+I*0
DIAGONAL    KAA/KDIAG/*SQUARE*/1.0 $
ADD         KAA,KDIAG/KAAD/(1.0,0.0)/(1.E-6,0.) $
RBMG2       KAAD/LLL $ FACTOR KAA
SSG3        LLL,KAAD,PL,LOO,KOO,PO/ULV,UOOV,RULV,RUOV/OMIT/
            V,Y,IRES=-1/1/S,N,EPSI $ STATIC SOLUTION
SDR1        USET,PG,ULV,UOOV,YS,GO,GM,PS,KFS,KSS,/UGV,PGG,QG/1/
            *BKL0* $ RECOVER DEPENDENT DISPLACEMENTS
TA1         ECT,EPT,BGPDT,SIL,GPTT,CSTM/X1,X2,X3,ECPT,GPCT/LUSET/
            NOSIMP/0/NOGENL/GENEL $ TABLES FOR DIFF. STIFF.
DSMG1       CASECC,GPTT,SIL,EDT,UGV,CSTM,MPT,ECPT,GPCT,DIT/KDGG/
            S,N,DSCOSET $ DIFF. STIFF. MATRIX
EQUIV       KDGG,KDNN/MPCF2 / MGG,MNN/MPCF2 $ EQUIV IF NO MPC'S
COND        LBL1D,MPCF2 $ TRANSFER IF NO MPC'S
MCE2        USET,GM,KDGG,,,/KDNN,,, $ MPC'S ON DIFF. STIFF.
LABEL       LBL1D $
EQUIV       KDNN,KDFF/SINGLE / MNN,MFF/SINGLE $ EQUIV. IF NO SPC'S
COND        LBL2D,SINGLE $ TRANSFER IF NO SPC'S
```

```
SCE1        USET,KDNN,,,/KDFF,KDFS,KDSS,,, $ SPC'S AND DIFF. STIFF.
LABEL       LBL2D $
EQUIV       KDFF,KDAA/OMIT / MFF,MAA/OMIT $ EQUIV. IF NO OMITS
COND        LBL3D,OMIT $ TRANSFER IF NO OMITS
SMP2        USET,GO,KDFF/KDAA $ OMITS AND DIFF. STIFF.
LABEL       LBL3D $
PARAMR      //*SUBC*////MHSPC//HSPC $ NEGATE HYDRO. P
ADD         KDD,KDAA/NEWKDD/(1.0,0.0)/MHSPC $
ADD         KFS,KDFS/NEWKFS/(1.0,0.0)/MHSPC $
EQUIV       NEWKDD,KDD//NEWKFS,KFS $
LABEL       LBL4D $ END OF DIFF. STIFF. EFFECTS (HSP)
DIAGONAL    KDD/IDENT/*SQUARE*/0. $ D-SET IDENTITY
ADD         IDENT,/IDM/(1.0,0.0) $ ANOTHER D-SET IDENTITY
ADD         IDENT,/ZERO/(0.0,0.0) $ D-SET ZERO MATRIX
FRRD        CASEXX,USETD,DLT,FRL,GMD,GOD,IDENT,ZERO,IDM,,DIT/
            UDVF,PSF,PDF,PPF/*DISP*/*DIRECT*/LUSETD/MPCF1/
            SINGLE/OMIT/NONCUP/FRQSET $ PDF, KDD=I, BDD=0, MDD=I
CHKPNT      MDD,KDD,BDD,PDF,PSF,PPF,EQDYN,USETD,GOD,GMD $
CHKPNT      KFS,BGPDT,ECT,EQEXIN,GPECT,SIL $
EXIT $
ENDALTER $
```

The above alter does not depend on whether the fluid is interior or exterior to the structure. The Step 2 alters, however, depend on whether an interior fluid is present. For Step 2A, the following alter is used:

```
ALTER       1 $ NASHUA STEP 2A, COSMIC 1988 RF8 (REVISED 11/7/89)
ALTER       2,167 $ REPLACE ALL AFTER 'BEGIN' AND BEFORE 'END'
INPUTT2     /DAT2,,,,//13 $ INTERNAL FLUID
INPUTT2     /DAT,,,,//11 $ READ SURF MATRIX FROM UT1
MATPRN      DAT,DAT2,,, $
PARAML      DAT//*DMI*/1/8/RIGD $ GET RIGID FLAG
PARAMR      //*EQ*//RIGD/0.////ELAST $ SET ELAST=-1 IF ELASTIC
COND        LBL9D,ELAST $ IF ELASTIC, JUMP OVER RIGID/SOFT
PARAMR      //*EQ*//RIGD/2.////SOFT $ SET SOFT=-1 IF SOFT BD.
COND        LBL9E,SOFT $ IF SOFT BOUNDARY, JUMP OVER RIGID
INPUTT2     /E,PI,VEKC,,//11 $ READ SURF MATRICES FROM UT1
OUTPUT2     PI,E,,, //-1 $ INPUTT2 FILE IS OVER-WRITTEN (UT1)
OUTPUT2     ,,,, //-9 $ EOF
CHKPNT      DAT,VEKC $
EXIT        $
LABEL       LBL9E $ BEGINNING OF SOFT ANALYSIS
INPUTT2     /CT,PI,VEKC,,//11 $ READ SURF MATRICES FROM UT1
TRNSP       CT/C $
ADD         PI,/MPI/(-1.0,0.0) $ NEGATE PI
OUTPUT2     MPI,C,,, //-1 $ INPUTT2 FILE IS OVER-WRITTEN (UT1)
OUTPUT2     ,,,, //-9 $ EOF
CHKPNT      DAT,VEKC $
EXIT        $
LABEL       LBL9D $ BEGINNING OF ELASTIC ANALYSIS
INPUTT5     /G2,A2,,,//14 $ INTERNAL FLUID
```

```
INPUTT2     /C2T,E2,PI2,VEKC2,//13 $ INTERNAL FLUID
INPUTT5     /G1,A1,,,//12 $ READ SURF MATRICES FROM UT2
INPUTT2     /C1T,E1,PI1,VEKC,FVEC//11 $ READ SURF MATRICES
MATPRN      FVEC,,,, $
MERGE       A1,,,A2,FVEC,/A/-1 $
MERGE       E1,,,E2,FVEC,/E/-1 $
MERGE       C1T,,,C2T,FVEC,/CT/-1 $
MERGE       G1,,G2,,FVEC,/G/0 $
MERGE       PI1,,,,,FVEC/PI/0 $
MATPRN      VECM,VECS,,, $
PARAML      DAT//*DMI*/1/2/FREQ $ GET FREQ FROM DAT
PARAMR      //*COMPLEX*//FREQ/0./FREQC $ FREQ+I*0
PARAMR      //*MPYC*////W/FREQC/(6.283185,0.) $ OMEGA
PARAMR      //*MPYC*////IW/W/(0.,1.) $ I*OMEGA
PARAMR      //*MPYC*////WW/W/W $ OMEGA**2
PARAMR      //*SUBC*////MWW//WW $ -OMEGA**2
ADD5        MDD,KDD,BDD,,/Y/MWW/(1.0,0.0)/IW $
MPYAD       G,A,/GA/0 $
DECOMP      Y/L,U/1//S,N,MINDIAG///S,N,SING $
FBS         L,U,GA/YIGA/1 $
FBS         L,U,PDF/YIF/1 $
ADD         YIGA,/ZIGA/IW $
ADD         YIF,/ZIF/IW $
MPYAD       G,ZIGA,/GTZIGA/1 $
MPYAD       CT,GTZIGA,E/H/1 $ LHS
MPYAD       G,ZIF,/GTZIF/1 $
MPYAD       CT,GTZIF,/Q/1 $ MECHANICAL RHS
MERGE       DUM,,PDF,,VECM,/PDF1/1 $ MERGE IN 0 COLUMNS
MERGE       DUM,,PSF,,VECM,/PSF1/1 $ MERGE IN 0 COLUMNS
MERGE       DUM,,PPF,,VECM,/PPF1/1 $ MERGE IN 0 COLUMNS
EQUIV       PDF1,PDF//PSF1,PSF//PPF1,PPF $
MERGE       DUM,,Q,,VECM,/RHS1/1 $ MERGE IN ZERO COLUMNS
MERGE       DUM,,GTZIF,,VECM,/GTZIFE/1 $ MERGE IN 0 COLUMNS
MERGE       DUM,,PI,,VECS,/RHS2/1 $ MERGE IN ZERO COLUMNS
ADD         RHS1,RHS2/RHS $ ADD MECH. AND INC. RHS
EQUIV       USETD,DUM1//GOD,DUM2//GMD,DUM3//KFS,DUM4 $
OUTPUT2     RHS,H,,, //-1 $ INPUTT2 FILE IS OVER-WRITTEN (UT1)
OUTPUT2     ,,,, //-9 $ EOF
CHKPNT      GTZIGA,GTZIFE,GA,PDF,L,U,PSF,DAT,VEKC,FVEC $
CHKPNT      USETD,GOD,GMD,KFS $
ENDALTER    $
```

The differences between this alter and one used for submerged evacuated structures are due to the need to read and combine two sets of SURF matrices, one for each fluid domain. For Step 2B, the following alter is used:

```
ALTER       1 $ NASHUA STEP 2B, COSMIC 1988 RF8 (REVISED 11/7/89)
ALTER       2,167 $ REPLACE ALL AFTER 'BEGIN' AND BEFORE 'END'
INPUTT2     /PC,,,,//11 $ READ PRESSURES FROM BLOCK SOLVER (UT1)
PARTN       PC,,FVEC/P1,,,/0 $ REMOVE INTERNAL FLUID DOF
PARTN       P1,,VEKC/P,,,/0 $ REMOVE CHIEF DOF FROM P
```

```
COND        LBL9D,ELAST $ IF ELASTIC, JUMP OVER RIGID/SOFT
OUTPUT2     DAT,P,,, //-1 $ INPUTT2 FILE IS OVER-WRITTEN (UT1)
OUTPUT2     ,,,, //-9 $ EOF
MATPRN      DAT,P,,, $ FOR SOFT BOUNDARY, P REPRESENTS VELOCITY
EXIT        $
LABEL       LBL9D $ ELASTIC ANALYSIS
MPYAD       GTZIGA,PC,GTZIFE/VNC/0/-1 $ NORMAL VELOCITIES
MPYAD       GA,PC,PDF/FA/0/-1 $ A-SET FORCES
FBS         L,U,FA/UDVF/1 $ A-SET DISPLACEMENTS
SDR1        USETD,,UDVF,,,GOD,GMD,PSF,KFS,,/UPVC,,QPC/1/
            *DYNAMICS* $
PARTN       VNC,,FVEC/V1,,,/0 $ REMOVE INTERNAL FLUID DOF
PARTN       V1,,VEKC/VN,,,/0 $ REMOVE CHIEF DOF FROM VN
OUTPUT2     DAT,P,VN,UPVC, //-1 $ INPUTT2 FILE IS OVER-WRITTEN
OUTPUT2     ,,,, //-9 $ EOF
MATPRN      DAT,P,VN,, $
ENDALTER    $
```

This alter differs from one for evacuated structures because of the presence of several matrix partitionings to remove the internal fluid DOF from the solution vectors before the solutions are merged with the results for other frequencies.

NUMERICAL EXAMPLE

Here we illustrate and validate the two-fluid boundary element formulation developed above by solving the problem of acoustic scattering from a submerged fluid-filled spherical thin shell. The incident loading is a time-harmonic planar wavetrain, as shown in Fig. 2. The specific problem solved has the following characteristics:

shell mean radius (a)	5 m
shell thickness (h)	0.15 m
shell Young's modulus (E)	2.07×10^{11} N/m^2
shell Poisson's ratio (ν)	0.3
shell density (ρ_s)	7669 kg/m^3
shell loss factor (η)	0.01
fluid density (ρ)	1000 kg/m^3
fluid sound speed (c)	1524 m/s

The same fluid is used for both the exterior and interior fluid domains. The solution of this problem exhibits rotational symmetry about the spherical axis parallel to the direction of wave propagation. The benchmark solution to which the numerical results will be compared is a series solution, the derivation of which is summarized in the next section.

Series Solution

The series solution for scattering from a submerged evacuated spherical thin shell was presented by Junger and Feit (Ref. 13). Here we summarize that solution and indicate the modification necessary to include the addition of an interior fluid which fills the spherical

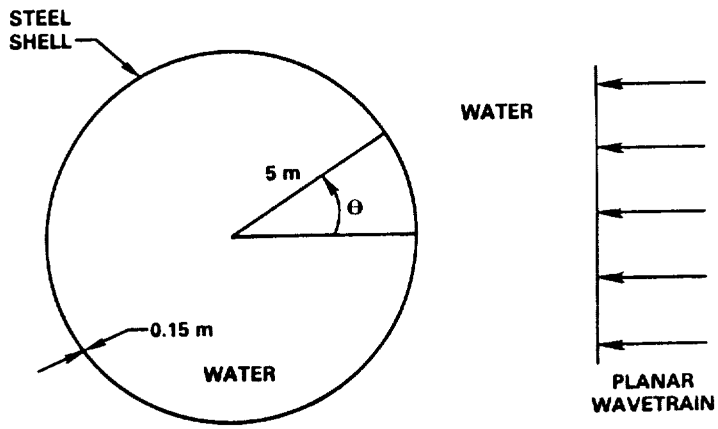

Fig. 2. Plane Wave Scattering from a Fluid-Filled Spherical Shell

volume.

In general, the series solution for plane wave scattering from a submerged, evacuated, spherical thin shell involves computing the impedances of the shell and exterior fluid, the scattered field due to rigid body effects, and the radiated field due to elastic (vibrational) effects. The shell impedance (the ratio of pressure to normal velocity) for the nth axisymmetric shell mode is

$$Z_n = -\frac{i\rho_s c_p}{\Omega}\,\frac{h}{a}\,\frac{[\Omega^2-(\Omega_n^{(1)})^2]\,[\Omega^2-(\Omega_n^{(2)})^2]}{[\Omega^2-(1+\beta^2)(\nu+\lambda_n-1]}, \tag{18}$$

where ρ_s is the structural mass density, $c_p = \sqrt{E/[\rho_s(1-\nu^2)]}$, E is Young's modulus, ν is Poisson's ratio, $\Omega = \omega a/c_p$ is the dimensionless frequency, h is the shell thickness, a is the shell mean radius, $\beta = h/(a\sqrt{12})$, and $\lambda_n = n(n+1)$. The quantities $\Omega_n^{(1)}$ and $\Omega_n^{(2)}$ are the upper and lower shell resonance dimensionless frequencies, respectively. They are the solutions of the characteristic equation

$$\Omega^4 - [1+3\nu+\lambda_n-\beta^2(1-\nu-\lambda_n^2-\nu\lambda_n]\,\Omega^2 + (\lambda_n-2)(1-\nu^2)+\beta^2[\lambda_n^3-4\lambda_n^2+\lambda(5-\nu^2)-2(1-\nu^2)] = 0. \tag{19}$$

The impedance of the exterior fluid, found by using the Green's function and identity for the exterior fluid, is

$$z_n = i\rho c\frac{h_n(ka)}{h'_n(ka)}, \tag{20}$$

where h_n is the Bessel's function of the third kind of order n.

Thus, Junger and Feit showed that the far-field scattered pressure is

$$p(R,\theta) = -\frac{ie^{ikR}p_o}{kR}\sum_{n=0}\frac{(2n+1)P_n(\cos\theta)}{h'_n(ka)}\left[j'_n(ka) - \frac{\rho c}{(Z_n+z_n)(ka)^2 h'_n(ka)}\right], \quad R>>a, \tag{21}$$

where R is the distance to the field point, θ is the angle from the z-axis, p_o is the incident pressure, P_n is the Legendre polynomial of order n, and j_n is the Bessel's function of the first kind of order n. The two terms in the bracketed expression correspond to rigid body and radiated effects, respectively.

The above expression for the pressure scattered from an evacuated shell can be extended to include the effects of the interior fluid merely by replacing the exterior fluid impedance z_n in Eq. 21 with the sum of the fluid impedances for the exterior and interior fluids. It can be shown, by using the Green's function and identity for the interior domain, that the interior impedance, denoted ς_n, is given by

$$\varsigma_n = -i\rho c\frac{j_n(ka)}{j'_n(ka)}. \tag{22}$$

We note the resemblance between Eqs. 20 and 22 for the exterior and interior domains, respectively.

The computer program used to evaluate this series solution is a modification of a program called SCATSPHERE written by F.M. Henderson, a retired employee of DTRC. SCATSPHERE in turn is a variant of an earlier program called RADSPHERE (Ref. 14) for computing the radiation from an internally-driven submerged spherical shell.

Numerical Solution

A NASTRAN finite element model of the spherical shell was prepared using 40 axisymmetric conical shell elements spanning the 180 degrees between the two poles of the sphere. Due to the axisymmetry of the incident pressure loading, only the N = 0 harmonic was required. Since all structural points are in contact with both interior and exterior fluids, the resulting model therefore had 205 independent structural degrees of freedom (DOF) and 41 fluid DOF for each of the two fluid domains. System matrices for the exterior fluid were also augmented by the addition of four constraint equations associated with interior Chief points to ensure uniqueness of the integral representation at the upper frequencies. The nondimensional frequency range $0<ka<5$ was swept using a frequency increment of about ka = 0.05 with NASHUA and ka = 0.005 with the series solution. Since the series solution is converged, we treat it as an "exact" solution for this problem.

The comparison between the computed and exact solutions is presented is Figs. 3 and 4, which plot the frequency response of the nondimensional scattered pressure $pr/(p_o a)$, where p is the far-field scattered pressure at distance r from the origin, p_o is the incident pressure, and a is the mean radius of the spherical shell. These two figures show very good agreement between the two scattering solutions in the backward ($\theta = 0$) and forward ($\theta = 180$ degrees) directions. In fact, the computed and series solutions are virtually indistinguishable from each other.

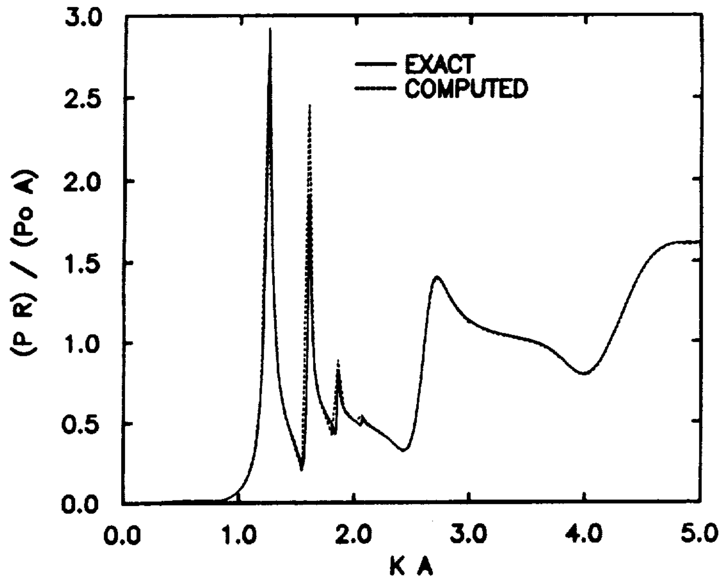

Fig. 3. Forward Scattering from a Fluid-Filled Spherical Shell

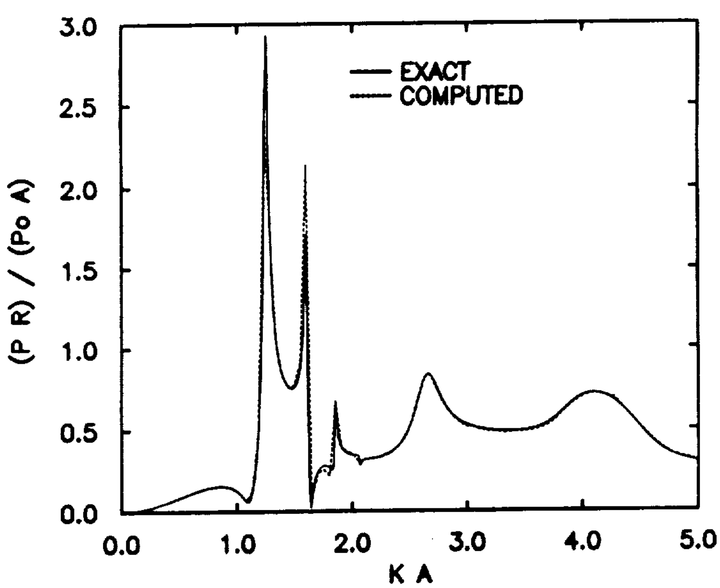

Fig. 4. Backward Scattering from a Fluid-Filled Spherical Shell

DISCUSSION

A very general computational capability has been described for predicting the sound pressure field radiated or scattered by arbitrary, submerged, fluid-filled, three-dimensional elastic structures subjected to time-harmonic loads. The structure is modeled with NASTRAN (in all the generality that NASTRAN allows) in combination with boundary element models of both interior and exterior fluid domains. Sufficient automation is provided so that, for many structures of practical interest, an existing structural model can be adapted for NASHUA acoustic analysis within a few hours.

One of the many benefits of having NASHUA linked with NASTRAN is the ability to integrate the acoustic analysis of a structure with other dynamic analyses. Thus the same finite element model can be used for modal analysis, frequency response analysis, linear shock analysis, and underwater acoustic analysis. In addition, many of the pre- and postprocessors developed for use with NASTRAN become available for NASHUA as well.

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