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An Investigation of a Mathematical Model of an Optically Pumped $Ti^{3+}:Al_2O_3$ Laser System

Dr. Lila F. Roberts
Assistant Professor
Department of Mathematics and Computer Science
Georgia Southern College
Statesboro, Georgia

Abstract

During the last several years, solid state lasers have been developed that have the potential for meeting rigorous performance requirements for space-based remote sensing of the atmosphere. In order to design a stable and efficient laser and to understand the effect on laser output of changes in the physical and design parameters, an understanding of the development of the dynamical processes of the laser is necessary.

Typically, the dynamical processes in a laser system are investigated via rate equations describing the evolution of the occupancy in the electronic levels and of the photon density in the laser cavity. There are two approaches to this type of study. Most often, for the sake of simplicity, the spatial variations of the dynamic variables in the laser system are disregarded and the mathematical model consists of a system of first order nonlinear ordinary differential equations(ODE). The potential disadvantage of this approach is that the spatial distribution may indeed be important, particularly when experimental techniques such as injection seeding are used in the laser system. In addition, this type of model treats the photon density as a superposition of left and right traveling photons, thus the individual dynamics of the left and right traveling photons are not accessible. The primary advantage to this model is that the rate equations can be solved numerically by readily available ordinary differential equation codes.

The second approach is to take into account both spatial and temporal variations in the dynamic variables in the laser cavity. The resulting model consists of a first order semilinear system of partial differential equations(PDE). Left and right traveling photons are treated separately and their developments in both the cavity and the active medium are described. Effects of the optical elements in the cavity and experimental techniques are taken into account by way of appropriate boundary conditions. The spatial and temporal model also can account for absorption of the pump by the active medium as pumping photons propagate through the material, an aspect which may not adequately be described by the simplified model. While this may be a more realistic approach, the primary disadvantage is that methods for the numerical solutions of systems of partial differential equations are not as easily available as those for ordinary differential equations.In fact, when such methods are available,the large amount of computing time necessary to obtain solutions may make the more complete model impractical.

The model which was studied was generic in the sense that it was a four-level laser system, but the parameters used in the numerical study were specific to Titanium-doped sapphire. For simplicity, a constant, spatially uniform pumping scheme was considered. In addition, a simplification of the model was made so that it treats a single lasing wavelength with a narrow bandwidth. The purpose of the work was to investigate both versions of the mathematical model and to determine whether the numerical solutions are similar both qualitatively and quantitatively.

Reduction of the spatial and temporal model to a temporal model has been treated in the literature a number of ways. First, a spatial average of the dynamic quantities may be taken over the length of the active region. Secondly, a spatial average may be taken over the optical length of the cavity, and finally, an average of the electronic populations over the length of the active medium may be taken and spatial variations in the photon flux are merely disregarded, i.e., the partial derivative of the photon flux with respect to the spatial variable is taken to be zero. The details of each of the above reductions were examined and the resulting systems of ODEs were solved numerically.

The systems of ordinary differential equations were solved numerically using a Runge-Kutta-Fehlberg algorithm which was very efficient for typical values of the physical parameters. A numerical scheme, based on the Modified Euler method, for computing solutions to the system of partial differential equations was developed and implemented. The computer code was written that solves the respective systems for both a Fabry -Perot cavity and a ring cavity configuration. The PDE model was solved numerically at the expense of greatly increased computer time.

Results from numerical computations demonstrate that there are differences in the computed solutions for the PDE and ODE systems. An important parameter which was varied was cavity length. For fixed crystal length and all other physical and design parameters held constant, the cavity length was varied and results for both systems were compared. Each of the ODE systems, except the system obtained from spatially averaging all variables over the length of the active medium, produced numerical results which were similar, qualitatively, to those obtained from the PDE system. However, the quantitative results were different. In the best case, the output predicted by one of the ODE systems was about 28% less than that predicted by the PDE system.

The computer codes for both the ODE and PDE systems have been extended so that output at multiple wavelengths is accessible. In order for the PDE system to be practical for this application, the code should be modified for vector processing. Also, the code to solve the system for an end-pumping scheme has been developed, but a comparison study is incomplete. The primary area in which this study should be extended is to compare the theoretical results to actual experimental data. In addition, there are some qualitative properties of the PDE system which are expected, but have not as yet been proved analytically. An investigation of stability of the PDE system under typical operating parameter values is a theoretical area which could be addressed.