N90-24650

MONITORING OF RITZ MODAL GENERATION

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In applying a Ritz modal expansion to the solution of a transient response, there is a problem as to how many modes are needed to obtain accuracy to within a specified percentage.

One of us, Chargin, has suggested a method based on the characteristics of the forcing function. The method can be incorporated into the Ritz generation algorithm such that it will automatically monitor, regulate and terminate the process according to a specified tolerance.

FORCING FUNCTION CHARACTERISTICS

Assume that the forcing function F(x,t) can be represented as a product of a spatial function and a temporal function; i.e.

(1) $F(x,t) = F(x) \, . \, f(t).$

Develop a criterion based upon measuring the amount of power developed in the forcing function F(x,t). The total power in F(x,t) is the product of the "power" in F(x) and f(t) owing to the assumption in equation (1). The scheme in outline is to measure the temporal power and the spatial power in F(x,t) separately then compare the corresponding power in the Ritz modes against power in each component of the forcing. Generation of additional Ritz modes will continue until the power criteria are met. First a measure is taken of the total power in the temporal function.

Temporal Power

$$(2) \qquad P(\tau) = \int_0^{\tau} \left[f(t)\right]^2 dt \quad ,$$

where τ is the interval over which the transient will act. There could very well be a separate temporal function for each spatial function. In that case of multiple loadings $P(\tau)$ of equation (2) would be a vector "l" long.

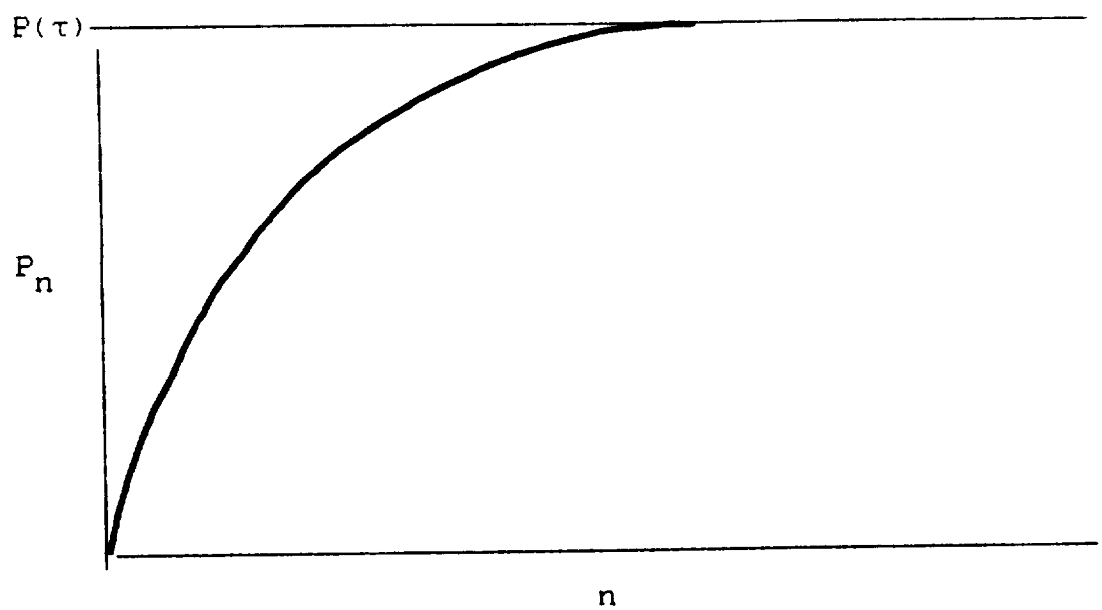
In order to tailor this power to our use as a guide in selecting Ritz modes it will be useful to measure the amount of temporal power as a function of frequency. Expand the temporal function in a Fourier Series and sum the power versus the expansion multiple:

$$(3) \qquad f(t) = a_o + a_1\cos\,\omega t + b_1\sin\,\omega t + a_2\cos\,2\omega t + b_2\sin\,2\omega t + \ldots\ldots + a_n\cos\,\omega t + b_n\sin\,\omega t \; .$$

The power within a band 0 to $n\omega$ is:

$$(4) \qquad P_n = \sum_{i=0}^{n} \left(a_i\cos\,i\omega t + b_i\sin\,i\omega t\right)^2 = \sum_{i=0}^{n} \left(a_i^2 + b_i^2\right) \; .$$

One can compare the amount of power within a given band P_n with the total power $P(\tau)$. When P_n is within close range of $P(\tau)$, say 1%, then the analyst can be satisfied that the frequency range of the truncated temporal forcing function is sufficiently broad to encompass the temporal requirements of the forcing.



For a certain frequency $f_n = \frac{n\omega}{2\pi}$; $\frac{P_n}{P(\tau)} = .99$. Let this frequency be f_o.

Spatial Power

Now turn to the spatial distribution of the forcing function F(x), and develop a measure that is called spatial power. Make an additional assumption that all points being loaded have mass.

$$(5) \qquad [\Pi(x)] = [F(x)]^T[M]\,[F(x)] = \left[C_{1a}\right]\left[F_{a1}(x)\right],$$

where $\left[C_{1a}\right] = \left[F(x)^T M\right]$ is a coefficient matrix that will be shown to be usefual later. $\Pi(\mathbf{x})$ is a matrix if there are a number of loading cases "1".

At this point we have 2 measures of the forcing fuction. We have f_0 reflecting the desired frequency content and $\Pi(\mathbf{x})$ reflecting the desired level of power to activate the structural mass. Now it is time to test the adequacy of the number of Ritz modes to repond to the forcing at these power levels.

First, get a spatial measure of the Ritz modes θ_k. A simple scheme is to pattern the measure after equation (5) but substitute the matrix of k spatial Ritz functions for the post-multiply operation instead of the spatial component of loading F(x).

$$(6) \qquad \left[{}^{\backslash}R_{r\backslash}\right] = \sum_{k=1}^{r} [F(x)]^T [M] \left[\theta_k\right] = \sum_{k=1}^{r} \left[C_{1a}\right]\left[\theta_k\right].$$

The way to use $\mathbf{R_r}$ is to compare each diagonal term of $\Pi(\mathbf{x})$ with the corresponding diagonal term of $\mathbf{R_r}$ to see if the ratio is within a specified tolerance; i.e.

$$(7) \qquad \left|\frac{\Pi_1 - R_{r1}}{\Pi_1}\right| \leq \varepsilon \text{ for every } 1.$$

Keep generating additional Ritz modes θ_k k > r until an r has been reached for which every diagonal term satisfies the criterion.

Monitoring Function

If the θ_k satisfy the spatial power criterion of F(x,t) it does not necessarily hold that θ_k will simultaneously satisfy the temporal requirements of F(x,t). Therefore, use the θ_k which have met the spatial power requirements and obtain an estimate of its frequency content by setting up the frequency equation. Use an estimator that is much less demanding than solving the eigenvalue problem. Merely do the following.

Compute a $k^{\underline{th}}$ order Ritz generalized mass and stiffness; i.e.

$$(8) \qquad \left[\nu_k\right] = \left[\theta_k^T\right][M]\left[\theta_k\right] \quad \text{and} \quad \left[\kappa_k\right] = \left[\theta_k^T\right][K]\left[\theta_k\right]$$

and construct a test matrix called $\left[S_k\right]$ which involves the threshold frequency, f_o, determined from the temporal power P_n.

$$(9) \qquad \left[\kappa_k - (2\pi f_o)^2 \nu_k\right] \equiv \left[S_k\right].$$

Decompose $\left[S_k\right]$ and extract the value of the output parameter NBRCHG issued by the DECOMP module. Param NBRCHG reports the number of negative values on the factor diagonal of $\mathbf{S_k}$ which is tantamount to the number of sign changes or zero crossings in the characteristic equation. If the value of NBRCHG = k, it indicates that the frequencies of all k Ritz modes are less than f_o. One would be inclined to want the frequency content of the Ritz modes to bracket f_o; i.e. some modes with a frequecy > f_o. This implies that one would seek to have the value of NBRCHG to be less than the order of matrix $\mathbf{S_k}$. This idea can be built into the Ritz generation routine as a test as to whether enough modes have been generated to within a certain margin α such that $f_n > \alpha f_o$, where the user specifies α.

Obviously, one would not want to repeat the eigenvalue estimate each time a new Ritz mode is obtained, because $\mathbf{\theta_k^T K \theta_k}$ and $\mathbf{\theta_k^T M \theta_k}$ could become expensive as k becomes large. Therefore develop a scheme whereby the spatial power is reduced by some factor $\gamma > 1$; i.e. $\varepsilon_{new} = \varepsilon_{old}/\gamma$. Typically, one can use a value of 2 to 4 for γ. Once the spatial error ε_{new} is satisfied, repeat the eigenvalue estimate test.

Conclusion

A scheme has been proposed to monitor the adequacy of a set of Ritz modes to represent a solution by comparing the quantity generated with certain properties involving the forcing function. In so doing an attempt has been made to keep this algorithm lean and efficient, so that it will be economical to apply. Using this monitoring scheme during Ritz Mode generation will automatically ensure that the k Ritz modes $\boldsymbol{\theta}_{\mathbf{k}}$ that are generated are adequate to represent both the spatial and temporal behavior of the structure when forced under the given transient condition defined by F(x,t).