Control Strategy for Cooperating Disparate Manipulators

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Abstract

To manipulate large payloads typical of space construction, the concept of a small arm mounted on the end of a large arm is introduced. The main purposes of such a configuration are 1) to increase the structural stiffness of the robot by bracing against or locking to a stationary frame, and 2) to maintain a firm position constraint between the robot's base and workpieces by grasping them. In this research, possible topologies for a combination of disparate large and small arms are discussed, and kinematics, dynamics, controls, and coordination of the two arms, especially when they brace at the tip of the small arm, are developed. In the thesis, the feasibility and improvement in performance will be verified, not only with analytical work and simulation results but also with experiments on the existing arrangement (RALF and SAM).
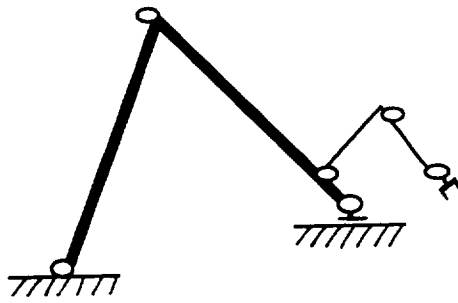
1. Introduction

1.1 Motive

The proposed research seeks higher performance manipulators in large workspaces, particularly for those robotic manipulators that require precise positioning and mating of relatively massive payloads. Demand for these manipulators can be found in common space maintenance and construction scenarios. The fundamental characteristics of such manipulators include their light weight and their flexibility. Also, fundamental is the need to maintain a firm position constraint between the robot's base and the payload or workpiece. This requires a more complicated manipulator topology than a single serial link arm, which cannot constrain two mating workpieces.
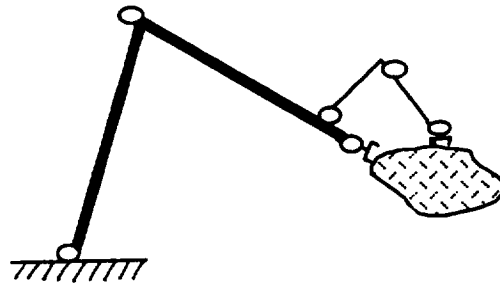
i) Bracing   ii) Grasping

Figure 1: Appications of Large/Small Manipulators

As a solution, the concept of a small arm mounted on the end of a large arm is introduced to provide precise motion as well as a large workspace [6]. Such a configuration forms a redundant arm. Thus, the number of extra degrees of freedom can be used to brace on a stationary frame or grasp a payload as shown in Figure 1. Bracing and grasping are used to ensure accurate positioning of the end effector and to support the force at the end effector due to the change in its stiffness caused by the kinematic closed chain. It has been proven that bracing reduces the positioning uncertainty [13] and increases the stiffness of the manipulator by the closed kinematic constraint [11].

From real world experience with crane-human coordination, when a heavy payload is unloaded (see Figure 2), we know that we can obtain precise positioning and high payload capacity. This crane-human configuration may be analogous to the topology of bracing at the tip of the small arm and having an end effector at the middle of the chain. In other words, the large arm provides the load capacity like a crane, and the small arm does the precise positioning like a human. This topology gives motivation to develop a coordination strategy for disparate large and small manipulators.
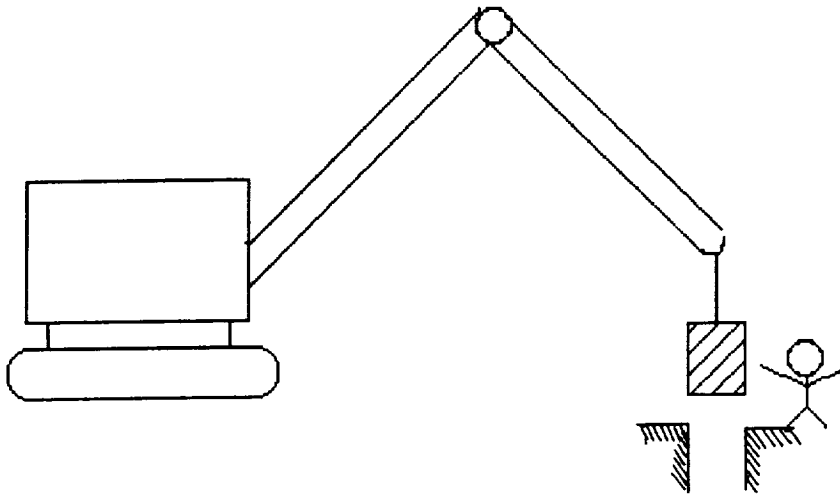
Figure 2   Crane -Human Coordination

1.2 Research Description

The small manipulator carried by the large arm forms a topology similar to a redundant arm. If contact with the environment occurs at a bracing point on the small arm, similarity to a dual arm topology exists. However, neither of these existing topologies exactly represent the large and small arm combination in some of the common space maintenance and construction scenarios. Also, when a closed chain is formed by bracing, the number of degrees of freedom are less than the total number of joints. Since the number of actuators will be greater than the number of degrees of freedom, the number of controlled active joints can be reduced, or a control strategy should be formulated to utilize the excess number of actuators.

The crane-human topology may be viewed as two arms holding a single object with one end of both arms attached to a stationary frame. However, control of this proposed topology is different from a common dual arm control in some ways. For example, the large arm is powerful and flexible, and the small one is capable of precise positioning and is rigid. To take full advantage of such disparate features, special control schemes can be applied. So far, no work or study for those control schemes has been done. As a first approach to develop a control strategy for coordinating disparate manipulators, an advanced master/slave control scheme is proposed. In this scheme, the small rigid arm performs precise position control, and the large flexible arm does force control to compensate for the external forces such as a payload weight as well as internal forces due to relative kinematic motion errors between the two arms.

The objective of this proposal is to present the theory and the related problem behind disparate large and small arm coordination. In this proposal, first the related literature is briefly reviewed. Second, the kinematic topology is synthesized for the combination of a large and small arm. Third, the kinematics for a large and small arm is studied when they are constrained by a closed chain, and some advantages of the proposed configuration are analytically proven. Finally, with an advanced master/slave approach, the control strategy for the two arms is discussed, and a non-colocated flexible arm force control law is designed.

2. Previous Related Work

2.1 Macro/Micro Manipulator

To reduce manipulator positioning error caused by unmeasured deflections of the structure and poor actuator servo resolution, Sharon & Hardt [6] introduced the concept of a micro-manipulator mounted on the end of a large manipulator. This configuration comprised a large robot carrying a micro-manipulator to the area of interest and used the micro-manipulator along with absolute endpoint position feedback for fine motion control necessary to eliminate static errors. Also, the authors concluded that the bandwidth of the micro-manipulator should be kept below the fundamental frequency of vibration of the basic robot structure to avoid exciting any natural frequencies. Also, Cannon [27] carried out experiments to study the dynamic interactions between the motion of a mini-manipulator and the structural flexibility of the main robot arm that carries it. In contrast to Hardt's result [6], the experiments demonstrated the feasibility of using an end-point sensor to control a flexible macro/mini-manipulator system at a bandwidth several times higher than the structural flexibility of the main manipulator. This capability will make possible very rapid operation of the macro/mini-manipulator using the dynamic interaction in the future.

2.2 Bracing Manipulator

West and Asada [11] described how the kinematics and statics of a manipulator were changed in terms of the stiffness, and showed how the performance of a robot could be improved in

4

terms of transmission ratio by bracing against a work surface. Also, they introduced essential and arbitrary variables of a task to reduce the number of degrees of freedom necessary to perform the task motion within the framework of Mason [2]. Kwon and Book [13] proposed bracing the arm to reduce the uncertainty in positioning a manipulator and compared the braced arm with an unbraced one. However, the bracing arm required more degrees of freedom at the end effector to achieve a desired task. This drawback of the bracing arm was not solved.

2.3 Redundant Manipulators

Redundant manipulators have more degrees of freedom than the dimension of the workspace so that the number of joint space coordinates, m, exceeds the number of workspace coordinates, n (m>n). In most work done in this area, the redundancy of such manipulators was effectively used to avoid obstacles [15], avoid singularities, and maintain a high degree of manipulability [21] while performing the desired end effector task. Also, the desired redundant joint velocity can be specified to optimize a cost function (e.g. force ellipsoid [14] or energy consumption [1]) over the configurations which are allowed by the extra degrees of freedom while achieving the end effector position.

2.4 Multiple Arms

The use of multiple arms on a robotic system makes it possible to perform various kinds of sophisticated tasks such as handling large, massive, or non-rigid objects, or modifying the grasping position of an object. These applications pose complex problems such as the analysis of a closed chain and the position/force control in using two arms handling the same object. Much of the early work in dual arm control dealt with the master/slave architecture [3,16] whereby one arm moves under kinematic position control while the other follows the first by way of force feedback. This control scheme permits compensating the positioning errors of the robots which induce undesired squeezing forces in the object. However, this approach seems to lack the primary benefit of multiple arm control since the arms do not truly share the load bearing task.

Hayati [4] has recently proposed an elegant control architecture for position/force control of multi-arm robots. His approach employs an extension of Mason's Hybrid position/force control [2]. However, from a practical point of view, the method may be difficult to

implement effectively because it requires accurate knowledge of the inertial properties of the arms and of the jointly manipulated object.   Also, the formulation requires the solution of the inverse dynamics.

3.   Characteristics of Proposed Manipulator

3.1 Topology

The topology of a mechanism can be defined by a set of parameters: the degree of freedom, the number of the links and joints, their connectivity, the types of joint, which links are grounded.   For a closed chain manipulator, it is important to determine which joints are active (or passive) and which links are grounded. For example, there are several possible topologies for the combination of large/small arms.   As shown as Figure 3, depending on what part of the manipulator is braced, different characteristics can be expected.

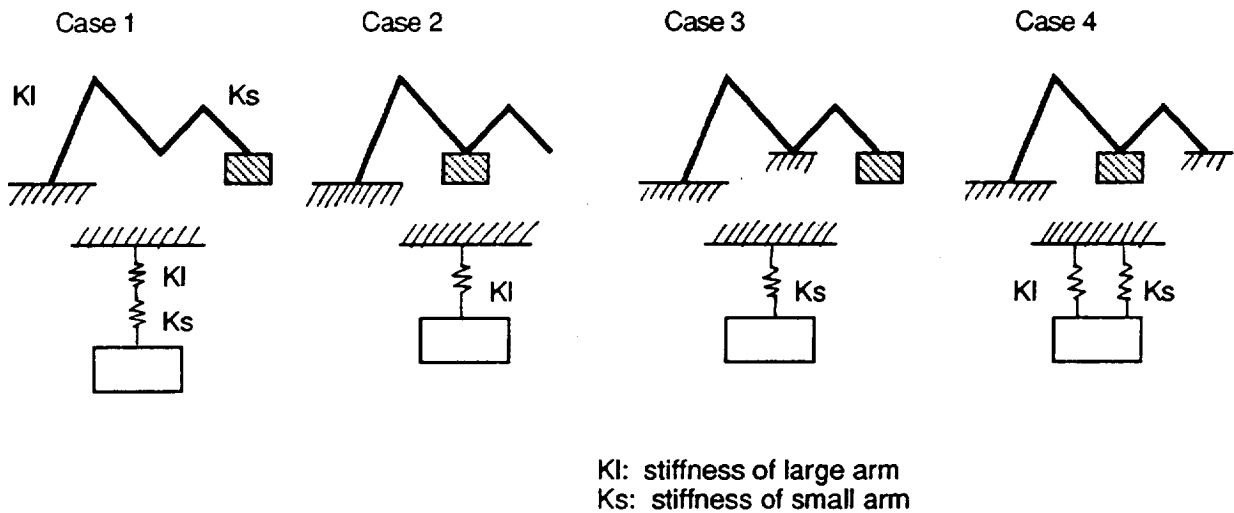KI: stiffness of large arm
Ks: stiffness of small arm

Figure 3   Schematic Representation for a Large and Small Arm

Schematic comparison in terms of the stiffness of the robots show that bracing increases the stiffness by a closed kinematic chain, and Case 4 can produce the largest stiffness and payload capacity among the four cases. Thus, one possible proposed topology would be a large arm carrying the payload plus a small arm (tip manipulator) which braces against a stationary frame or grasps to the payload for precision adjustment of relative payload position.

Elementary kinematics shows that constraining a manipulator by bracing at the tip of the small arm reduces the number of degrees of freedom of motion of the manipulator. In other words, redundancy in actuation occurs. This means that all the joints do not have to be controlled actively to obtain the desired motion for the closed chain large/small arm configuration. From mechanism theory of closed chain kinematics, "mobility" is defined as the number of input parameters which must be independently controlled in order to bring the device into a particular position. Thus, we can compute "mobility" of the proposed topology using Gruebler's formula;

$$M = \lambda \ (I - j - 1) + \sum_{i=1}^{j} f_i$$

where M is the mobility of a mechanism, λ is 6 for a spatial mechanism or 3 for a planar mechanism, I is the number of links, j is the number of joints, and f_i the number of degrees of freedom of each joint. Therefore, based on this formula, we can determine the smallest number of active joints to move the end effector to desired positions.

For the most general case, let us consider the spatial motion of large/small manipulators. We assume that each large and small arm has 6 degrees of the freedom (DOF) with rotational joints only because it requires 6 DOF for the position and orientation of the end effector, and another 6 DOF for the position and orientation of the bracing device. As shown as Figure 4, the small arm is mounted on the tip of the large arm, and their joint connectivity is described in Table 1. For closed chain arm, the mobility M becomes 3 by computing Gruebler's formula since I = 5, j = 5, and f_i = 1, 2, or 3. Thus, we can move the end effector to the desired position with three actively controlled joints. The joint 6, the orientation of a payload (3 DOF), is not included in this analysis since it is controlled independently by the wrist joint. However, the large arm is flexible in most cases. Therefore, the number of active joints (actuators) should be larger than the mobility of the mechanism so that the rigid arm can constrain undesired motions (3 DOF) such as vibration and deflection and share the weight of a payload. For example, only 6 joints are powered, and 6 joints can be left unpowered, which means joint 3, 4, and 5 can be passive, and still it can achieve the desired position and orientation. However, this configuration cannot constrain the position of the end effector due to the flexibility of link 1 and link 2. Therefore, 9 powered joints with 3 unpowered joints (only joint 3 is passive) is a

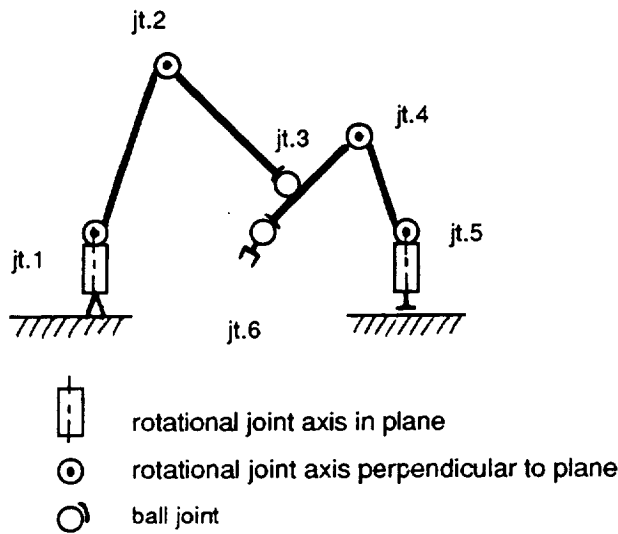reasonable choice to ensure the position of the end effector with the least number of the actuated joints.

Figure 4 Topology for a Large and Small Arm

Table 1 Joint Connectivity

joint # (i)	joint type	d.o.f. (fi)
1	collar	2
2	pin	1
3	ball	3
4	pin	1
5	collar	2
6	ball	3

3.2 Kinematics

A redundant arm in contact with its environment is characterized by closed chains in the structure and redundancy in actuation. In other words, the number of actuators typically exceeds the mobility. However, the general case of the Jacobian matrix can be applied with kinematic constraints. We assume that the joints are frictionless, that the link inertial forces are negligible, and that all the links are rigid to simplify kinematic analysis.

Let the kinematic transformation from joint space to task space be given by

$$X = X (q)$$

Where X contains the task coordinates and q contains the joint coordinates. Differentiating with respect to time, we obtain

$$\dot{X} = J_i(q)\ \dot{q}_i \qquad\qquad (1)$$

where $J(q)$ is the Jacobian matrix with elements given by $J_{m,n} = \dfrac{\partial X_m}{\partial q_n}$

The following subscript, i, denotes the large (L) and small (S) arms.

Let us assume that neither arm is in a singular position and that J_i is square matrix. Then, we can obtain the joint rates required to perform a desired control task in the task coordinates.

$$\dot{q}_i = J_i(q)^{-1} \dot{X} \tag{2}$$

If Eqn. (2) is combined with the kinematic constraint that $\dot{X}$ is same for both arms, we can write the following form of a new Jacobian, H, for the closed chain arm.

$$\begin{bmatrix} \dot{q}_L \\ \dot{q}_S \end{bmatrix} = \begin{bmatrix} (J_L)^{-1} \\ (J_S)^{-1} \end{bmatrix} \dot{X} \tag{3}$$

or

$$\dot{q} = H \dot{X} \tag{4}$$

where $q = [(q_L)^T, (q_S)^T]^T$ and $H = [\ [(J_L)^{-1}]^T, \ [(J_S)^{-1}]^T \ ]^T$

Similarly, using the principle of virtual work in the static case, we can derive the following relationship.

$$F = H^T T \tag{5}$$

$$= \left[[(J_L)^{-1}]^T, [(J_S)^{-1}]^T \right] \begin{bmatrix} T_L \\ T_S \end{bmatrix} \tag{6}$$

$$= F_L + F_S \tag{7}$$

where T_i is the vector of actuator torques, and F_i is the vector of forces and moments exerted by arm i on the end effector. Since the load capacity in the task coordinates is the sum of the two arms' capacities, Eqn. (7) is true also.

Since the matrix J_i is square matrix (nxn), the H^T is then n x 2n. We have to find joint torques that will exactly balance forces at the end effector in the static situation. Most research [4,7] for optimal load distribution for multiple manipulators has been devoted to solving Eqn. (5) to determine the redundant actuator torque for the task space load. For

example, desired criteria such as energy consumption can be minimized with the kinematic constraint equation, or the optimal load distribution can be obtained by using a weighted pseudo-inverse which gives the minimized solution for weighted torque.

3.2.1 Load Capacity

Intuitively we know that closed chain manipulators have greater load capacity than open chain manipulators. However, we need to show analytically how much load they can handle. In this section, using the idea of a force ellipsoid from the serial chain redundant manipulator, an index for measuring the force transmission at a given posture for the closed chain manipulator is developed. Also, these indices for both closed and open chain arms are computed and compared.

From Eqn.(5), we see that the Jacobian is simply a linear transformation that maps the joint torque T in R^m into a task force F in R^n. The unit sphere in R^m defined by

$$T^T T \leq 1 \tag{8}$$

is mapped into an ellipsoid in R^n defined by

$$F^T (H^T H)^{-1} F \leq 1$$
$$or \quad F^T (J_L J_L^T + J_s J_s^T) F \leq 1 \tag{9}$$

This ellipsoid has principal axes $\lambda_1 u_1, \cdots\cdots, \lambda_n u_n$ where $u_i \in R^n$ and λ_i is an eigenvalue of $(H^T H)^{-1}$. Chiu [14] call this a "force ellipsoid", and Yoshikawa[21] used the volume of a velocity ellipsoid as an index of manipulability for open chain redundant arms. Similarly, the volume of the force ellipsoid is used as a measurement of a manipulator's payload capacity at a given posture.
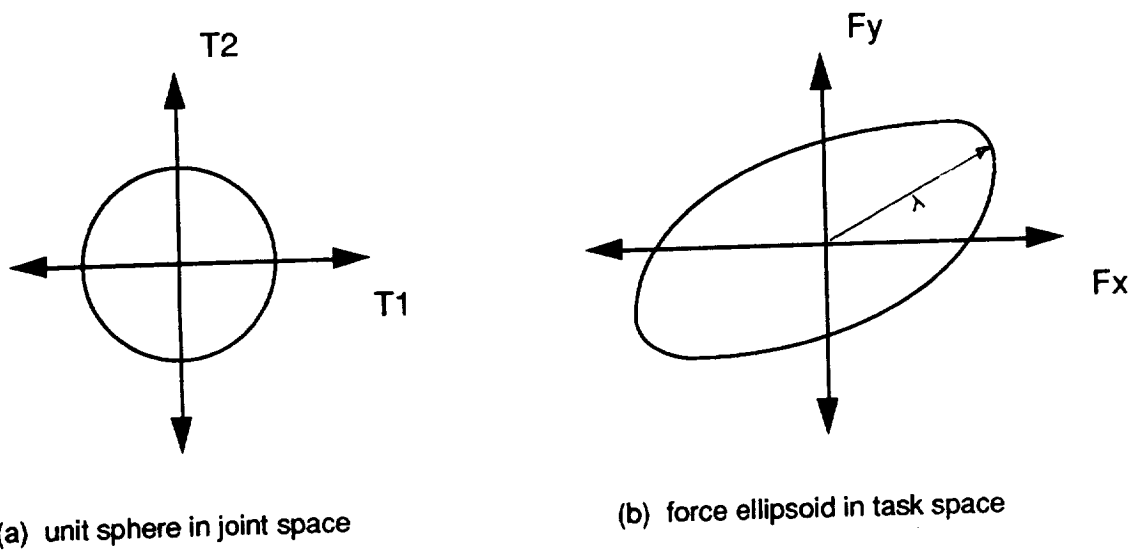
(a) unit sphere in joint space

(b) force ellipsoid in task space

Figure 5 Force Ellipsoid

The payload capacity index for the closed chain is;

$$
\begin{aligned}
W_{L+S} &= \lambda_1 \cdots \lambda_n \\
&= \det(H^T H)^{-1} \\
&= \det(J_L J_L^T + J_S J_S^T) \\
&= W_L + W_S + (\text{non-negative term})
\end{aligned}
\tag{10}
$$

This result can easily be obtained from a property of determinants [21]. If we carry out the computation for W_{L+S}, we can notice that W_{L+S} is the summation of volume of the force ellipsoid of the large arm(W_L), the small arm(W_S), and a non-negative term from Eqn. (10). Physically, this means that the closed chain arm has a larger load capacity than either the the large or small arm, and the volume index of the closed chain is larger than the summation of the two indices for the large arm and the small arm by a non-negative term which came from the process of squaring the Jacobian.

4 Proposed Work

The structure of the dynamic equations of the closed chain motion of two disparate manipulators holding a single object are presented in this section and a control law is proposed. Under the assumption that the large arm is flexible, the equations of motion for

the closed chain mechanism are determined. The control scheme that utilizes the payload capacity of the large arm and the position accuracy of the small arm is introduced

4.1 Dynamics

In the first phase of this work, the dynamics of the proposed disparate manipulators system is derived. Its topology can be considered as two cooperating robots holding a single object as shown in Figure 6. The motion of the manipulators is dynamically coupled because the generalized forces are interrelated through the common rigid object. Since joint P is a ball joint, only forces can be transmitted through it. The forces that the large arm exerts on joint P are denoted by a 3x1 vector F_L, and the forces that the small arm exerts are denoted by a 3x1 vector F_S. We can decompose the system into three parts (large arm, small arm, and payload) and derive the equations of motion for each body.
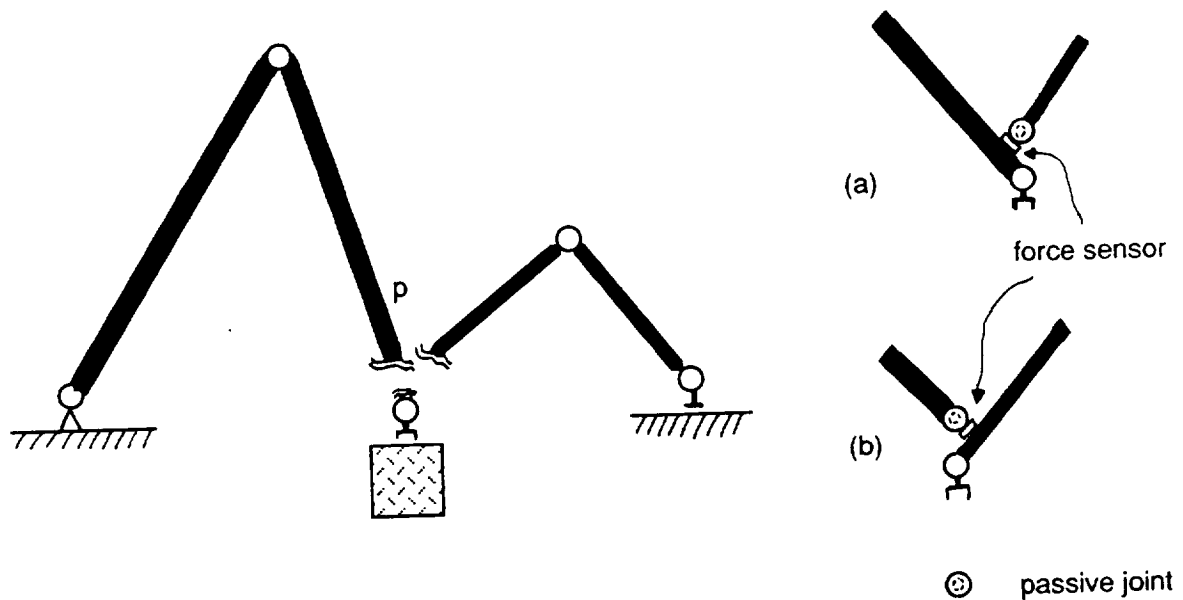

Figure 6   Proposed Disparate Manipualators with
Alternative Topologies

First, modeling a large arm, which is assumed to be flexible, is complicated. The flexible deflection of the arm is approximated to be a finite series of separable modes which are the product of admissible shape functions $\psi_{ij}(x)$ and time dependent generalized coordinates $\delta_{ij}(t)$.

$$u_i(x,t) = \sum_{j=1}^{m} \psi_{ij}(x) \, \delta_{ij}(t) \qquad (11)$$

The equation of the flexible arm motion can be derived from several techniques, but the Lagrange's formulation [12] is known for its simplicity and systematic approach. Using Jacobians [20], the kinematic and potential energies have been obtained by integrating the velocity and position of points over the total system. These energies were used in Lagrange's equations. For the large arm, $q_L = [\,(q_r)^T, (q_f)^T]^T$

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix}\begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} q_r \\ q_f \end{bmatrix} + \text{Nonlinear Term} = \begin{bmatrix} T_L \\ 0 \end{bmatrix} - \begin{bmatrix} J_r^T \\ J_f^T \end{bmatrix} F_L \qquad (12)$$

Where $X = X(q_r, q_f)$, $J_r = \dfrac{\partial X}{\partial q_r}$, and $J_f = \dfrac{\partial X}{\partial q_f}$

Again q_r contains the generalized rigid joint coordinates, and q_f contains the generalized flexible coordinates. The nonlinear term is negligible under the fine motion control.

For the small arm, which is assumed to be rigid, q_s contains the generalized joint coordinates.

$$M_s(q_s)\ddot{q}_s + N_s(q_s,\dot{q}_s) = T_s - J_s^T F_s \qquad (13)$$

where M_s is the inertia matrix of the manipulator, N_s is the nonlinear term including centrifugal force, Coriolis force and gravity term, and T_s is the joint torque.

For the rigid body payload dynamics,

$$\begin{bmatrix} m_p & 0 \\ 0 & I_p \end{bmatrix}\begin{bmatrix} \ddot{X} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -m_p G \\ \dot{\theta} \times (I_p \dot{\theta}) \end{bmatrix} = \begin{bmatrix} F_L + F_s \\ I_L \times F_L + I_s \times F_s + T_3 \end{bmatrix} \qquad (14)$$

where X is the position of the payload in the task coordinates, and θ is the orientation of the payload. m_p is the diagonal matrix of payload, and I_p is the inertia matrix of the payload. F_i is the desired force on the payload in the task space, and I_i is distance vector between the mass center and the force sensor. Later, we assume that the orientation of the payload is controlled independently by the wrist joint torque.

4.2 Control Strategy

With manipulator dynamics available, alternative control strategies will be studied. This will be one of the major contributions of the thesis.

Unusual characteristics of the proposed topology compared to common dual arm control are :

* One arm is large, and the other is small.
* One is flexible, and the other is rigid.
* One is coarse (actuator and sensors), and the other is precise.
* One is strong (high forces and torques), and the other is less strong.

To take full advantage of the complimentary features of the large and small manipulators, the control strategy for each arm can be unsymmetric. In other words, each arm's role should be different rather than just sharing a payload. Thus, one proposed control strategy is the master/slave scheme whereby the force-controlled slave arm (large arm) follows the position-controlled master arm (small arm). When using kinematic control of a closed chain that is redundant in actuation, geometric imperfections in the robot structures along with other sources of the kinematic error and feedback position control error results in undesired internal forces between two arms. Therefore, force control by feedback from the slave arm's tip mounted force sensors offers an approach to eliminating these unwanted forces. The rigid arm, which is capable of precise positioning, is position servoed and follows a preplanned trajectory. The flexible arm, which is capable of handling a massive payload, is force servoed to balance the load and accommodate any internal forces that may arise.

Also, the location of a force sensor in relation to the physical world under the master/slave scheme could be an interesting research issue . For example, if the force sensor is mounted on the tip of the large arm (Fig.6 (b)), we may interpret that the small arm (lady) is trying to move the payload (luggage) to a desired position, but it is too heavy to be moved by the small arm itself. Thus, the large arm (gentleman) helps to lift the payload by the force control. On the other hand, if the force sensor is located at the tip of the small arm (Fig.6 (a)), then the large arm (blind) lifts the payload (luggage) by itself and is guided by the small arm (dog) to move to the desired position. These two approaches have their own advantages and disadvantages depending on the surrounding conditions such as the resolution

of the force sensor, the power ratio of two arms, and the weight of the payload. Comparing these two approaches and identifying their applications for better usage will be one of the contributions in this research.

4.3 Simplified Modeling

In a real system, physical properties may be nonlinear and distributed throughout the manipulator. However, a lumped approximation often gives useful insights into how systems behave. By understanding how to obtain the desired performance with this simple system, we can devise conceptional control strategies for more complex actual manipulators. The rigid arm with the position feedback controller can be modeled as the unit mass system with a damper (Kd) and a spring (Kp) when the computed torque method, the so called, "control law partitioning" is applied [19,22]. The force sensor can be modelled as a linear spring with stiffness Ks. The flexible arm itself is represented by two masses, m1 and m2, and the flexibility between them is given by K. The damping is negligible.

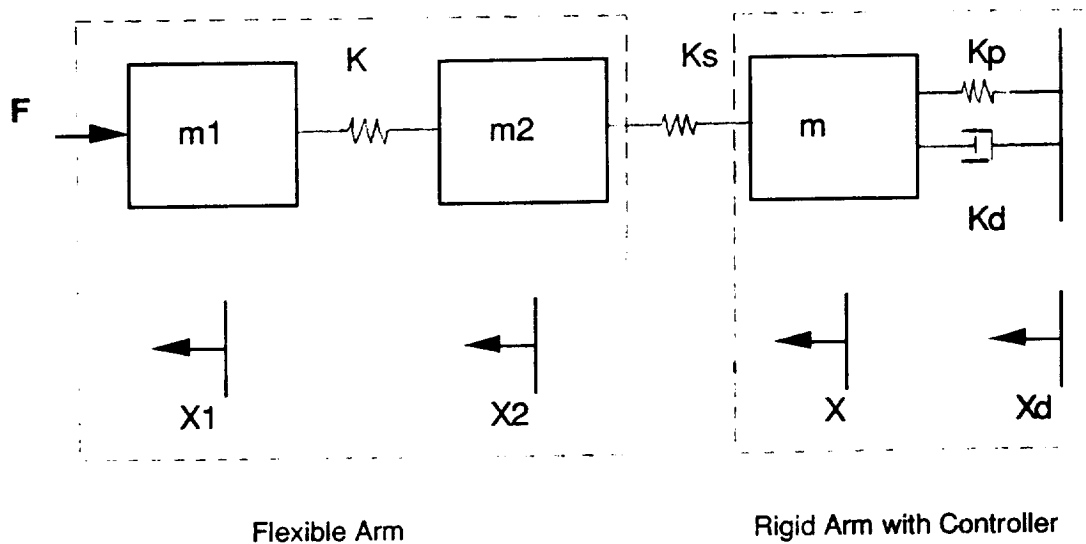Flexible Arm   Rigid Arm with Controller

Figure 7 Simplified Model of a Large and Small Arm

Since the actuators on the large (flexible) arm and the force sensor are physically located at different points, the performance of the force controller is severely limited by unstable poles as the feedback gain increases. Some people call this situation "non-colocated control" [23]. As we see from the root locus in Figure 8, with only force feedback the system

becomes unstable with high gain. Physically, the behavior of non-colocated control means that the flexibility in the system increases the number of poles in the system, and eventually these poles become unstable with high feedback gain, although those poles remain stable in colocated control. To solve this problem, the extra sensors or a dynamic observer should be added. Thus, all the states become available, and undesired poles can be moved to where the designer wants by state feedback. (It becomes controllable.) As shown in Figure 7, due to the flexibilities of the large arm, two masses are used to describe the dynamic behavior of the arm instead of one mass like a rigid arm. If we install strain gages to monitor flexible motion, we can manipulate the position of closed loop poles using feedback gains.
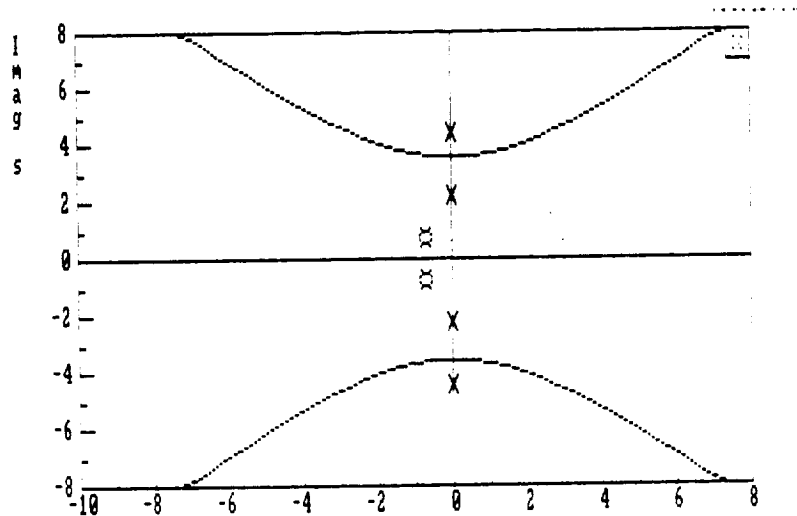

Figure 8   Root Locus of Simplified Model with Force Feedback

The dynamics of the two arms are coupled, which make the system more complicated. However if we apply the master/slave scheme, the coupling force between two arms, which is an internal force, is force-controlled to be as small as possible. Also, since we can measure the uncontrolled internal force from the force sensor, two arms can be dynamically decoupled by feeding forward coupling term using the small arm controller. This decoupling will be imperfect in any phsical apparatus and must be evaluated with experiments and/or simulations. Thus, we have two independent systems.

By reasoning about the master/slave scheme for the simplified lumped model, premises have been developed: (1) Decouple the system by feeding forward. Then, design a stable motion control law without the consideration of the force control, and design a stable force

control law by treating the large arm as a decoupled independent system; (2) Make all the states available for flexible arm force control. Thus, the system will be controllable, which means that the poles of the system can be moved to any desired location by state feedback.

4.4   Master controller design

The equations of motion of the system are derived in section 4.1.   The equations for the small arm and the payload are shown in Eqn. (13) and (14). They are coupled dynamically. Since the orientation of the payload is controlled independently, only the position of the payload is considered. If we combine two equations, we may rewrite.

$$M_s \, \ddot{q}_s + N_s(q_s, \dot{q}_s) = T_s - J_s^T (m_p \, \ddot{X} - m_p \, G - F_L) \qquad (16)$$

Recall $\quad \ddot{X} = \dot{J}_s \, \dot{q}_s + J_s \, \ddot{q}_s \qquad\qquad (17)$

If we substitute Eqn. (17) into Eqn. (16),

$$(M_s + J_s^T \, m_p \, J_s) \, \ddot{q}_s + J_s^T \, m_p \, \dot{J}_s \, \dot{q}_s + N_s(q_s, \dot{q}_s) - J_s^T \, m_p \, G$$
$$= T_s + J_s^T \, F_L \qquad (18)$$

Here, F_L is the force applied by the large arm and is under force control. However, the components of F_L will be

$$F_L = F_D + F_U \qquad\qquad (19)$$

where F_D is the desired internal force to match the predicted gravity term and/or the nonlinearity term, and F_U is the undesired internal force due to the control error or the kinematic measurement error. Now, if we decide to compensate only the weight of the payload by the large arm, then $F_D = -m_p G$. Substitute Eqn. (19) into Eqn. (18).

$$(M_s + J_s^T \, m_p \, J_s) \, \ddot{q}_s + J_s^T \, m_p \, \dot{J}_s \, \dot{q}_s + N_s(q_s, \dot{q}_s) + J_s^T \, F_U$$
$$= T_s \qquad\qquad (20)$$

F_u can be measured by the tip-mounted force sensor . Thus we can decouple the system by feeding forward the undesired force $-J_s{}^T F_u$. Then the small arm becomes an independent nonlinear system. To design a position controller for this system, many control laws can be applied depending on what kind of assumptions are made. One simple and practical controller is the so-called, "control law partitioning" [19,22]. Under the assumption that we know all the dynamic parameters of the arm, we can feed back nonlinear terms so that the dynamics of the small arm becomes linear, and then we can design a linear controller. The control law for the small arm is then:

$$T_s = \text{decoupling force} + \text{nonlinear term feedback} + \text{linear controller}$$
$$= - J_s{}^T F_u + J_s{}^T m_p \dot{J}_s \dot{q}_s + N_s(q_s, \dot{q}_s) + \text{linear controller} \qquad (21)$$

Substitute Eqn. (21) into Eqn. (20) and apply the linear controller. Then, the overall system equation becomes

$$M_{sp} \ddot{q}_s = M_{sp} \{ \ddot{q}_d + K_d(\dot{q}_d - \dot{q}_s) + K_p (q_d - q_s) \}$$

where $M_{sp} = M_s + J_s{}^T m_p J_s$ and q_d is the desired trajectory.

Therefore, the error dynamics become $\ddot{e} + K_d \dot{e} + K_p e = 0$ where $e = q_d - q_s$. Thus, we can choose K_d and K_p depending on the desired performance. Finally, the controller for the small arm is

$$T_s = - J_s{}^T F_u + J_s{}^T m_p \dot{J}_s \dot{q}_s + N_s(q_s, \dot{q}_s)$$
$$+ M_{sp} \{ \ddot{q}_d + K_d(\dot{q}_d - \dot{q}_s) + K_p (q_d - q_s) \}$$

4.5   Slave controller design

The purpose of the slave controller is to compensate for external forces such as the payload and eliminate undesired internal forces. In this section, the force controller for the flexible arm is designed. When two arms cooperate, the large arm moves very small angle in the joint coordinates due to the size difference of two arms. Therefore, the dynamics of the flexible arm is assumed to be linear under fine motion, and can be written as a simpler form from Eqn. (12).

$$M_L(q_L)\,\ddot{q}_L + K(q_L)\,q_L = \begin{bmatrix} I \\ 0 \end{bmatrix} T_L - J_L^T(q_L)\,F_L \qquad (22)$$

Since the force sensor is modeled as a linear spring K_{fs},

$$F_L = K_{fs}\,(X_L - X_S) \qquad (23)$$

where X_S is the position of the tip of the small arm in the Cartesian coordinates, and X_L is the position of the tip of the large arm in the Cartesian coordinates. Let $E = X_L - X_S$. Then, $\dot{E} = J_L\,\dot{q}_L - J_S\,\dot{q}_S$ Since, the large arm is in the quasi-static fine motion, we may assume that $M_L(q_L) = M_L$ and $J_L(q_L) = J_L$. (i.e. They remain constant.)

If we rewrite Eqn. (22) and (23) together as a state space form,

$$\frac{d}{dt}\begin{bmatrix} E \\ q_L \\ \dot{q}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 & J_L \\ 0 & 0 & I \\ -M_L^{-1}J_L^T K_{fs} & -M_L^{-1}K & 0 \end{bmatrix}\begin{bmatrix} E \\ q_L \\ \dot{q}_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -M_L^{-1} \end{bmatrix}\begin{bmatrix} I \\ 0 \end{bmatrix} T_L + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} J_s\,\dot{q}_s$$

or $\dot{X}_o = A X_o + B U + F X_d \qquad (24)$

$\quad Y = F_L = [K_{fs},0,0]\,X_o$

where $X_o = \begin{bmatrix} E & q_L & \dot{q}_L \end{bmatrix}^T$, $U = T_L$, and $X_d = J_s\,\dot{q}_s$

The force control is a kind of high gain position control [18]. Therefore, our control objective may be translated as a tracking problem to maintain the relative distance between two arms' tip positions E. As seen in Eqn. (24), the system is a linear time invariant system, and X_d is the disturbance input which is known from the master arm's position controller design. Therefore, we can design a tracking and disturbance rejection controller that uses the exogenous variables [26]. To formulate the problem purely in terms of state variables, it is often expedient to assume that the disturbance X_d satisfy known differential equations: $\dot{X}_d = A_d\,X_d$ To the differential equation (24) is adjoined the equations for the "exogenous" disturbance input to produce a system having the "metastate" vector $X = [X, X_d]^T$ and satisfying the "metastate equation"

$$\dot{X} = A\,X + B\,U \qquad (25)$$

where $A = \begin{bmatrix} A & 0 \\ 0 & A_d \end{bmatrix}$, $B = \begin{bmatrix} B \\ 0 \end{bmatrix}$

Now, based on the system equation (25), we can apply linear control theory to design a feedback controller

4.6 Experiments

A large experimental arm designated RALF (Robotic Arm, Large and Flexible) has been constructed and placed under computer control. Also a small experimental arm, SAM (Small Articulated Manipulator) has been constructed and is being tested under single joint control. It will be mounted on the end of RALF. This will enable experimental studies in the proposed research so that the feasibility of the concepts can be verified.

5. Prospective Contributions

Main contributions of the proposed research are anticipated in the following areas:
(1) Derivation of the dynamic equations for the constrained flexible and rigid manipulators
(2) Coordination of flexible and rigid arms for manipulating an object using the master/slave or other appropriate scheme
(3) Force control for a flexible manipulator exhibiting the non-colocated sensor and actuator problem
(4) Actual implementation of a strain and force feedback controller

I also anticipate the following by-products from the proposed research:
(1) Kinematic topology synthesis in closed chain manipulators
(2) Analytical comparison of load capacity in closed chain vs. open chain using the force ellipsoid

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