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COSMIC STRINGS AND BARYON DECAY

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ABSTRACT

We briefly review the current literature on baryon decay catalysis by cosmic strings, presenting a summary of the main arguments for decay catalysis.

Introduction.

Several years ago Callan[1] and Rubakov[2] showed how it was possible for a grand unified monopole to catalyse proton decay, and that the catalysis would occur with an enhanced cross-section: the inverse square of the proton rather than the grand unified mass scale. This result had immediate cosmological implications. Assuming a standard grand unified picture, monopoles would be produced in some primordial phase transition, these would then be able to catalyse baryon decay. The known baryon to entropy ratio then places constraints on the number density of such monopoles[3] which are far more stringent than conventional bounds. Usually, inflation is invoked to explain the dilution of the monopole density. Cosmic strings[4] are also topological defects of grand unified theories; thus like monopoles, they too can catalyse baryon decay. If however, one intends to use cosmic strings as a means of explaining structure formation, then inflation must not dilute the string network. Therefore the question of baryon decay becomes important: If cosmic strings, like monopoles, give an enhanced cross-section, then their density would also be severely constrained, and they could be ruled out as a means of forming structure.

Baryon decay can be catalysed by grand unified topological defects, because in the core of such defects the grand unified symmetry is restored, and baryon number violating processes can occur. However, the baryon must be able to reach this core. Due to the spin of the baryon, there is a natural supression of its wave function near the core, thus, without any 'long range force' to attract it to the defect, we expect that the cross-section will be given at most by the geometrical cross-section of the defect, a grand unified cross-section. For the monopole, the long range force which leads to amplification is the coupling of the magnetic moment to the magnetic field. This enhances the wave function of the baryon near the core, and leads to an expected cross-section of the order of the baryon cross-section. At first sight, for strings it would appear that there is no long range force. The 'magnetic' fields either vanish (as in the Nielsen-Olesen string) or are perpendicular to the magnetic moment of the baryon. However, this reasoning is somewhat naive, and can break down if the string has fractional charge. In this case, a 'long range' Aharonov-Bohm effect[5,6] takes over, and enhancement does occur.

In this review, we present some recent arguments concerning enhancement or otherwise of string catalysed baryon decay. We will only summarise the arguments, referring the reader to the original literature for the detail.

Catalysis in the quark picture.

The first argument concerning cosmic string catalysis of baryon decay was due to Brandenberger, Davis and Matheson[7]. They considered a quark-string interaction, and calculated the cross-section using free field wave functions, multiplying by the ratio of the interacting wave function to the free wave functions to get the cross section in the presence of the gauge fields.

In order to estimate the free field catalysis cross-section, for the sake of simplicity they considered the transition amplitude between an initial single quark state, $|q\rangle$, and a single final lepton state, $\langle l|$. The interaction lagrangian allowing for baryon number violating processes is of the form

$$e\bar{\psi}A\!\!\!/\psi \tag{1}$$

where we have supressed the internal $SU(5)$ indices. Thus to first order in perturbation theory, one can represent the transition amplitude

$$\mathcal{A} = < f|i > = e\int d^4x < l|\bar{\psi}A\!\!\!/\psi|q > \tag{2}$$

where the spatial integral is over the core of the defect. Since we are only interested in the dependence of the cross-section on the defect mass M and the fermion mass m, we count the relevant orders of magnitude. The gauge field introduces a factor of M, whereas integration over the core gives M^{-3} for the monopole, and M^{-2} for the string. The sum over the spins in $\bar{\psi}\gamma^\mu\psi$ gives a factor of m, whence one obtains

$$\mathcal{A} \sim \begin{cases} \frac{e}{M^2} m\delta^{(4)}(\Sigma p_i - \Sigma p_f) & \text{for the monopole, and} \\ \frac{e}{M} m\delta^{(4)}(\Sigma p_i - \Sigma p_f) & \text{for the string.} \end{cases} \tag{3}$$

The cross section is then given by

$$\sigma \sim \frac{1}{VT}\frac{1}{m^2}\sum_f \int \frac{d^3\mathbf{p}_f}{2p_f^0}|\mathcal{A}|^2 \ , \tag{4}$$

where the sum is taken over the final momenta. Thus

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &\sim \frac{e}{M^2}\left(\frac{m}{M}\right)^2 \qquad \text{for the monopole, and} \\
\frac{d\sigma}{d\Omega dl} &\sim \frac{e}{M}\left(\frac{m}{M}\right) \qquad \text{for the string.}
\end{aligned} \tag{5}
$$

Notice that both of these are grand unified cross sections.

Now we examine the boundary conditions at the surface of the defect to see if there is any amplification of the fermion wave function near the core. Since it is A_ϕ that is non-zero, if we write

$$A^2 = \frac{\bar{\psi}\gamma^\phi\psi_{\text{DEFECT}}}{\bar{\psi}\gamma^\phi\psi_{\text{FREE}}}\,, \tag{6}$$

then the differential cross-section will become multiplied by A^4.

The key point that makes calculation of A^2 non-trivial is that in the presence of a long range gauge field, the orbital angular momentum becomes modified:

$$\mathbf{L} = \mathbf{r} \times (\mathbf{p} + e\mathbf{A}), \tag{7}$$

thus the spectrum of eigenfunctions and eigenvalues of the Hamiltonian and $\mathbf{J}^2$ may change.

In the case of the monopole, the eigenvalues of angular momentum shift by $eg = 1/2$, and thus the admissable j values are integers. Therefore there exists a $j = 0$ partial wave which can penetrate the core. The lowest 'free field' mode on the other hand has a radial behaviour proportional to r, hence the overall amplification factor, A^4, is $(M/m)^4$.

For the string, $A_\mu = -\frac{1}{g}\partial_\mu\phi$ in cylindrical polar coordinates, thus the component of $\mathbf{J}_z$ is shifted by e/g. Conventionally we set $e/g = 1$, so the eigenfunctions are unchanged (although the eigenvalues 'shift' by 1).

In order to see this more clearly, let us explicitly solve the Dirac equation in the presence of the string:

$$(\not{D} - m)\psi = 0 \tag{8}$$

If one uses the representation

$$\gamma^0 = \begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix},\ \gamma^1 = \begin{pmatrix} i\sigma_y & 0 \\ 0 & -i\sigma_y \end{pmatrix},\ \gamma^2 = \begin{pmatrix} -i\sigma_x & 0 \\ 0 & i\sigma_x \end{pmatrix},\ \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{9}$$

the Dirac equation separates into two 2-component spinor equations, the positive energy one being of the form:

$$(\sigma_z\partial_t + i\sigma_y\partial_z - i\sigma_z\partial_y + \frac{ie}{r}A_\phi(-i\sin\phi\sigma_y + i\cos\phi\sigma_z) - m)\chi = 0. \tag{10}$$

This has solutions:

$$\chi_\pm \propto \begin{pmatrix} J_{\pm\nu} \\ \frac{\pm ik}{m+\omega} J_{\pm(\nu+1)} e^{i\phi} \end{pmatrix} e^{-i\omega t} e^{in\phi} \tag{11}$$

where $\nu = n + e/g$. Now, in order to decide which solutions to choose one usually imposes regularity of the eigenfunctions at the origin. If $e/g \in \mathbb{Z}$, this uniquely specifies the χ_n:

$$\chi_n \propto \begin{pmatrix} J_{|\nu|} \\ \frac{\eta ik}{m+\omega} J_{|\nu+1|} e^{i\phi} \end{pmatrix} e^{-i\omega t} e^{in\phi}, \tag{12}$$

(where $\eta = |n|/n$) and thus the dominant partial wave, $n = -e/g$, has a behaviour

$$\bar{\psi}\gamma^\phi\psi \propto r \ \text{ as } r \to 0. \tag{13}$$

The amplitude of the wave function is therefore unchanged, and there is no amplification factor.

The conclusion of Brandenberger et al. was therefore that the catalysis cross-section for cosmic strings was a grand unified cross-section. The corresponding proof for (integer charged) superconducting strings[8] is more subtle, but the same conclusion was shown to hold[9].

However the question later arose as to what would happen if cosmic strings were not integrally charged. In the case of monopoles, the Dirac quantisation condition implies a fixed set of values for eg_m, however, for strings, no such quantisation condition need hold, models can be found in which the 'charge' of the string is fractional[6]. In the context of the above calculation, e/g being fractional implies that regularity turns out to be too strong a requirement, as the Hamiltonian ceases to be self-adoint[10,11]. Square integrability on the other hand is too weak a requirement, since for the mode $n = -N-1$, (where N is the greatest integer less than e/g) both eigenfunctions contain a divergent component, both are square integrable, yet including both violates self-adjointness of the Hamiltonian. In order to choose the correct χ_n, one needs to consider the boundary conditions at the core of the string. There is currently no entirely satisfactory resolution of this question, since it seems to depend on the composition of the interior of the core[12]. If there exists an excited scalar field in the core, one obtains a different result than if only gauge fields are excited. For the sake of argument, we will impose square integrability of the wave function and finiteness of the spatial probability current at the origin as our boundary conditions, since this is independent of the core composition. With these boundary conditions, we obtain

$$\bar{\psi}\gamma^{\phi}\psi \simeq \begin{cases} r^{(1-2\alpha)} & \alpha < 1/2 \\ r^{(2\alpha-1)} & \alpha > 1/2. \end{cases} \tag{14}$$

Thus

$$\frac{\bar{\psi}\gamma^{\phi}\psi_{\mathrm{STRING}}}{\bar{\psi}\gamma^{\phi}\psi_{\mathrm{FREE}}} \simeq \begin{cases} r^{-2\alpha} & \alpha < 1/2 \\ r^{-2+2\alpha} & \alpha > 1/2 \end{cases}$$

$$\Rightarrow \frac{d\sigma}{d\Omega dl} \simeq \begin{cases} M^{4\alpha-2} & \alpha < 1/2 \\ M^{2-4\alpha} & \alpha > 1/2 \end{cases} \tag{15}$$

(This coincides with Alford et al. who had a scalar condensate in the core of the string.)

Therefore it would appear that fractionally charged cosmic strings can have a significant enhancement of the catalysis cross-section. We must stress that even if we had imposed different boundary conditions at the core of the string (thus choosing a different χ_n) this enhancement would still be present, if not in certain cases more marked.

Cosmic string catalysed skyrmion decay.

So far we have presented a high energy picture of baryon decay, however, in order to understand catalysis it is important also to develop a low energy picture. One such model was investigated[13] based on work by Callan and Witten[14] who examined a skyrmion decay process in the presence of a monopole. We will summarise the process for a string, showing that we are forced to consider a vortex model for the string in order to obtain catalysis in the string core. The analysis gives a heuristic explanation of the enhancement factor with monopoles, as we will show.

Let us first highlight the features of the Skyrme model relevant to the catalysis procedure. The Skyrme model is a sigma model with stable soliton solutions otherwise known as skyrmions. In the case of two quark flavours (which we will be assuming here for simplicity), the pion field content is contained in an SU(2) field $U = \exp\{\frac{2i}{f_\pi}\, \underline{\tau}.\underline{\pi}\}$, where $\underline{\tau} = (\tau_1, \tau_2, \tau_3)$ are the three generators of SU(2). The field space is thus isomorphic to S^3. Since finiteness of the energy requires that $U(\underline{x}) \rightarrow$ const. as $|\underline{x}| \rightarrow \infty$, we can think of a soliton field configuration as a map from compactified three-space ($\mathbb{R}^3 \cup \{\infty\} \cong S^3$) to the three-sphere of SU(2). Such maps may be classified according to the homotopy equivalence class to which they belong. Since $\Pi_3(S^3) \cong \mathbb{Z}$, we may conclude that soliton field configurations are labelled uniquely by an integer value, N_B (the baryon number), which is the degree of the map. In a dynamical theory, the continuity of the fields implies that N_B is a continuous function of time and hence constant. The baryon number may be more familiarly represented as the charge associated with the conserved baryon current

$$B^\mu_s = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\Big(U^{-1}\partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U\Big). \tag{16}$$

In the presence of electromagnetism, the model must be generalised to allow for the nucleon charge and magnetic moment interaction. Taking into account QCD anomalies, Witten[15] showed that the baryon current becomes modified to

$$B^\mu = B^\mu_s + \frac{ie}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu [A_\rho \mathrm{Tr} Q (U^{-1}\partial_\sigma U + \partial_\sigma U U^{-1})] \ , \tag{17}$$

where Q is the quark charge matrix ($Q = \frac{1}{6} I_2 + \frac{1}{2}\tau_3$). The new A_μ dependent term is a divergence so provided there are no singularities in A_μ, and that surface terms vanish, the baryon number is still integral. In terms of the topological picture presented previously, provided there are no singularities, $U_{sol}(x)$ is still a map from $S^3 \rightarrow S^3$ and thus the classification of maps into equivalence classes labelled by baryon number still holds.

For the case Callan and Witten considered, a skyrmion interacting with a Dirac monopole, the gauge potential is singular on the line $\theta = \pi$, although the electromagnetic flux is finite. This singularity is a gauge artefact, the Dirac string, which can readily be removed[16] if one chooses two coordinate patches for $\mathbb{R}^3 - \{0\}$, each with an associated A_μ, relating the two different 'branches' of A_μ by a gauge tranformation on the overlap.

One includes the SU(2) field, U, in this picture by using the transformation induced by the gauge transformation on A_μ. This gives a consistent, singularity free picture of the nucleon on the background field of the monopole. However, since there is a non-trivial transformation for U in the overlap of the two coordinate patches, in the classification of field configurations according to homotopy equivalence there is a shuffling of members of the baryon equivalence classes. In other words, baryon number is different. In particular, Callan and Witten found that a pure π^0 radial configuration, the radial kink, carried baryon number 1. Since the wave functions of charged pions are suppressed near the core, but those of uncharged particles are not, the nucleon can now approach the monopole core by deforming into the radial kink. Then, provided the boundary conditions at the monopole core allow baryon non-conservation, the proton can decay. Thus, monopoles can catalyse skyrmion decay.

We now turn to the case of a skyrmion interacting with a cosmic string. In contrast to the monopole, in this case the string has a well defined gauge field without invoking coordinate patches. Thus the gauge field for a cosmic string exhibits no singularities, the additional term in the baryon current is once more a total divergence, and baryon number is unchanged. Alternatively, if there are no gauge singularities, the equivalence classes of the soliton maps are unchanged. We therefore expect that in this case the radial kink will not carry baryon number (as was shown to be true in ref. 13). Rather like the monopole case, charged fields pick up extra "angular momentum" around the z-axis due to the presence of a non-zero A_ϕ. For the infinitesimally thin string, the radial part of the wave equation for the lowest angular momentum eigenstate tends to zero as least as quickly as ρ near $\rho = 0$. Therefore, without introducing core structure, we cannot obtain skyrmion catalysis. In order to be more physically realistic, we need to consider a vortex model for the string. To illustrate the salient features of skyrmion catalysis by cosmic strings it is only necessary to consider an abelian theory: the Nielsen-Olesen vortex[17]. This corresponds to an infinite, straight static string aligned with the z-axis. In this case, we can choose a gauge in which

$$\phi = \eta X(\rho) e^{i\phi} \quad ; \quad A^\mu = \frac{1}{e}[P(\rho) - 1]\nabla^\mu \phi. \tag{18}$$

This string has winding number one. Near the origin, X and P take the form:

$$X \propto \rho \quad ; \quad P = 1 + O(\rho^2) \quad \text{as } \rho \to 0. \tag{19}$$

Using the asymptotic form for P in the Klein-Gordon equation implies that the radial equation for the lowest angular momentum eigenstate becomes

$$\rho\partial_\rho\rho\partial_\rho\varphi(\rho) = O(\rho^4)\varphi(\rho), \tag{20}$$

allowing $\varphi \sim$ const. as $\rho \to 0$. Thus, on the scale of the core of the string, we need not have total suppression of charged particle wave functions.

Clearly, as before, the radial kink cannot carry baryon number. However, this is no longer critical for we can have all three pion fields approaching the core. Once the skyrmion is in contact with the core of the string, where the grand unified symmetry is essentially restored, the possibility of decay arises.

One can explicitly show this by making the the nucleon field configuration depend on time:

$$U_N(\underline{x}, t) = \exp[iF(r,t)\hat{\underline{x}}.\underline{\tau}], \tag{21}$$

and allow F to change at the core. The skyrmion can then unwind, leaving a residual field configuration which is a topologically trivial excitation of the pion fields and dissipates.

Thus strings can catalyse skyrmion decay. The picture however relies fundamentally on taking a vortex model for the string, i.e. one in which the string has a finite thickness. A model of the string with infinitesimal thickness (the wire model) gives no catalysis. A similar argument applies for superconducting strings: a wire model gives no catalysis, but a vortex model does.

Notice that the monopole argument was conducted exclusively within the approximation of the Dirac monopole; the only place we needed a grand unified monopole was in invoking baryon number non-conserving boundary conditions. By contrast, a thick string or vortex model was required in order to get catalysis to occur at all in the string picture. Thus in the monopole picture, the only scale we have is the skyrmion scale whereas the inescapability of the vortex model in the string case suggests that the reaction is occurring on the scale of the string thus giving a grand unified cross-section. For more detailed arguments, see refs. 13 and 18.

The skyrmion argument thus reinforces the quark scattering picture developed earlier, and provides an interesting alternative description of baryon decay. However, to date, no corresponding picture of decay for fractionally charged strings has been developed.

Summary.

To summarise: we have seen that in both the free quark and skyrmion pictures, cosmic strings and superconducting cosmic strings catalyse baryon decay with a grand unified cross-section. This constrasts with the monopole case, where the Callan-Rubakov effect indicates a cross-section on the scale of the proton. This difference can be understood physically in terms of the presence of a magnetic moment interaction which acts as a 'long range force' in the case of the monopole. In addition, we have seen that non-integrally charged cosmic strings display some enhancement over their integrally charged cousins in the catalysis cross-section.

These arguments illustrate well the current interplay between theoretical physics and cosmology. The catalysis of proton decay by strings could have ruled out the string model of galaxy formation had there been a large enhancement. Even with the current cross-sections, some restrictions can be placed on strings using baryogenesis bounds[19]. Interestingly, there are arguments that the conical structure of spacetime surrounding strings may further suppress the cross-section[20]. This latter result could be particularly relevant for superconducting strings which have far stronger gravitational effects. In conclusion, these studies have shown a suprising amount of variety in cosmic string behaviour, and have further added to their interest as cosmologically viable objects.

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