NASA Technical Memorandum 102695

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July 1990

(NASA-TM-102695) INTERGRATION OF SYSTEM IDENTIFICATION AND ROBUST CONTROLLER DESIGNS FOR FLEXIBLE STRUCTURES IN SPACE (NASA) 17 p CSCL 20K

N90-26362

Unclas

G3/39 0294714

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Space Administration

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Integration of System Identification and Robust Controller Designs for Flexible Structures in Space

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Abstract

A novel approach is developed using experimental data from the structural testing of a physical system to identify a reduced order model and its error for a robust controller design. There are three steps involved in the approach. First, an approximately balanced model is identified using the Eigensystem Realization Algorithm which is an identification algorithm. Second, the model error is calculated and described in frequency domain in terms of the H_∞ norm. Third, a pole placement technique in combination with a H_∞ control method is applied to design a controller for the considered system. A set of experimental data from an existing setup, namely the Mini-Mast system, is used to illustrate and verify the approach developed in this paper.

Introduction

Constructing a suitable mathematical model for a dynamic system is a major task in the fields of structure analysis and control design for vibration suppression. The difficulty of estimating and describing the model error,[1] which is a necessity for a robust controller design,[2–12] is even more severe than that of constructing a mathematical model. In the past, system identification techniques such as the Eigensystem Realization Algorithm (ERA)[13–18] have been developed, using experimental data from the modal testing of large space structures, for constructing a mathematical model either for a controller design or for modal parameter identification including frequencies, damping and mode shapes. Yet, the model error issue has rarely been addressed.

The multi-input/multi-output time domain ERA technique was originally developed for modal parameter identification and successfully applied to the modal testing of large space structures. The ERA technique[13,14] uses the singular values of a finite Hankel matrix formed by impulse response functions to determine the system order and further reduce the noise effect on the realized system model by truncating some small singular values. The retained singular values are dominated by the system signals and the order of the system is determined by the number of the retained singular values. On the other hand, the truncated singular values are dominated by noise and hence may be used to estimate a noise-related model error bound. Defining the model error bound to be the maximum truncated singular value is equivalent to defining a H_∞ error bound for the realized system model. Once the H_∞ error bound is obtained, a H_∞ control method[3] may be readily applied to derive a robust controller design for the system represented by the realized model. General speaking, the H_∞ control methods are optimal frequency domain algorithms which compute the feedback control law by minimizing the maximum singular value of the system transfer function. It is known that the H_∞ control methods address the full range of stability margin, sensitivity, and robust optimization. This paper is motivated by the strong connection between the ERA system identification method and the H_∞ control methods. It seems natural to integrate together the system identification method with the control methods so that a robust controller design can be achieved directly using the experimental data.

The objective of this paper is to develop a method using experimental data from measurements of a structural testing to identify a reduced order model and its error for a robust controller design to suppress the vibrational motion of the structure. The approach is to integrate the Eigensystem Realization Algorithm with the H_∞ robust control methods. This paper is written to outline the fundamental concepts from different disciplines and then integrate them together, while including advanced materials. In the first section, Hankel singular values of a linear system in frequency domain are discussed and the relation with the ERA Finite-Hankel singular values is described. In the second section, the ERA algorithm is briefly introduced and the convergence of its model realization to the balanced realization[19,20] is described and discussed. The balanced realization has been widely used for model reduction[20] and robust controller designs. In the third section, the H_∞ control methods are briefly described. In the fourth section, the ERA/H_∞ control algorithm is shown

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including two numerical examples. The ERA reduced realized model is chosen as the H_∞ control design model and the estimated model error is used to compute the weighting matrices for a robust controller design. In the fifth section, an example is given using an existing experimental setup, the NASA Langley Mini-Mast system, to illustrate the ERA/H_∞ control algorithm.

Hankel and Finite-Hankel Singular Values of a Linear System

Computation of the model error bound is a key element in developing robust control systems. The model error bound can be expressed and described in many different ways including the Hankel singular values of the system transfer function matrix. In this section, the Hankel and Finite-Hankel singular values of a linear, time-invariant dynamical system are discussed. Consider the linear system

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) \tag{1}$$

$$\underline{y}(t) = C\underline{x}(t) \tag{2}$$

where $\underline{x}$ is an $n \times 1$ state vector, $\underline{u}$ is an $m \times 1$ input vector, $\underline{y}$ is a $l \times 1$ output vector and A, B, and C are $n \times n, n \times m$, and $l \times n$ system matrices, respectively. The transfer function of this system is

$$G(s) = C(sI - A)^{-1}B. \tag{3}$$

If the system is stable, i.e. all the eigenvalues of A are in the left half-plane, then the controllability and the observability grammians are defined as

$$P = \int_0^\infty exp(At)BB^* exp(A^*t)dt$$

and

$$Q = \int_0^\infty exp(A^*t)C^*C exp(At)dt \tag{4}$$

where P and Q satisfy the Lyapunov equations

$$AP + PA^* + BB^* = 0$$

and

$$A^*Q + QA + C^*C = 0. \tag{5}$$

Here the superscript * is used herein to indicate the complex conjugate of the variable so marked.

Hankel Singular Values

The Hankel singular values of this system are defined as

$$\sigma_i(G(s)) = (\lambda_i(PQ))^{1/2}, \quad \lambda_i \geq \lambda_{i+1} \tag{6}$$

where λ_i is the ith eigenvalue. Now, consider the discrete-time state-space model

$$\underline{x}(t+1) = \hat{A}\underline{x}(t) + \hat{B}\underline{u}(t) \tag{7}$$

$$\underline{y}(t) = \hat{C}\underline{x}(t). \tag{8}$$

If this system is stable then the discrete controllability and observability grammians are defined as

$$\hat{P} = \sum_{0 \leq k < \infty} \hat{A}^k \hat{B}\hat{B}^* \hat{A}^{*k}$$

and

$$\hat{Q} = \sum_{0 \leq k < \infty} \hat{A}^{*k}\hat{C}^*\hat{C}\hat{A}^k \tag{9}$$

which satisfy the discrete Lyapunov equations

$$\hat{P} - \hat{A}\hat{P}\hat{A}^* = \hat{B}\hat{B}^*$$

and

$$\hat{Q} - \hat{A}^*\hat{P}\hat{A} = \hat{C}^*\hat{C}. \tag{10}$$

The z transform of this discrete system is given as

$$G(z) = \hat{C}(zI - \hat{A})^{-1}\hat{B} \tag{11}$$

or in series expansion

$$G(z) = \sum_{i=0}^{\infty} Y_i z^{-i-1}$$

where $Y_i = \hat{C}\hat{A}^i\hat{B}$, $i = 0, 1, 2, \ldots$ are impulse response functions and usually referred to as the Markov parameters associated with the model ($\hat{A}$, $\hat{B}$, $\hat{C}$). The Hankel singular values of $G(z)$ are defined as

$$\sigma_i(G(z)) = (\lambda_i(\hat{P}\hat{Q}))^{1/2}. \tag{12}$$

Because a discrete-time model is related to a continuous-time model by an invariant step time response, the grammians in the discrete time domain are equivalent to the grammians in the continuous time domain.[1,20] Therefore the Hankel singular values for the continuous-time and discrete-time models are identical.

Next it will be shown how the Hankel singular values are related to the singular values of a Hankel matrix in the discrete-time case. Consider a discrete-time infinite Hankel matrix whose (i,j)th block is the Markov parameter $Y_{i+j-2} = \hat{C}\hat{A}^{i+j-2}\hat{B}$. The Hankel matrix is written as

$$H = W_oW_c = \begin{pmatrix} Y_0 & Y_1 & \ldots & Y_{s-1} & \ldots \\ Y_1 & Y_2 & \ldots & Y_s & \ldots \\ \vdots & \vdots & \ddots & \vdots & \\ Y_{r-1} & Y_r & \ldots & Y_{r+s-2} & \\ \vdots & \vdots & & & \ddots \end{pmatrix} \tag{13}$$

where

$$W_o = [\hat{C}^*, \hat{A}^*\hat{C}^*, ..., \hat{A}^{*k}\hat{C}^*, ...]^*$$

and

$$W_c = [\hat{B}, \hat{A}\hat{B}, ..., \hat{A}^k\hat{B}, ...]. \tag{14}$$

It can be shown that $\hat{P} = W_cW_c^*$ and that $\hat{Q} = W_o^*W_o$ satisfy Eq. (10). The singular values of H are

$$\sigma_i(H) = (\lambda_i(H^*H))^{1/2} = (\lambda_i(W_c^*W_o^*W_oW_c))^{1/2}. \tag{15}$$

In this equation, $\lambda_i(W_c^*W_o^*W_oW_c)$ means that

$$W_c^*W_o^*W_oW_c\underline{u}_i = \lambda_i\underline{u}_i \tag{16}$$

where $\underline{u}_i$ is the eigenvector corresponding to the eigenvalue λ_i. Pre-multiplying Eq. (16) by W_c and defining $\underline{x}_i = W_c\underline{u}_i$ yields

$$W_cW_c^*W_o^*W_o\underline{x}_i = \lambda_i\underline{x}_i. \tag{17}$$

Because Eqs. (16) and (17) produce the same eigenvalues, Eq. (15) can be written as

$$\sigma_i(H) = (\lambda_i(W_cW_c^*W_o^*W_o))^{1/2} = (\lambda_i(\hat{P}\hat{Q}))^{1/2}. \tag{18}$$

Hence the Hankel singular values of $G(z)$, Eq. (11), which is formed from impulse response functions, are the singular values of H, Eq. (13). Again note that the Hankel matrix H in (13) is infinite dimensional. It is, of course, impractical to solve for the Hankel singular values from an infinite dimensional Hankel matrix H. If system matrices A, B, and C are exactly known, the Hankel singular values can be obtained by solving Eq. (12). In practice, however, flexible structures are subjected to uncertainties to certain extent. Experiments are conducted to verify the matrices A, B and C. In general, experimental data of inputs and outputs are stored in terms of the system transfer function (frequency domain) which is equivalent to the impulse response functions (Markov parameter) (discrete-time domain). Therefore, from a practical point of view, the Hankel singular values can be approximately computed from a Hankel matrix that is formed by a finite number of Markov parameters. The question arises how much error one expects to have if the Hankel singular values are computed from a finite-dimensional Hankel matrix. This question will be answered in the following section.

Finite-Hankel Singular Values

In this section, the singular values obtained from a Finite-Hankel matrix are compared with the Hankel singular values. First a Finite-Hankel matrix is defined as

$$H(0) = \begin{pmatrix} Y_0 & Y_1 & \cdots & Y_{s-1} \\ Y_1 & Y_2 & \cdots & Y_s \\ \vdots & \vdots & \ddots & \vdots \\ Y_{r-1} & Y_r & \cdots & Y_{r+s-2} \end{pmatrix} \tag{19}$$

where Y_i is the Markov parameter. An extended matrix of the Finite-Hankel matrix $H(0)$ is defined as

$$H_{rs}^0 = \begin{pmatrix} Y_0 & Y_1 & \cdots & Y_{s-1} & 0 & \cdots \\ Y_1 & Y_2 & \cdots & Y_s & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \\ Y_{r-1} & Y_r & \cdots & Y_{r+s-2} & 0 & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \\ \vdots & \vdots & & \vdots & & \ddots \end{pmatrix}. \tag{20}$$

It is obvious that

$$\sigma_i(H_{rs}^0) = \sigma_i(H(0)). \tag{21}$$

The Hankel matrix H in Eq. (13) can be written as

$$H = H_{rs}^0 + H_{rs}' \tag{22}$$

where

$$H_{rs}' = \begin{pmatrix} 0 & \cdots & 0 & Y_s & \cdots \\ 0 & \cdots & 0 & Y_{s+1} & \cdots \\ \vdots & & \ddots & \vdots & \\ 0 & \cdots & 0 & Y_{r+s-1} & \cdots \\ Y_r & \cdots & Y_{r+s-1} & Y_{r+s} & \\ \vdots & & \vdots & & \ddots \end{pmatrix}, \tag{23}$$

the left upper $r \times s$ blocks of H_{rs}' are 0 and the other blocks are the same as H. Next it will be shown that the Finite-Hankel singular values converges to the Hankel singular values. Consider the system with the single-input and single-output transfer function $G(s)$ that is real rational, scalar-valued, strictly proper and analytic in Re $s > 0$. Then the Markov parameter Y_i can be expressed as

$$Y_i = \sum_{j=1}^{k} c_j e^{i\lambda_j \Delta t} \tag{24}$$

where k is the system order, λ_j is the $j-th$ eigenvalue with negative real part and Δt is the measurement time increment. The norm of Y_i satisfies the following inequality

$$|Y_i| \leq ce^{-\lambda t_i}, \quad t_i = i\Delta t, \ \ i \geq n_i \tag{25}$$

where c and λ are positive real constants, and n_i is a positive integer. Using singular value inequalities[21] yields

$$\sigma_i(H) \leq \sigma_i(H_{rs}^0) + \sigma_1(H_{rs}'). \tag{26}$$

Let $r > n_i + 1$ and $s > n_i + 1$ then

$$\sigma_1(H'_{rs}) \le \|H'_{rs}\| = \max_{j \ge s} \sum_{i=1}^{\infty} |Y_{i+j-1}|$$
$$= \sum_{i=s}^{\infty} |Y_i| \le \sum_{i=n_i+1}^{\infty} |Y_i| \le \sum_{i=n_i+1}^{\infty} |ce^{-\lambda t_i}| \quad (27)$$
$$\le \frac{1}{\Delta t} \int_{t_{n_i}}^{\infty} ce^{-\lambda t} dt = \frac{c}{\lambda \Delta t} e^{-\lambda t_{n_i}} \le \varepsilon$$

where ε is a positive real number. For any small $\varepsilon > 0$, there exists an n_i which makes Eq. (27) true. Combination of Eqs. (21), (26) and (27) yields

$$\sigma_i(H) - \sigma_1(H'_{rs}) \le \sigma_i(H^0_{rs}) \le \sigma_i(H) + \sigma_1(-H'_{rs})$$

or

$$\sigma_i(H) - \varepsilon \le \sigma_i(H(0)) \le \sigma_i(H) + \varepsilon. \quad (28)$$

Equation (28) shows that the Finite-Hankel singular values converge to the Hankel singular values. The proof can be easily extended for the multi-input/multi-output system.

The finite-Hankel matrix is the basis for the Eigensystem Realization Algorithm (ERA) which can be used to identify a system model for modal parameter (system eigenvalues) identification or controller designs. The Markov parameters Y_i are the impulse response functions which may be obtained experimentally.

Eigensystem Realization Algorithm (ERA) and Model Reduction

The problem of constructing a suitable mathematical model for a dynamic system is a major task for a control engineer as well as a structural engineer. For structural analyses and controller designs experimental data from structural testing are used to either verify an analytical model or directly determine a mathematical model. The ERA is an algorithm which computes a mathematical model directly from experimental data. In this section, the basic ERA formulations are briefly discussed and the relation between the ERA realization and the balanced realization is derived. Note that balanced realization has been frequently used for model reduction,

Basic ERA Formulations

The ERA algorithm uses the singular value decomposition of a Hankel matrix to determine a system model under test. Consider the discrete-time state-space model as given by Eq. (8). Let the input $\underline{u}$ be the impulse input at an initial time such that its components satisfy $u_i(0) = 1$, $(i = 1, ..., m)$ and $\underline{u}(k) = 0$, $(k = 1, ...)$. The time domain description is given by the Markov parameters[14], i.e. impulse response functions,

$$Y_k = \hat{C}\hat{A}^k\hat{B}. \quad (29)$$

Note that there are many other ways to obtain the impulse response functions experimentally. For example, inverting a transfer function from the frequency domain to the time domain will generate the impulse response functions. The algorithm begins by forming the $r \times s$ block matrix (generalized Finite-Hankel matrix)

$$H(k) = \begin{pmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+s-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+s} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+r-1} & Y_{k+r} & \cdots & Y_{k+r+s-2} \end{pmatrix}. \quad (30)$$

The Finite-Hankel matrix $H(k)$ can be expressed as

$$H(k) = V_r\hat{A}^kW_s \quad (31)$$

where

$$V_r = \begin{pmatrix} \hat{C} \\ \hat{C}\hat{A} \\ \vdots \\ \hat{C}\hat{A}^{r-1} \end{pmatrix}, \quad W_s = [\hat{B}\ \hat{A}\hat{B}\ \ldots\ \hat{A}^{s-1}\hat{B}],$$

V_r and W_s are the observability and controllability matrices, respectively. Assume that there exists a matrix H', such that

$$W_sH'V_r = I_n \quad (32)$$

where I_n is an identity matrix of order n. Equations (31) and (32) imply

$$H(0)H'H(0) = V_rW_sH'V_rW_s = V_rW_s = H(0). \quad (33)$$

Thus H' is the pseudo inverse of $H(0)$. A solution for H' can be obtained as follows. Use singular value decomposition to the matrix $H(0)$

$$H(0) = U\Sigma V^T \quad (34)$$

where the columns of U and V are orthonormal and Σ is diagonal with positive elements $[\sigma_1, \sigma_2, ..., \sigma_n]$. The pseudo inverse can be computed as

$$H' = [V][\Sigma^{-1}U^T]. \quad (35)$$

Define 0_m as the null matrix of order m, I_m an identity matrix, $E_m^T = [I_m, 0_m, ..., 0_m]$. From Eqs. (31), (32), and

(35), a minimal realization can be obtained from

$$\begin{aligned} Y(k+1) &= E_q^T H(k) E_p = E_q^T V_r \hat{A}^k W_s E_p \\ &= E_q^T V_r W_s H' V_r \hat{A}^k W_s H' V_r W_s E_p \\ &= E_q^T U \Sigma^{1/2} [\Sigma^{-1/2} U^T H(1) V \Sigma^{-1/2}]^k \Sigma^{1/2} V^T E_p. \end{aligned} \tag{36}$$

The triple $[\Sigma^{-1/2} U^T H(1) V \Sigma^{-1/2},\ \Sigma^{1/2} V^T E_p,\ E_q^T U \Sigma^{1/2}]$ is a minimum realization. This is the basic ERA formulation.

For a finite dimensional and linear time-invariant system, an exact system realization can be obtained by the ERA algorithm from noise-free measurements.[14] The ERA algorithm is accurate and efficient, particularly for low noise levels, and produces a minimum order realization. It is a powerful identification algorithm. However, if significant noises are present in the measurements, caution must be taken to identify a proper order for the system model.

Relation Between ERA and Balanced Realizations

In this section, the relation between ERA and Balanced Realizations is derived. Examination of Eqs. (31) and (34) indicates that the observability matrix and controllability matrix can be written as

$$V_r = U \Sigma^{1/2} \ \ and \ \ W_s = \Sigma^{1/2} V^T. \tag{37}$$

The equality $U^T U = I = V^T V$ implies that

$$V_r^T V_r = \Sigma^{1/2} U^T U \Sigma^{1/2} = \Sigma$$

and

$$W_s W_s^T = \Sigma^{1/2} V^T V \Sigma^{1/2} = \Sigma. \tag{38}$$

Combination of Eqs. (14) and (31) yields

$$W_o = \begin{pmatrix} \hat{C} \\ \hat{C}\hat{A} \\ \vdots \\ \hat{C}\hat{A}^{r-1} \\ \hat{C}\hat{A}^{r} \\ \vdots \\ \hat{C}\hat{A}^{2r-1} \\ \hat{C}\hat{A}^{2r} \\ \vdots \end{pmatrix} = \begin{pmatrix} V_r \\ V_r \hat{A}^r \\ V_r \hat{A}^{2r} \\ \vdots \end{pmatrix}. \tag{39}$$

and

$$\begin{aligned} W_c &= [\hat{B}\ \ \hat{A}\hat{B}\ \ldots\ \hat{A}^{s-1}\hat{B}\ \ \hat{A}^s\hat{B}\ \ldots\ \hat{A}^{2s-1}\hat{B}\ \ \hat{A}^{2s}\hat{B} \ldots] \\ &= [W_s\ \ \hat{A}^s W_s\ \ \hat{A}^{2s} W_s \ldots]. \end{aligned}$$

The observability and controllability grammians can thus be written as

$$\begin{aligned} \hat{Q} &= W_o^T W_o \\ &= V_r^T V_r + (\hat{A}^r)^T V_r^T V_r \hat{A}^r + (\hat{A}^{2r})^T V_r^T V_r \hat{A}^{2r} + \ldots \\ &= \Sigma + (\hat{A}^r)^T \Sigma \hat{A}^r + (\hat{A}^{2r})^T \Sigma \hat{A}^{2r} + \ldots \end{aligned}$$

and

$$\begin{aligned} \hat{P} &= W_c W_c^T \\ &= W_s W_s^T + \hat{A}^s W_s W_s^T (\hat{A}^s)^T + \hat{A}^{2s} W_s W_s^T (\hat{A}^{2s})^T + \ldots \\ &= \Sigma + \hat{A}^s \Sigma (\hat{A}^s)^T + \hat{A}^{2s} \Sigma (\hat{A}^{2s})^T + \ldots \end{aligned} \tag{40}$$

If the system is stable, all the eigenvalues of $\hat{A}$ should be inside the unit circle, i.e., $|\lambda_i(A)| < 1$. Therefore, for a stable system, there exists a value of n such that $|\lambda_i(\hat{A}^n)|$ is less than ε for a given ε. Equation (28) implies

$$\lim_{r\to\infty,\ s\to\infty} \Sigma = \Sigma_\infty \tag{41}$$

where Σ_∞ is the Σ in Eq. (34) when r and s approach ∞, which, in turn from Eq. (40), yields

$$\lim_{r\to\infty,\ s\to\infty} \hat{P} = \lim_{r\to\infty,\ s\to\infty} \Sigma = \Sigma_\infty$$

and

$$\lim_{r\to\infty,\ s\to\infty} \hat{Q} = \lim_{r\to\infty,\ s\to\infty} \Sigma = \Sigma_\infty. \tag{42}$$

Thus the ERA realization converges to the balanced realization. So far the realization order is assumed to be identical to the true one. If the realization system order γ is lower than the true one n, Eq. (36) implies that C_γ is the $l \times \gamma$ left block of C_n, A_γ is the $\gamma \times \gamma$ left-upper block of A_n and B_γ is the $\gamma \times m$ upper block of B_n. Since the ERA realization $[A_n,\ B_n,\ C_n]$ converges to the balanced realization $[A_b, B_b, C_b]$, A_γ, B_γ and C_γ should converge to the corresponding blocks of A_b, B_b and C_b, respectively. From Eq. (42) and the balanced realization[19,20], one obtains

$$\lim_{r\to\infty,\ s\to\infty} \hat{P}_\gamma = \Sigma_\infty^\gamma \ \lim_{r\to\infty,\ s\to\infty} \hat{Q}_\gamma = \Sigma_\infty^\gamma \tag{43}$$

where $\hat{P}_\gamma$ and $\hat{Q}_\gamma$ are the controllability and observability grammians of $[A_\gamma,\ B_\gamma,\ C_\gamma]$ and Σ_∞^γ is the $\gamma \times \gamma$ left-upper block matrix of Σ_∞. The γ-order ERA realization converges to the γ-order balanced realization. It is concluded that the reduced ERA model converges to the reduced balanced model.

H_∞ Robust Controller Designs

The H_∞ control methods are optimal frequency domain algorithms[2-4], which address the full range of stability margin, sensitivity, and robust criteria. In this section, a brief description of H^∞ control methods is given.

A block diagram of the H_∞ design algorithm is presented in Fig. 1. The closed-loop transfer function matrices (u_1 to three outputs y_2, u_2 and y) are expressed as

$$S(s) = (I + L(s))^{-1} \tag{44}$$

$$R(s) = F(s)(I + L(s))^{-1} \tag{45}$$

$$T(s) = L(s)(I + L(s))^{-1} \tag{46}$$

where $L(s) = G(s)F(s)$, $G(s)$ is defined in Eq. (3), and $F(s)$ is the controller transfer function matrix. The matrices $S(s)$ and $T(s)$ are known as the sensitivity and complementary sensitivity, respectively. The singular values of these matrices can be used to quantify the stability margins and performance of the system. In the H_∞ controller design, the weighting function matrix $W_1(jw)$ is used to weight the sensitivity so that

$$\bar{\sigma}(S(jw)) \leq |W_1^{-1}(jw)| \tag{47}$$

where $\bar{\sigma}(\ )$ means the maximum singular values of ( ). The quantity $|W_1^{-1}(jw)|$ is the desired disturbance attenuation.

The plant uncertainty can be considered as additive uncertainty Δ_A or multiplicative uncertainty Δ_M. The stability margin of the additive uncertainty must satisfy

$$\bar{\sigma}(\Delta_A(jw))\bar{\sigma}(R(jw)) \leq 1. \tag{48}$$

If the weighting function matrix $W_2(jw)$ is chosen such that

$$|W_2(jw)| \geq \bar{\sigma}(\Delta_A(jw))$$

and

$$|W_2(jw)|\bar{\sigma}(R(jw)) \leq 1 \tag{49}$$

then the inequality Eq. (48) holds. The disturbance attenuation and additive stability margin designs can be combined into a single H_∞ problem as

$$\|T_{y_1u_1}\|_\infty \leq 1 \tag{50}$$

where

$$T_{y_1u_1} = \begin{pmatrix} W_1S \\ W_2R \end{pmatrix} \tag{51}$$

and this combination is called the H_∞ additive perturbation control design. Similarly, the H_∞ multiplicative perturbation control design can be derived as

$$\|T_{y_1u_1}\|_\infty \leq 1 \tag{52}$$

where

$$T_{y_1u_1} = \begin{pmatrix} W_1S \\ W_3T \end{pmatrix}. \tag{53}$$

A detailed examination of H_∞ design methods can be found in Ref. 7. In the next section, a combined ERA/H_∞ control algorithm is developed and discussed including some numerical examples.

ERA/H_∞ Control Algorithm

This section shows a control design algorithm which combines the features of both ERA and H_∞ control methods. The flowchart of this algorithm is shown in Fig. 2.

To use the ERA/H_∞ algorithm, a set of experimental input and output data is required. The model error between the reduced ERA model and the full ERA model is used for a H_∞ controller design. The full ERA model is defined as the model of maximum order which can be obtained by the ERA. For noise-free measurements, if not impossible in practice, the full ERA model is the exact system model. For noisy measurements, the order of the full ERA model is equal to either the number of columns or the number of rows in the Finite-Hankel matrix whichever is less. The reduced ERA model is the model reduced from the full model such as the triple $[A_\gamma, B_\gamma, C_\gamma]$ shown in the last paragraph of the ERA Section.

It has been shown that the ERA models converge to the balanced models. The γ-th order reduced balanced model $G_\gamma(s)$ of an n-th full order system $G(s)$ satisfies the following error bound[1]

$$\bar{\sigma}(G(jw) - G_k(jw)) \leq 2\sum_{i=k+1}^{n} \sigma_i \tag{54}$$

where $\bar{\sigma}(\ )$ is the maximum singular value of ( ) and σ_i is the ith Hankel singular value of $G(s)$. This equation provides the H_∞ error bound for the reduced model. There are two different H_∞ control designs, namely the additive perturbation control design[11] and the multiplicative perturbation control design.[12] Numerical examples are used in the following to illustrate the ERA/H_∞ control algorithm for the two different designs.

ERA/H_∞ Additive Perturbation Control Design

In this design the closed-loop transfer function must satisfy the inequality

$$\|T_{yu}\| \leq 1 \tag{55}$$

where

$$T_{yu} = \begin{pmatrix} W_1S \\ W_2R \end{pmatrix} \tag{56}$$

and u is the input, y is the output, and W_1 and W_2 are appropriate weighting functions. The sensitivity matrix S tends to be dominated by the sensitivity of low frequency modes (or dominant modes). An appropriate weighting function W_1 is the inverse of the desired sensitivity. The weighting function matrix W_2 is chosen to penalize the control action at high frequencies (or residual modes) as well as incorporate other robustness constraints. One of the design goals is to assure that unmodelled high frequency dynamics will not destabilize the closed-loop systems. Some examples of choosing weighting functions can be found in Refs. 10 and 11.

Let the system impulse response measurement be generated by the following equation

$$y(t_i) = \sum_{i=1}^{4} a_i exp(\zeta_i t_i) sin(w_i t_i) \tag{57}$$

where the system parameters are listed in Table 1.

TABLE 1

i	1	2	3	4
a_i	1.0	0.2	0.04	0.01
ζ_i	-0.2	-0.3	-0.4	-0.5
w_i	3.0	5.0	7.0	8.0

For illustration, the measurements are assumed to be noise free. The Finite-Hankel matrix size is chosen as 40×45. Figure 3 shows the singular-value Bode plots of the full model transfer function G, the reduced model transfer function G_γ and the model error $\Delta G = G - G_\gamma$. Here G_γ of order 4 is the transfer function obtained by the ERA algorithm. The model error ΔG is an additive model error. The additive robustness weighting function W_2 and the sensitivity weighting function W_1 are chosen as

$$W_1 = \frac{0.005s + 1}{0.4s + 0.01}, \quad W_2 = \frac{0.04s + 0.005}{0.001s + 1}.$$

Figure 4 shows the Bode plots of ΔG and W_2, whereas Fig. 5 shows the Bode plots of W_1 and W_2. The Bode plot of T_{yu} is shown in Fig. 6, in which the gain margin is 5.2 db and the phase margin is 56.6 degree.

ERA/H_∞ Multiplicative Perturbation Control Design

In this design, the closed-loop transfer function must satisfy

$$||T_{yu}|| \leq 1 \tag{58}$$

where

$$T_{yu} = \begin{pmatrix} W_1 S \\ W_3 T \end{pmatrix} \tag{59}$$

and W_1 and W_3 are appropriate weighting functions. The system measurements and the Finite-Hankel matrix are the same as in the above example. Figure 7 shows the design robustness $W_3 G_\gamma$ and model error. The design weighting matrices are chosen as

$$W_1 = \frac{0.005s + 1}{0.2s + 0.01}, \quad W_2 = 1.0 \times 10^{-7}, \quad W_3 = \frac{0.04s + 0.01}{0.001s + 1}.$$

Figure 8 shows the Bode plots of the weighting matrices. The Bode plot of the closed-loop transfer function T_{yu} is shown in Figure 9, in which the gain margin is 6.0 db and the phase margin is 54.2 degree.

The ERA/H_∞ control algorithm provides robust control designs directly from experimental input and output data. First, the ERA algorithm is used to compute an appropriate reduced balanced model and its error in terms of the H_∞ norm. Then, the model error norm is used to select the H_∞ control weighting matrices for a robust controller design. In the next section, the ERA/H_∞ algorithm is applied to an experimental setup, i.e. the Mini-Mast system[22], for further illustration of the concept developed in this paper.

ERA/H_∞ Application: The Mini-Mast System

In this section, the ERA/H_∞ algorithm is used to design robust controllers for the Mini-Mast system[22] with two inputs (torgue wheels) and two outputs (displacement sensors). Mini-Mast is a 20-meter-long, deployable/retractable truss located in the Structural Dynamics Research Laboratory at NASA Langley Research Center. Mini-Mast was designed and built to high standards typical of spacecraft hardware which was constructed from graphite-epoxy tubes, titanium joints, and precision fabrication techniques.[23] It is used as a ground test article for research in the areas of structural analysis, system identification,[24] and control of large space structures. The sinusoid inputs with the frequencies close to the system natural frequencies are used to excite the system. The system has five modes less than 7 Hz and the lowest frequency is about 0.86 Hz. The design strategy is as follows:

1. A time domain least squares technique[23] is used to compute Markov parameters from input and output measurement data.
2. A Finite-Hankel matrix is formed from the Markov parameters and the ERA is used to compute a reduced model and its model error.
3. A H_∞ control method is applied to design a robust controller. When applying the ERA, the discrete time increment should be adjusted to compute an accurate model within the frequency range of interest. Figure 10 shows the singular value Bode plot of the ERA model transfer function. The singular value Bode plots of the model error (solid lines) and the designed additive robustness (dashed lines) are shown in Fig. 11.

In *MATLAB*[7], the H_∞ control computations are performed by using the recent Glover-Doyle "2-Riccati" formulae.[12] After the H_∞ control design is executed, the closed-loop system keeps the open-loop stable complex eigenvalue pairs and reflects the open-loop unstable complex eigenvalue pairs. Because the identified open-loop model of the Mini-Mast system has small positive damping (system modes) or negative damping (noise modes) for most complex eigenvalues, the H_∞ closed-loop system will then have low damping. To improve this condition, a multivariable bilinear transform[8] can be used to shift the open-loop eigenvalues before applying the H_∞ control methods. The process of the bilinear transform is described below:

1. The system model and the design weighting functions are shifted with a real value α.
2. The H_∞ design methods are used to design a controller for the shifted system.
3. Either the closed-loop system is shifted back by $-\alpha$ or the controller is used in the original nonshifted system.
4. Check the singular values of the closed-loop transfer function matrix T_{yu}, Eq. (56), which should be less than one.

In the process of applying the bilinear transform, several problems listed below are found.

1. The original system has a H_∞ control solution with appropriate design weighting functions, but the shifted system with the shifted weighting functions does not always have a H_∞ control design solution.
2. The singular values of the closed-loop system increase significantly when the imaginary axis has a positive shift.

To improve the above problems, a different technique which is equivalent to a pole placement technique is developed yielding freedom for assigning each closed-loop eigenvalue independently. Let the system state matrix be diagonalized such that

$$\bar{A} = \Psi^{-1} A \Psi \tag{60}$$

where Ψ is the eigenvector matrix. Let the closed-loop state matrix be represented as

$$\hat{A} = \Psi(\bar{A} + A1)\Psi^{-1} \tag{61}$$

where $A1$ is a semipositive diagonal matrix. The design strategy is as follows.

1. Use $\hat{A}$ as the state matrix for the H_∞ controller design.
2. Apply the controller to the original system.
3. Check whether this control design satisfies the H_∞ constraint equation, Eq. (55).

Figure 12 shows the singular value Bode plot of a H_∞ design transfer function T_{yu} with a specified matrix $A1$ which is not shown. The matrix $A1$ provides more than 10% damping for the first mode. Next, the computational simulation is used to check the performance of the above design. The solid lines in Fig. 13 are the first 5-sec experimental outputs. In these plots, y_{ij} represents the ith output with the jth input. The outputs from the ERA reduced model using the same inputs as the experimental data are plotted in dashed lines. The inputs are turned off after 5 seconds. The open-loop and closed-loop responses are compared in Fig. 14. The solid lines in Fig. 14 are the open-loop free decay responses, whereas the closed-loop free responses are plotted in dashed lines. The first closed-loop mode has more than 10% damping.

In this ERA/H_∞ control design, the following items are observed.

1. This algorithm can be used to design a robust controller directly from a set of experimental input and output data.
2. The controller order is the sum of the model order and weighting function order. Economical controllers may be obtainable through the use of appropriate model reduction techniques.
3. If the identified low frequency modes are somewhat accurate, the robustness weighting can cover significant high frequency model errors.
4. The additive matrix $A1$ can be used to assign the closed-loop dampings and frequencies for the modes of interest.
5. An analytical model is not required.

Concluding Remarks

In this paper, the Eigensystem Realization Algorithm is integrated with the H_∞ control methods to develop a technique for controller designs of flexible structures. The technique starts with a set of input and output data obtained from vibration tests of a physical system. The set of data is then used to identify a reduced model and assess its error bound. The reduced model describes the physical system, whereas the error bound characterizes the system and measurement uncertainties. Both reduced model and error bound are used to design a robust controller to attenuate the vibrational motion of the physical system. Since experimental data are used, actuator dynamics, sensor dynamics and filter dynamics are included in the identification of the reduced model and thus in the controller designs. This approach is believed to provide the reduced model with fidelity. Numerical simulations for control of an experimental setup (the Mini-Mast system) indicate that this approach is very promising for use in control of flexible structures. Note that experimental models are used in these simulations. The controllers which are successfully demonstrated in the simulations will be tested in the Mini-Mast system to verify the approach developed in this paper.

Acknowledgement

The authors wish to thank Jeffrey L. Sulla of Lockheed Engineering & Sciences Company, a NASA Langley Mini-Mast team member, for providing them with the open-loop experimental data.

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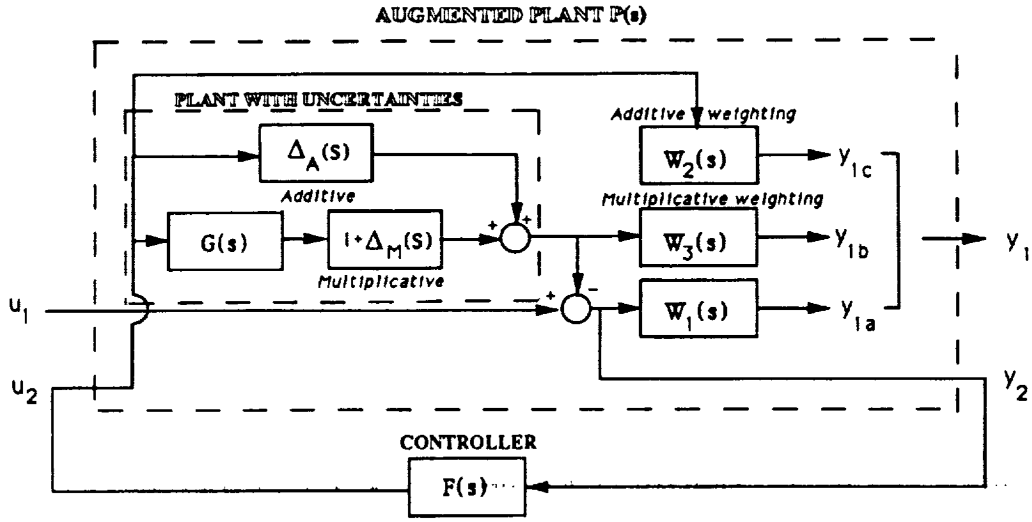

Figure 1 Weighted sensitivity and complementary sensitivity problem

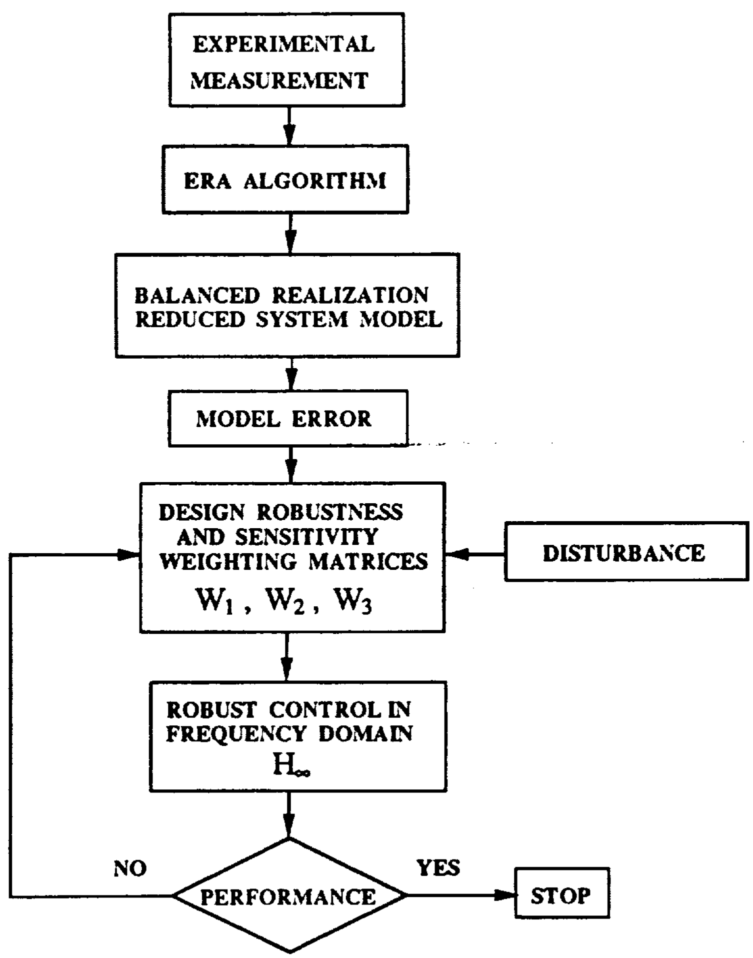

Figure 2 ERA/H_∞ Design Flowchart

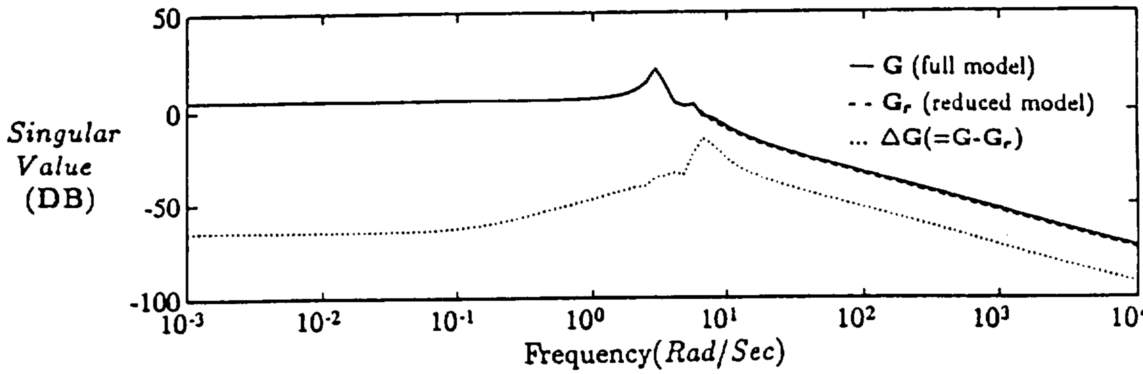

Figure 3 Full Model, Reduced Model and Model Error

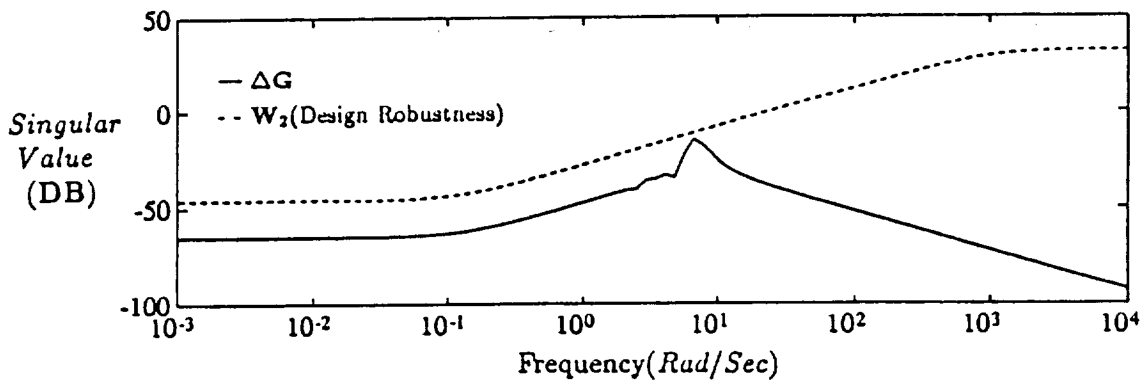

Figure 4 Design Robustness and Model Error

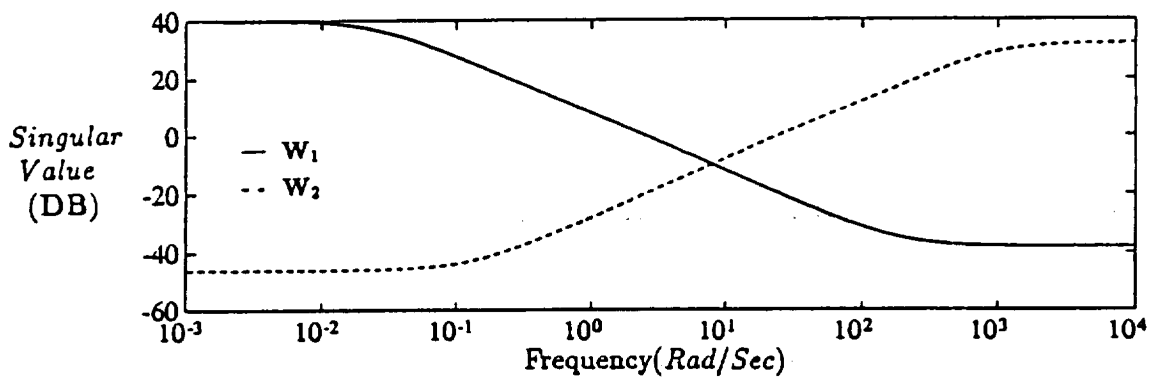

Figure 5 Weighting Functions

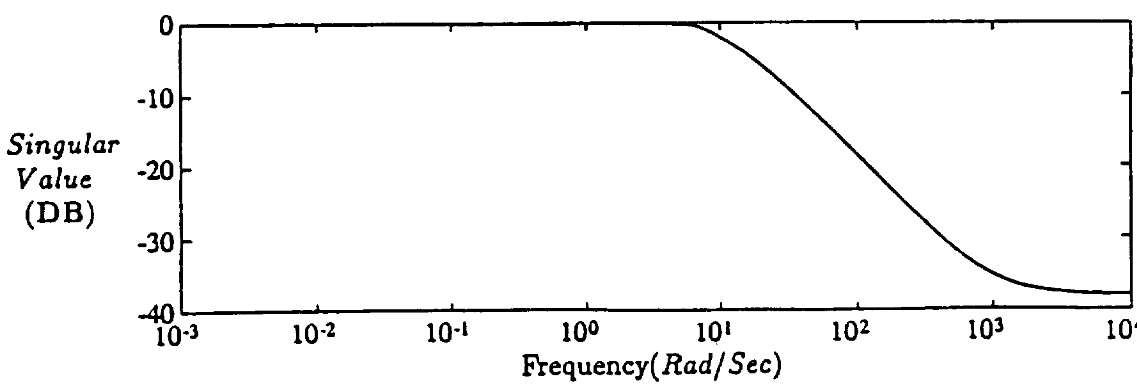

Figure 6 Closed-loop Transfer Function T_{yu}

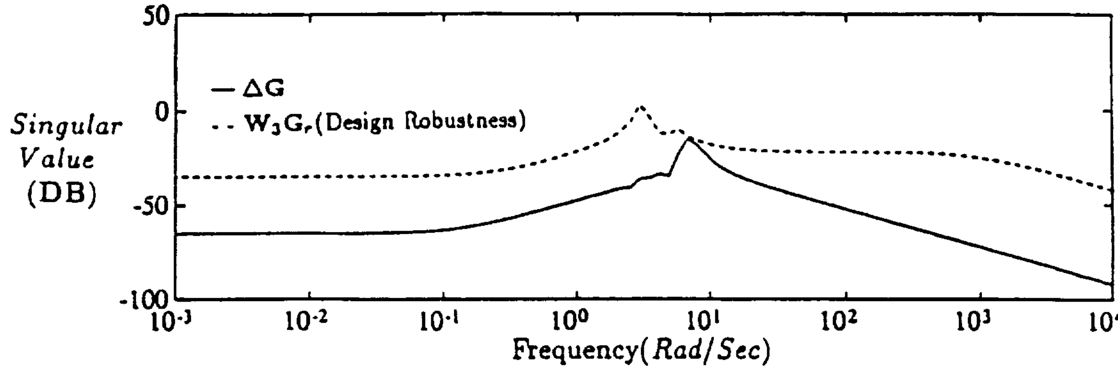

Figure 7 Design Robustness and Model Error

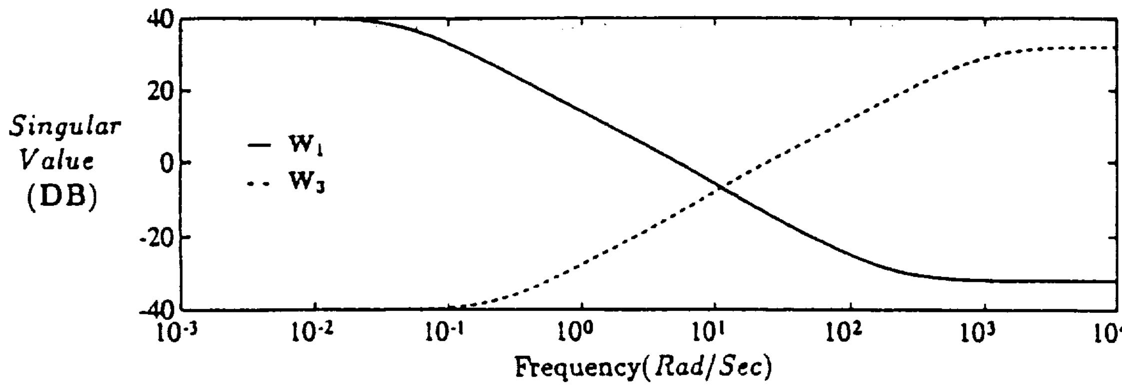

Figure 8 Weighting Functions

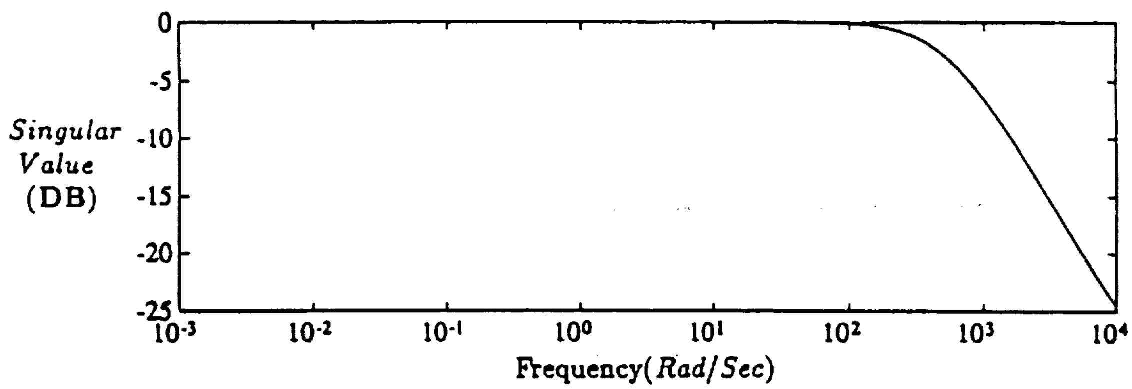

Figure 9 Closed-loop Transfer Function T_{yu}

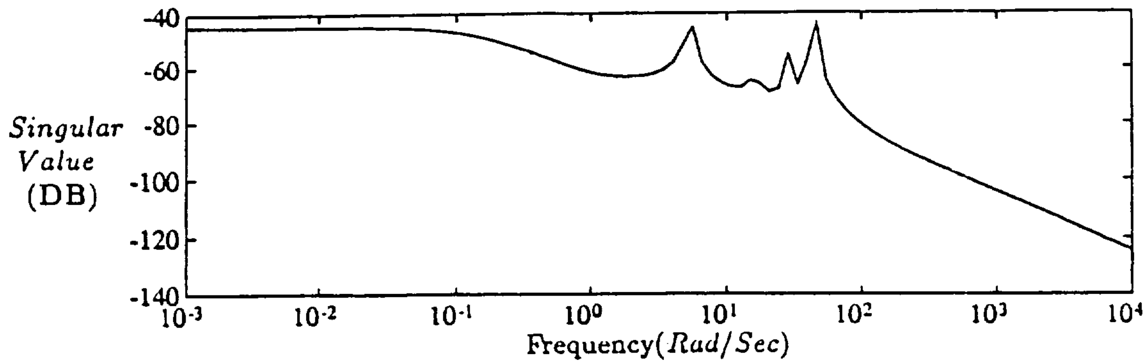

Figure 10 System Model

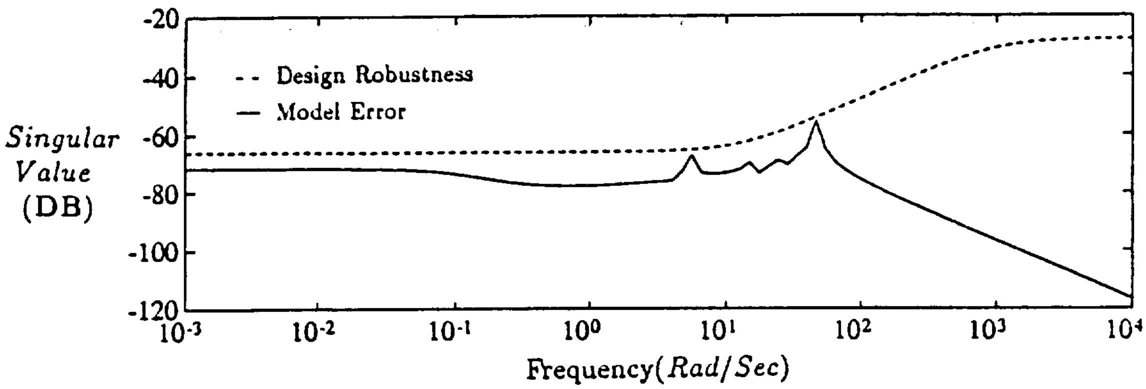

Figure 11 Design Robustness and Model Error

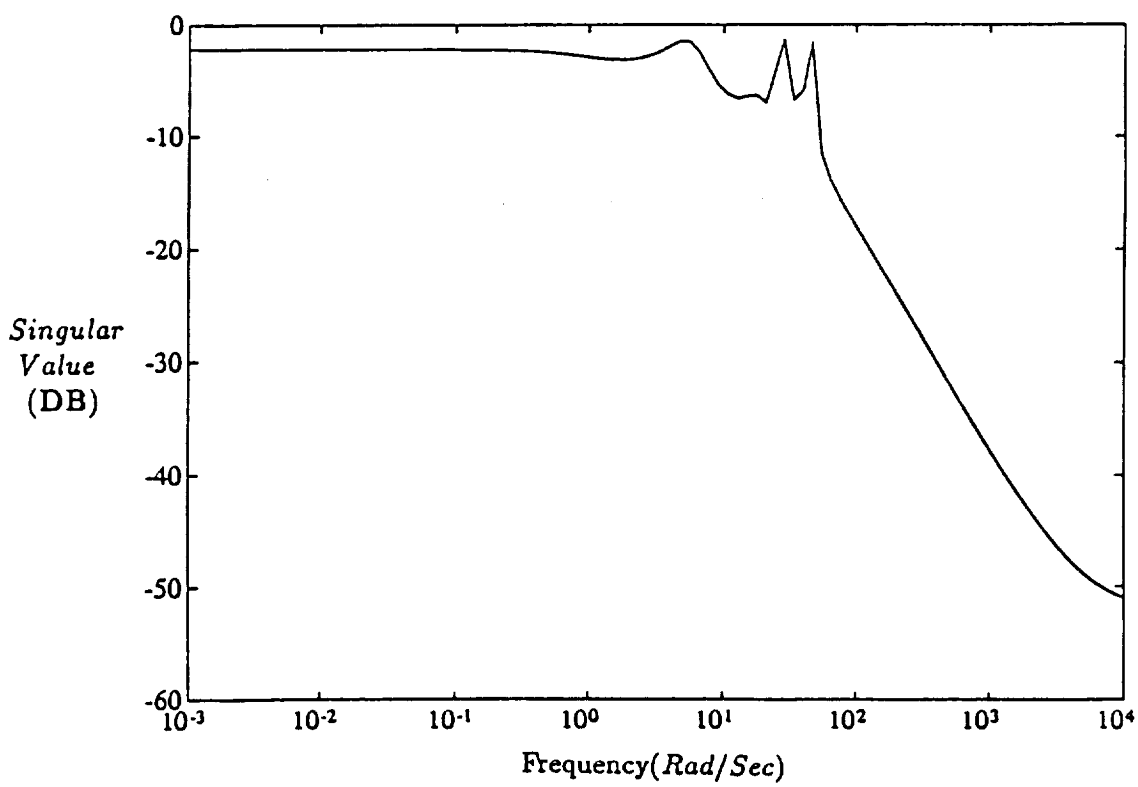

Figure 12 Closed-loop Transfer Function T_{yu}

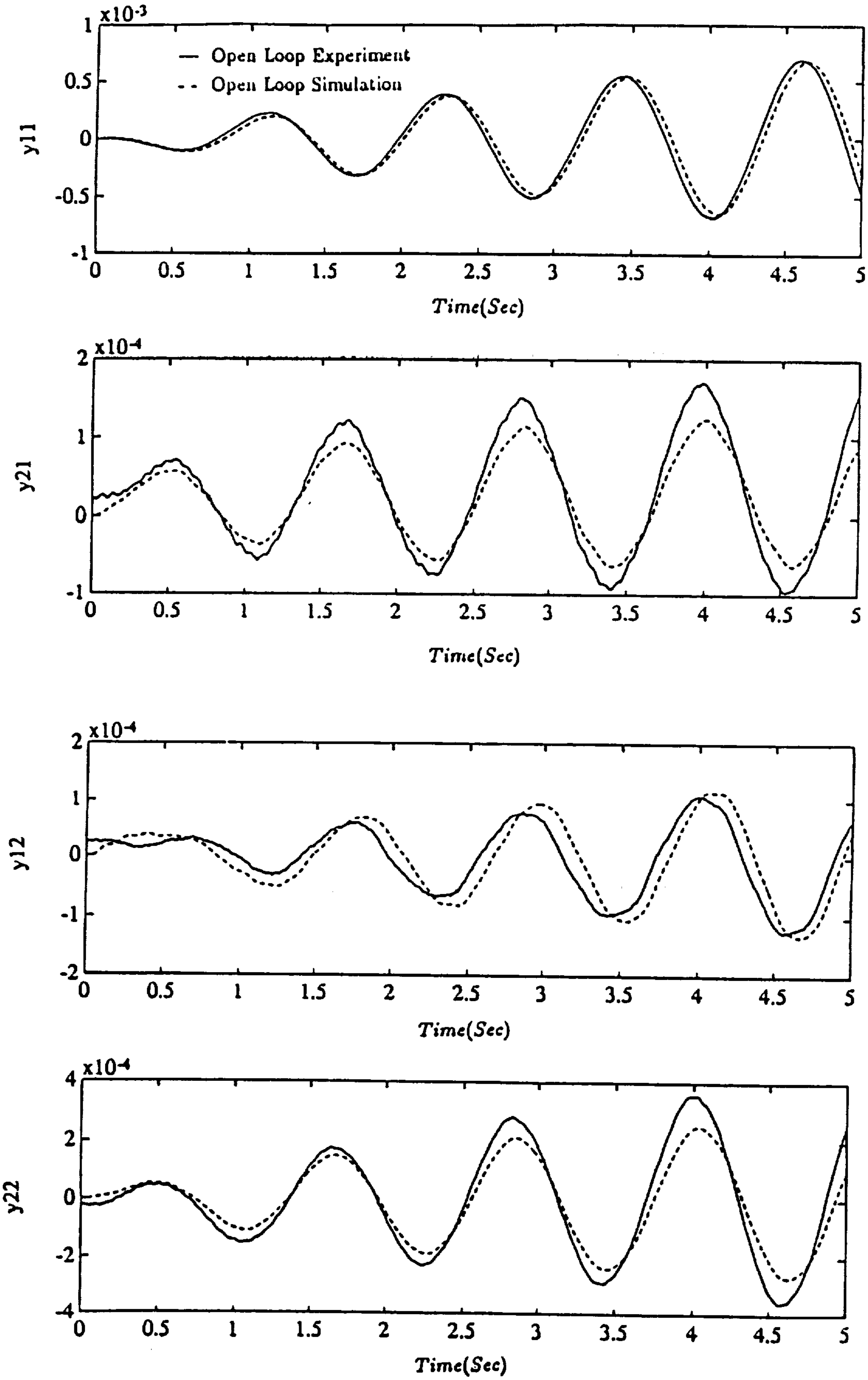

Figure 13 Experimental and Identified System Outputs

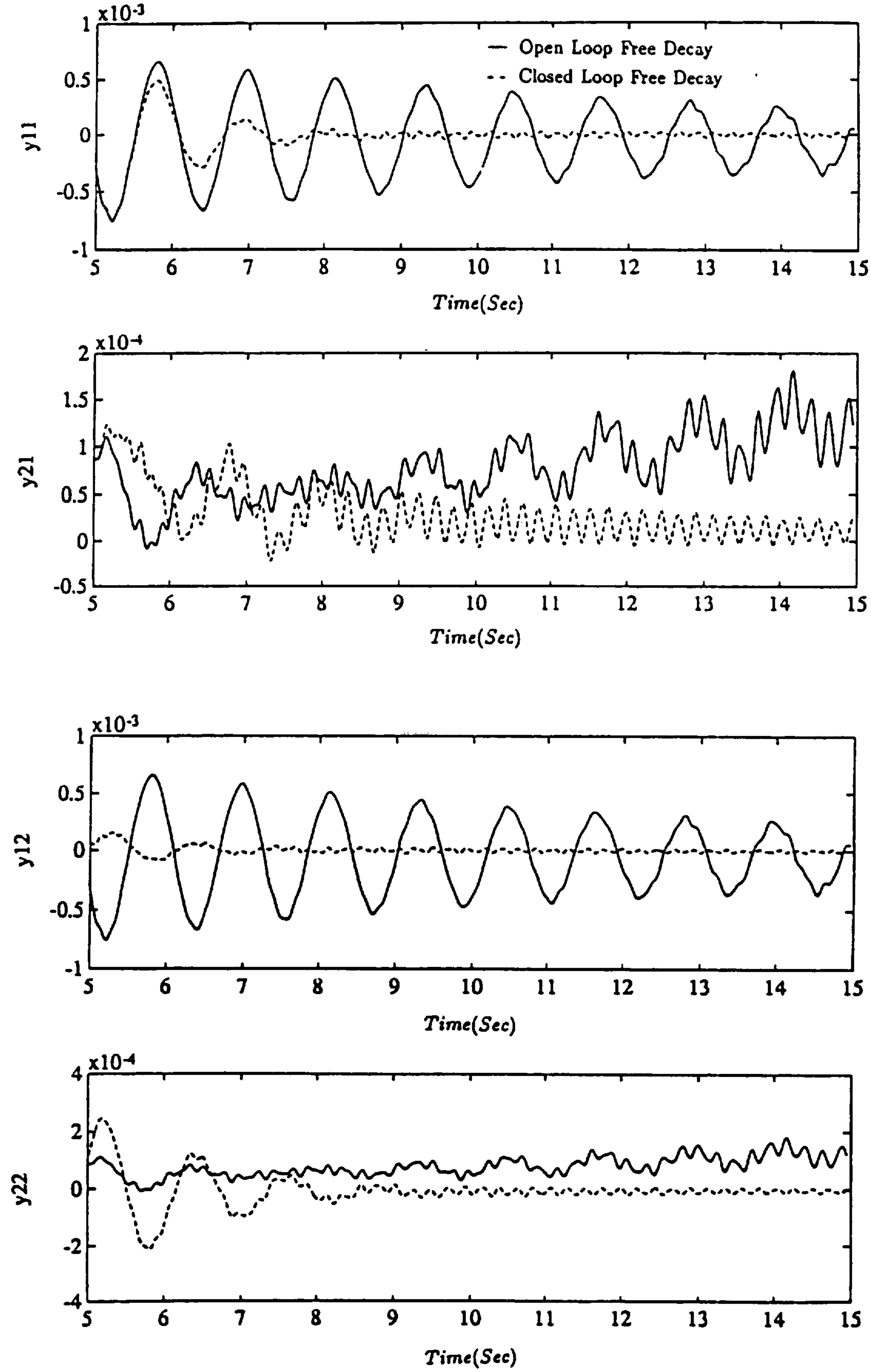

Figure 14 Closed-loop and Open-loop Free Decay Responses

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Report Documentation Page

1. Report No. NASA TM - 102695	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Integration of System Identification and Robust Controller Designs for Flexible Structures in Space		5. Report Date July 1990
		6. Performing Organization Code
7. Author(s) Jer-Nan Juang and Jiann-Shiun Lew		8. Performing Organization Report No.
		10. Work Unit No. 590-14-61-01
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A novel approach is developed using experimental data to identify a reduced-order model and its model error for a robust controller design. There are three steps involved in the approach. First, an approximately balanced model is identified using the Eigensystem Realization Algorithm, which is an identification algorithm. Second, the model error is calculated and described in frequency domain in terms of the H_∞ norm. Third, a pole placement technique in combination with a H_∞ control method is applied to design a controller for the considered system. A set of experimental data from an existing setup, namely the Mini-Mast system, is used to illustrate and verify the approach developed in this paper.

17. Key Words (Suggested by Author(s)) System Identification, Large Space Structures, Robust Controller Design, Closed-Loop Eigenvalue Assignment	18. Distribution Statement Unclassified--Unlimited Subject Category 39

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 16	22. Price A03

NASA FORM 1626 OCT 86