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# An Improved $k$-$\epsilon$ Model for Near–Wall Turbulence and Comparison with Direct Numerical Simulations

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This paper presents an improved $k$-$\epsilon$ model for low -Reynolds number turbulence near a wall. The near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the turbulent kinetic energy equation is analyzed. Based on this analysis, a modified eddy viscosity model, having correct near-wall behavior, is suggested, and a model for the pressure transport term in the $k$-equation is proposed. In addition, a modeled dissipation rate equation is reformulated. We use fully developed channel flows for model testing. The calculations using various $k$-$\epsilon$ models are compared with direct numerical simulations. The results show that the present $k$-$\epsilon$ model performs well in predicting the behavior of near-wall turbulence. Significant improvement over previous $k$-$\epsilon$ models is obtained.

## 1. Introduction

The $k$-$\epsilon$ model is one of the most widely utilized turbulence models for various turbulent flows of engineering interest. Patel et al.[1] extensively reviewed two-equation models which can be integrated down to the wall. One of their conclusions was that the damping functions used in turbulence models, especially the one for the eddy viscosity, need to be further modified in order to improve model performance. In fact, as we shall see later, many existing $k$-$\epsilon$ models do not provide correct near-wall behavior of the eddy viscosity. In addition, an asymptotic analysis of near-wall behavior of turbulence shows that the pressure transport term in the turbulent kinetic energy equation is much larger than the turbulent transport term. This near-wall behavior is also observed in direct numerical simulation of fully developed channel flows (Mansour et al.[2], Kim et al.[3]). However, in

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existing $k$-$\epsilon$ models, this pressure transport term is either ignored or included in a turbulent transport model.

In this paper, we will first analyze the near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the $k$-equation, and then, in sections 2 and 3, propose models according to their near-wall behavior. In addition, in section 3, we reformulate a model equation for the dissipation rate following an argument similar to that of Lumley.[4] In section 4, we present the results of the calculations with various $k$-$\epsilon$ models and compare them with direct numerical simulations.

To analyze the near-wall asymptotic behavior of the eddy viscosity and other turbulent quantities, we expand the fluctuating velocities and pressure in Taylor series about the wall distance as follows:

$$\begin{aligned} u_1 &= b_1 y + c_1 y^2 + d_1 y^3 + \dots \\ u_2 &= c_2 y^2 + d_2 y^3 + \dots \\ u_3 &= b_3 y + c_3 y^2 + d_3 y^3 + \dots \\ p &= a_p + b_p y + c_p y^2 + d_p y^3 + \dots \end{aligned} \qquad (1)$$

where $u_1$, $u_2$ and $u_3$ are the velocity components in the direction of $x$, $y$ and $z$. $y$ is normal to the wall and $x, z$ are parallel to the wall. The coefficients $a_p, b_1, c_2, \dots$ are functions of $x, z$ and $t$, where $t$ is the time. Using the continuity and momentum equations, Mansour et al.[2] showed the following relations between the coefficients,

$$\begin{aligned} 2c_2 &= -(b_{1,1} + b_{3,3}) \\ a_{p,1} &= 2\nu c_1 \\ a_{p,3} &= 2\nu c_3 \end{aligned} \qquad (2)$$

where $()_{,i}$ represents a derevative with respect to $x_i$. The eddy viscosity is usually defined as

$$-\langle u_i u_j \rangle = \nu_T (U_{i,j} + U_{j,i}) - \frac{2}{3} k \delta_{ij} \qquad (3)$$

where $\langle\ \rangle$ stands for ensemble average and $k \equiv \langle u_i u_i \rangle/2$ is the turbulent kinetic energy. For plane shear flows, we can write from Eq. (3)

$$\nu_T = \frac{-\langle uv \rangle}{\partial U/\partial y} \tag{4}$$

and using Eq. (1), we obtain the near-wall asymptotic behavior of the eddy viscosity:

$$\nu_T \frac{\partial U}{\partial y} = -\langle b_1 c_2 \rangle y^3 + (-\langle b_1 d_2 + c_1 c_2 \rangle + 2\langle b_1 c_2 \rangle \langle c_1 \rangle) y^4 + O(y^5) \tag{5}$$

That is, near the wall $\nu_T$ is $O(y^3)$, because $\partial U/\partial y$ is usually O(1). Any reasonable eddy viscosity model should have this near-wall behavior. We shall see later that many existing models do not have this near-wall behavior. For later use, let us examine also the near-wall asymptotic behavior of the turbulent kinetic energy $k$ and its dissipation rate $\epsilon \equiv \nu\langle u_{i,j} u_{i,j} \rangle$. Using Eq. (1), we obtain the following relations for the $k$ and $\epsilon$:

$$k = \frac{\langle b_1^2 \rangle + \langle b_3^2 \rangle}{2} y^2 + (\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle) y^3 + O(y^4) \tag{6}$$

$$\frac{\epsilon}{\nu} = \langle b_1^2 \rangle + \langle b_3^2 \rangle + 4(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle) y + O(y^2) \tag{7}$$

In addition, the pressure transport term in the $k$-equation, $\Pi \equiv -\frac{1}{\rho}\langle u_i p_{,i} \rangle$, becomes (using Eq.s (1) and (2))

$$\Pi = -2\nu(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle) y + O(y^2) \tag{8}$$

while the turbulent transport term in the $k$-equation, $-\langle k u_i \rangle_{,i}$, can be estimated as $O(y^3)$. Therefore, the pressure transport term is much larger than the turbulent transport term near the wall.

## 2. Eddy viscosity model

In this section, we will propose a model for the eddy viscosity using its near-wall behavior described in the previous section. The eddy viscosity model can be in general written as

$$\nu_T = c\, u' \ell' \tag{9}$$

where $u'$ and $\ell'$ are the turbulent characteristic velocity and length scale, respectively. Depending on the way to specify velocity and length scales, the eddy viscosity model can be a mixing length model, a one-equation ($k$) model or a two-equation (e.g. $k$-$\epsilon$) model. For example, in plane shear flows, Prandtl's mixing length model specifies the characteristic velocity with $\ell' \partial U / \partial y$. For near wall turbulence, the Van Driest mixing length model further damps the length scale to $y[1 - \exp(-y^+/A)]$ where $y^+ \equiv u_\tau y / \nu$ and $u_\tau$ is the friction velocity. For more advanced mixing length models, see Baldwin and Lomax[5], and King[6]. One-equation ($k$) models use $k^{1/2}$ as the characteristic velocity, which is determined by the turbulent kinetic energy equation. In two-equation models, e.g. $k$-$\epsilon$ models, the length scale is usually specified by $k^{3/2}/\epsilon$, where $\epsilon$ is determined by a dissipation rate equation. In this paper we will concentrate on two-equation models, which are usually written as

$$\nu_T = C_\mu f_\mu \frac{k^2}{\epsilon} \tag{10}$$

where $C_\mu = 0.09$, and $f_\mu$ is a damping function. The form of the damping function is quite critical to the prediction of the mean flow field. In fact, the mean velocity field mainly depends on the eddy viscosity model. Therefore it is important for an eddy viscosity model to have a proper near-wall behavior. We have examined the near-wall behavior of eddy viscosity models based on various $k$-$\epsilon$ model equations. The results are listed in Table 1, which shows that some of the $k$-$\epsilon$ models do not have the correct near-wall behavior of the eddy viscosity: $\nu_t = O(y^3)$.

The quantity $k^{3/2}/\epsilon$ is usually considered as a characteristic length scale (or the size) of the energy containing eddies, $\ell'$. One expects that near the wall, the size of these eddies should be order of the wall distance $O(y)$. However, Eq.s (6) and (7) show that $k^{3/2}/\epsilon$ is $O(y^3)$. Hence, $k^{3/2}/\epsilon$ is not an appropriate quantity to represent the length scale of the large eddies near the wall. However, we can define a variable $\tilde{\epsilon}$ as

$$\tilde{\epsilon} = \epsilon - \nu \frac{\partial k / \partial x_i \, \partial k / \partial x_i}{2k} \tag{11}$$

which has a nice property: $\tilde{\epsilon}$ approaches $\epsilon$ away from the wall and is $O(y^2)$ near the wall, according to the Eqs. (6) and (7). Therefore, $k^{3/2}/\tilde{\epsilon}$ is $O(y)$ near the wall, and is a proper quantity to characterize the length scale of the large eddies. With this length scale, the eddy viscosity should be written as

$$\nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\epsilon}} \tag{12}$$

Now in order for $\nu_T$ to have correct near-wall behavior, the damping function $f_\mu$ must be $O(y)$ near the wall and approaches 1 away from the wall. The damping functions used in various $k$-$\epsilon$ models are listed in Table 2. If we consider the presence of the wall as the main effect on the eddy viscosity, then we may assume $f_\mu$ is mainly a function of $y^+$ (defined as $u_\tau y/\nu$, where $u_\tau$ the friction velocity). The form of $f_\mu$ can be determined quite accurately if we know $\nu_T$, $k$ and $\tilde{\epsilon}$, for example, from the direct numerical simulations. We may also optimize the following simple form by numerical experiments:

$$f_\mu = 1 - \exp(-a_1 y^+ - a_2 {y^+}^2 - a_3 {y^+}^3 - a_4 {y^+}^4) \tag{13}$$

The optimal values for channel flows are $a_1 = 6 \times 10^{-3}, a_2 = 4 \times 10^{-4}, a_3 = -2.5 \times 10^{-6}, a_4 = 4 \times 10^{-9}$. This form, Eq.(13), does provide the required near-wall behavior. It can be further optimized using the direct numerical simulation data.

### 3. Modeled $k$-$\epsilon$ equation

To complete the eddy viscosity model, we need the modeled equations for the turbulent kinetic energy and its dissipation rate. This section will analyze the near-wall behavior of the $k$-equation and propose a model for the pressure transport term with a proper near-wall behavior. The equation for the dissipation rate is also reformulated with a formal invariant analysis.

Let us start with the equation for the turbulent kinetic energy,

$$k_{,t} + U_j k_{,j} = D_\nu + T + \Pi + P - \epsilon \tag{14}$$

where $D_\nu$, $T$ and $\Pi$ represent the transport of the turbulent kinetic energy due to the viscosity, turbulent velocity and pressure, respectively. $P$ and $\epsilon$ are the rate of production and dissipation of the turbulent kinetic energy. The terms on the right hand side of Eq. (14) are defined as follows:

$$\begin{aligned} D_\nu &= \nu k_{,jj} \\ T &= -\langle k u_j \rangle_{,j} \\ \Pi &= -\frac{1}{\rho}\langle p u_j \rangle_{,j} \\ P &= -\langle u_i u_j \rangle U_{i,j} \end{aligned} \tag{15}$$

Using Eq.s (1) and (2), we obtain the budget of the $k$-equation near the wall,

$$\begin{aligned} \frac{Dk}{Dt} &= O(y^3) \\ D_\nu &= \nu(\langle b_1^2 \rangle + \langle b_3^2 \rangle) + 6\nu(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y + O(y^2) \\ T &= O(y^3) \\ \Pi &= -2\nu(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y + O(y^2) \\ P &= O(y^3) \\ \epsilon &= \nu(\langle b_1^2 \rangle + \langle b_3^2 \rangle) + 4\nu(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y + O(y^2) \end{aligned} \tag{16}$$

This budget shows that the term $\Pi$ is much larger than the term $T$, and $\Pi$ cannot be neglected if we want the $k$-equation be balanced at the level of $O(y)$. However, existing models do not consider this term or simply combine it with the term $T$ and model them as

$$-\langle k u_j \rangle_{,j} = \left\{ \frac{\nu_T}{\sigma_k} k_{,j} \right\}_{,j} \tag{17}$$

In this paper, we propose a model for the pressure transport term $\Pi$ which has a similar form to that of the standard turbulent transport model, but with a coefficient to ensure its correct near -wall behavior, Eq. (8). The proposed model form of $\Pi$ is

$$\Pi = \left\{ \frac{C_0}{f_\mu[1 - \exp(-y^+)]} \frac{\nu_T}{\sigma_k} k_{,j} \right\}_{,j} \tag{18}$$

where $C_0$ is an adjustable model constant. Its optimal value for channel flows is 0.05. In addition, in some existing $k$-$\epsilon$ models, it is assumed that $\epsilon = 0$ at the wall. In that case, in order to balance the term $D_\nu$, a nonzero artificial term D must be added to the $k$-equation. The form of $D$ for various $k$-$\epsilon$ models is listed in Table 3. Finally, the modeled $k$-equation becomes

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_j}] + \Pi + \nu_T(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j} - \epsilon + D \tag{19}$$

In the present model, $D = 0$, since $\epsilon$ is nonzero at the wall.

The exact dissipation rate equation is

$$\epsilon_{,t} + U_j \epsilon_{,j} = D_\nu^\epsilon + T^\epsilon + \Pi^\epsilon + PMD \tag{20}$$

where $D_\nu^\epsilon$, $T^\epsilon$ and $\Pi^\epsilon$ represent the diffusion rate of the dissipation rate due to the viscosity, turbulent velocity and pressure, respectively. $PMD$ stands for the entire mechanism of the production and destruction of the dissipation rate $\epsilon$. The terms on the right hand side of the above equation are identified as follows:

$$\begin{aligned} D_\nu^\epsilon &= \nu \epsilon_{,jj} \\ T^\epsilon &= -\nu \langle u_{i,k} u_{i,k} u_j \rangle_{,j} \\ \Pi^\epsilon &= -\frac{2\nu}{\rho}\langle p_{,k} u_{j,k} \rangle_{,j} \\ PMD &= -2\nu(\langle u_{i,k} u_{j,k} \rangle + \langle u_{k,i} u_{k,j} \rangle) U_{i,j} - 2\nu \langle u_j u_{i,k} \rangle U_{i,kj} \\ &\quad - 2\nu \langle u_{i,k} u_{j,k} u_{i,j} \rangle - 2\nu^2 \langle u_{i,kj} u_{i,kj} \rangle \end{aligned} \tag{21}$$

The term $\Pi^\epsilon$ is usually neglected or combined with the term $T^\epsilon$ and modeled as

$$T^\epsilon = \frac{\nu_T}{\sigma_\epsilon} \epsilon_{,jj} \tag{22}$$

To model $PMD$, we define $\Psi$ by

$$PMD = -\frac{\epsilon \tilde{\epsilon}}{k} \Psi$$

At the level of the $k$-$\epsilon$ model, we assume $\Psi$ is a function of $\nu$, $\nu_T$, k, $\epsilon$, $\tilde{\epsilon}$, $U_{i,j}$ and $U_{i,jk}$. Since $\Psi$ is an invariant, it must be a function of the invariants that can be constructed from these quantities. Therefore we can write

$$\Psi = \Psi(R_t, \frac{\nu_T U_{i,j} U_{i,j}}{\tilde{\epsilon}}, \nu\nu_T U_{i,jk} U_{i,jk} \frac{k}{\epsilon\tilde{\epsilon}})$$

where $R_t$ is the turbulent Reynolds number $k^2/\nu\epsilon$. We expand $\Psi$ in a Taylor series about $\nu_T U_{i,j} U_{i,j}/\tilde{\epsilon}$ and $\nu\nu_T U_{i,jk} U_{i,jk} k/\epsilon\tilde{\epsilon}$, and take only the linear terms. We obtain

$$\Psi = \psi_0 + \psi_1 \frac{\nu_T U_{i,j} U_{i,j}}{\tilde{\epsilon}} + \psi_2 \nu\nu_T U_{i,jk} U_{i,jk} \frac{k}{\epsilon\tilde{\epsilon}} \tag{23}$$

where the coefficients $\psi_0$, $\psi_1$ and $\psi_2$ are in general functions of $R_t$. Finally, the modeled dissipation rate equation becomes

$$\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j}[(\nu + \frac{\nu_T}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial x_j}] + C_1 f_1 \frac{\epsilon}{k} \nu_T U_{i,j} U_{i,j} - C_2 f_2 \frac{\epsilon\tilde{\epsilon}}{k} + E \tag{24}$$

where, $C_1$ and $C_2$ are the model constants, and $f_1$ and $f_2$ are functions of $R_t$. The term E in the present model comes from the last term in Eq. (23):

$$E = \nu\nu_T U_{i,jk} U_{i,jk} \tag{25}$$

where we have taken $\psi_2 = -1$. The form of $E$ and $C_1$, $C_2$, $f_1$ and $f_2$ for various $k$-$\epsilon$ models are listed in Tables 3 and 4.

## 4. Fully developed turbulent channel flow

Flows which have self-similar solutions are particularly useful for accurate model testing, because their solutions are independent of initial conditions. Therefore, we do not need to accurately choose the initial conditions for the $k$ and $\epsilon$. In this paper, we use a fully developed channel flow for model testing. This flow is the simplest wall bounded turbulent shear flow with a self-similar solution. However, the complex features of the turbulence, for example, the effect of the wall on shear turbulence, are remained. In addition, the

$k$-$\epsilon$ equations for the channel flow are exactly one-dimensional steady problems, numerical calculations will be easy and accurate. Recently, the measurements[7] well verified the direct numerical simulation of a fully developed channel flow.[3] These data are available for comparison with model predictions.

Let $h$ be the half width of the channel, $u_\tau$ the friction velocity and $Re_\tau$ the Reynolds number defined as $u_\tau h/\nu$. Let $U, k, \epsilon, \nu_T$ and $y$ be the non-dimensional quantities, normalized by $u_\tau, u_\tau^2, u_\tau^3/h, \nu$ and $h$, respectively. The modeled equations for the channel flow become

$$\frac{dU}{dy} = Re_\tau \frac{1-y}{1+\nu_T} \tag{26}$$

$$\frac{d}{dy}\{\frac{1}{Re_\tau}[1+(1+C)\frac{\nu_T}{\sigma_k}]\frac{dk}{dy}\} + \nu_T(\frac{dU}{dy})^2\frac{1}{Re_\tau} - \epsilon = 0 \tag{27}$$

$$\frac{d}{dy}\{\frac{1}{Re_\tau}(1+\frac{\nu_T}{\sigma_\epsilon})\frac{d\epsilon}{dy}\} + C_1\frac{\epsilon}{k}\nu_T(\frac{dU}{dy})^2\frac{1}{Re_\tau} - C_2 f_2\frac{\epsilon\tilde{\epsilon}}{k} + \nu_T(\frac{d^2U}{dy^2})^2\frac{1}{Re_\tau^2} = 0 \tag{28}$$

where

$$\begin{aligned}
\nu_T &= C_\mu f_\mu Re_\tau \frac{k^2}{\tilde{\epsilon}} \\
\tilde{\epsilon} &= \epsilon - \frac{(\frac{dk}{dy})^2}{2k\ Re_\tau} \\
f_\mu &= \text{equation}(13) \\
f_2 &= 1 - \frac{0.4}{1.8}\exp[-(\frac{Re_\tau k^2}{6\epsilon})^2] \\
C &= \frac{C_0}{f_\mu[1-\exp(-y^+)]}
\end{aligned} \tag{29}$$

The boundary conditions are simple. At the wall,

$$\begin{aligned}
U &= k = 0 \\
\epsilon &= \frac{(\frac{dk}{dy})^2}{2k\ Re_\tau}
\end{aligned} \tag{30}$$

and at the center of the channel,

$$\frac{dk}{dy} = \frac{d\epsilon}{dy} = 0 \tag{31}$$

The numerical solutions with various $k$-$\epsilon$ models are obtained using Patankar and Spalding's method.[8] We use 65 nodes stretched in the half width of the channel in the same way as done in the direct numerical simulations:

$$y_i = 1 - cos\left(\frac{(i-1)\pi}{2(N-1)}\right) \quad i = 1, 2, ...65 \tag{32}$$

The main results from different $k$-$\epsilon$ models for $Re_\tau = 180$ are plotted in figures 1 – 6. All the calculations are compared with the direct numerical simulation data. Figure 1 shows that the model of Jones and Launder[9] (JL) underpredicts the mean velocity as well as the peak value of the turbulent kinetic energy. In figure 2, Chien's model[10] performs better than the JL model, but it overpredicts the mean velocity near the center of the channel as well as the turbulent kinetic energy. In these two models, $\epsilon = 0$ at the wall is used as the boundary condition, so the dissipation rate near the wall cannot be correctly predicted. Lam and Bremhorst[11] use a nonzero boundary condition for $\epsilon$ and have made some improvement for the mean velocity and turbulent kinetic energy compared with the results of the JL model, see figure 3. However, the mean velocity is still underpredicted near the center of the channel, and the dissipation rate near the wall is not correct. In figure 4, the model of Nagano and Hishida[12] presents a very good prediction for the mean velocity and shear stress, while the peak value of $k$ is underpredicted. Their main modification to the JL model is a change in the damping function $f_\mu$ and the form of $E$. A zero dissipation rate at the wall is used. Figure 5 presents the results of the present $k$-$\epsilon$ model, which shows the improvement in the prediction of all quantities, including the dissipation rate. The eddy viscosity profiles for various $k$-$\epsilon$ models are shown in figure 6. Overall, the present eddy viscosity model has better behavior than others. In figure 7, we show some results of present model for several high Reynolds number flows.

From the model testing, we conclude that the present $k$ -$\epsilon$ model has made a significant improvement over previous $k$-$\epsilon$ models according to the comparison with the direct numerical simulations. We find that the improvement is mainly due to the modified eddy viscosity model and the model of the pressure transport term in the $k$-equation. The proposed dissipation rate equation also shows a better near-wall behavior than the previous ones as shown in figures 1 – 5.

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**References**


1. Patel, V.C., Rodi, W. and Scheuerer, G., "Turbulence models for near–wall and low–Reynolds–number flows: A Review," *AIAA Journal*, 23, 1985, pp. 1308–1319.

2. Mansour, N.N., Kim,J. and Moin. P., "Reynolds–Stress and Dissipation Rate Budgets in a Turbulent Channel Flow," *J. Fluid Mech.*, Vol. 194, 1988, pp. 15–44.

3. Kim, J., Moin, P. and Moser, Robert., "Turbulent statistics in fully developed channel flow at low Reynolds number," *J. Fluid Mech.*, 177, 1987, pp. 133–166.

4. Lumley, J.L., "Computational modeling of turbulent flows," in *Advances in Applied Mechanics*, Vol. 18, 1978, Academic Press, New York, pp.123–176.

5. Baldwin, B.S. and Lomax H., "Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows,"*AIAA Paper* no. 78-257, Huntsville. Ala., 1978.

6. King, L.S., "A comparison of turbulence closure models for transonic flows about airfoils," *AIAA Paper* no. 87-0418, Reno, Nevada, 1987.

7. Nishino, K. and Kasagi, N., "Turbulent statistics measurement in a two dimensional channel flow using a three-dimensional particle tracking velocimeter," *Seventh Symposium on Turbulent Shear Flows*, Stanford University, 1989.

8. Spalding, D.B., GENMIX: "A General Computer Program for Two-dimensional Parabolic Phenomena," 1977, Pergamon Press.

9. Jones, W.P. and Launder, B.E., "The Calculation of Low -Reynolds Number Phenomena with a Two-Equation model of Turbulence," *International Journal of Heat and Mass Transfer*, Vol. 16, 1973, pp. 1119-1130.

10. Chien, K.-Y., "Predictions of Channel and Boundary -Layer Flow with a Low-Reynolds-Number Turbulence Model," *AIAA Journal*, Vol. 20, Jan. 1982, pp. 33-38.

11. Lam, C.K.G. and Bremhorst, K., "A Modified Form of the $K-\epsilon$ Model for Predicting Wall Turbulence,"*ASME Transactions, Journal of Fluids Engineering*, Vol. 103, Sept. 1981, pp.456-460.

12. Nagano, Y. and Hishida, M., "Improved Form of the $K-\epsilon$ Model for Wall Turbulent Shear Flows,"*ASME Transaction, Journal of Fluids Engineering*, Vol. 109, June 1987.

**Table 1 Eddy viscosity and boundary condition for $\epsilon$ in various $k$-$\epsilon$ models**

| Model | $\nu_T$ | BC: $\epsilon_w$ |
|---|---|---|
| JL | $O(y^3)$ | 0 |
| Reynolds | $O(y^5)$ | $\nu\frac{\partial^2 k}{\partial y^2}$ |
| LB | $O(y^4)$ | $\nu\frac{\partial^2 k}{\partial y^2}$ |
| Chien | $O(y^3)$ | 0 |
| NH | $O(y^4)$ | 0 |
| Present | $O(y^3)$ | $\nu\frac{\partial^2 k}{\partial y^2}$ |

**Table 2 Damping functions used in various $k$-$\epsilon$ models**

| Model | $f_\mu$ | $f_1$ | $f_2$ |
|---|---|---|---|
| JL | $\exp(\frac{-2.5}{1+R_t/50})$ | 1.0 | $1-.3\exp(-R_t^2)$ |
| Reynolds | $1-\exp(-.0198R_k)$ | 1.0 | $[1-.3\exp(-R_t^2/9)]f_\mu$ |
| LB | $[1-\exp(-.0165R_k)]^2$ $\times(1+\frac{20.5}{R_t})$ | $1+(\frac{.05}{f_\mu})^3$ | $1-\exp(-R_t^2)$ |
| Chien | $1-\exp(-.0115y^+)$ | 1.0 | $1-.22\exp(-R_t^2/36)$ |
| NH | $[1-\exp(-y^+/26.5)]^2$ | 1.0 | $1-.3\exp(-R_t^2)$ |
| Present | Eq. (13) | 1.0 | $1-.22\exp(-R_t^2/36)$ |

**Table 3 Model terms in various $k$-$\epsilon$ models**

| Model | $\Pi$ | $D$ | $E$ |
|---|---|---|---|
| JL | 0 | $-2\nu\left(\frac{\partial\sqrt{K}}{\partial y}\right)^2$ | $2\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$ |
| Reynolds | 0 | 0 | 0 |
| LB | 0 | 0 | 0 |
| Chien | 0 | $-\frac{2\nu K}{y^2}$ | $-\frac{2\nu\epsilon}{y^2}\exp(-.5y^+)$ |
| NH | 0 | $-2\nu\left(\frac{\partial\sqrt{K}}{\partial y}\right)^2$ | $\nu\nu_T(1-f_\mu)\left(\frac{\partial^2 U}{\partial y^2}\right)^2$ |
| Present | Eq. (18) | 0 | $\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$ |

**Table 4 Model constants in various $k$-$\epsilon$ models**

| Model | $C_\mu$ | $C_1$ | $C_2$ | $\sigma_k$ | $\sigma_\epsilon$ |
|---|---|---|---|---|---|
| JL | .09 | 1.45 | 2.0 | 1.0 | 1.3 |
| Reynolds | .084 | 1.0 | 1.83 | 1.69 | 1.3 |
| LB | .09 | 1.44 | 1.92 | 1.0 | 1.3 |
| Chien | .09 | 1.35 | 1.8 | 1.0 | 1.3 |
| NH | .09 | 1.45 | 1.9 | 1.0 | 1.3 |
| Present | .09 | 1.45 | 2.0 | 1.3 | 1.3 |

$R_t = K^2/\nu\epsilon$, $R_k = \sqrt{K}y/\nu$, $y^+ = u_\tau y/\nu$.

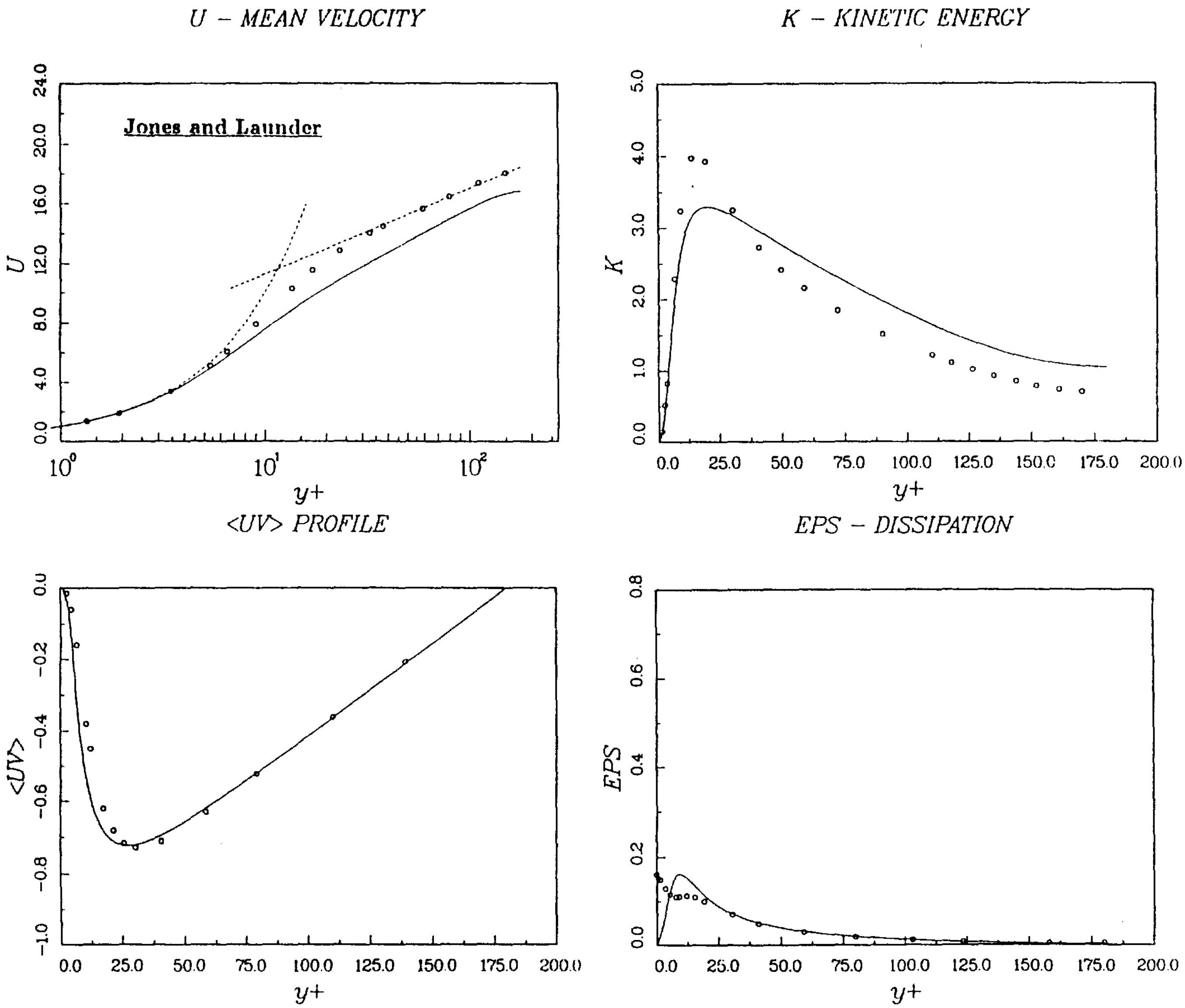


FIGURE 1. - COMPARISON OF k-ε MODEL WITH DIRECT NUMERICAL SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW. —— JONES AND LAUNDER; O DNS, KIM ET AL.

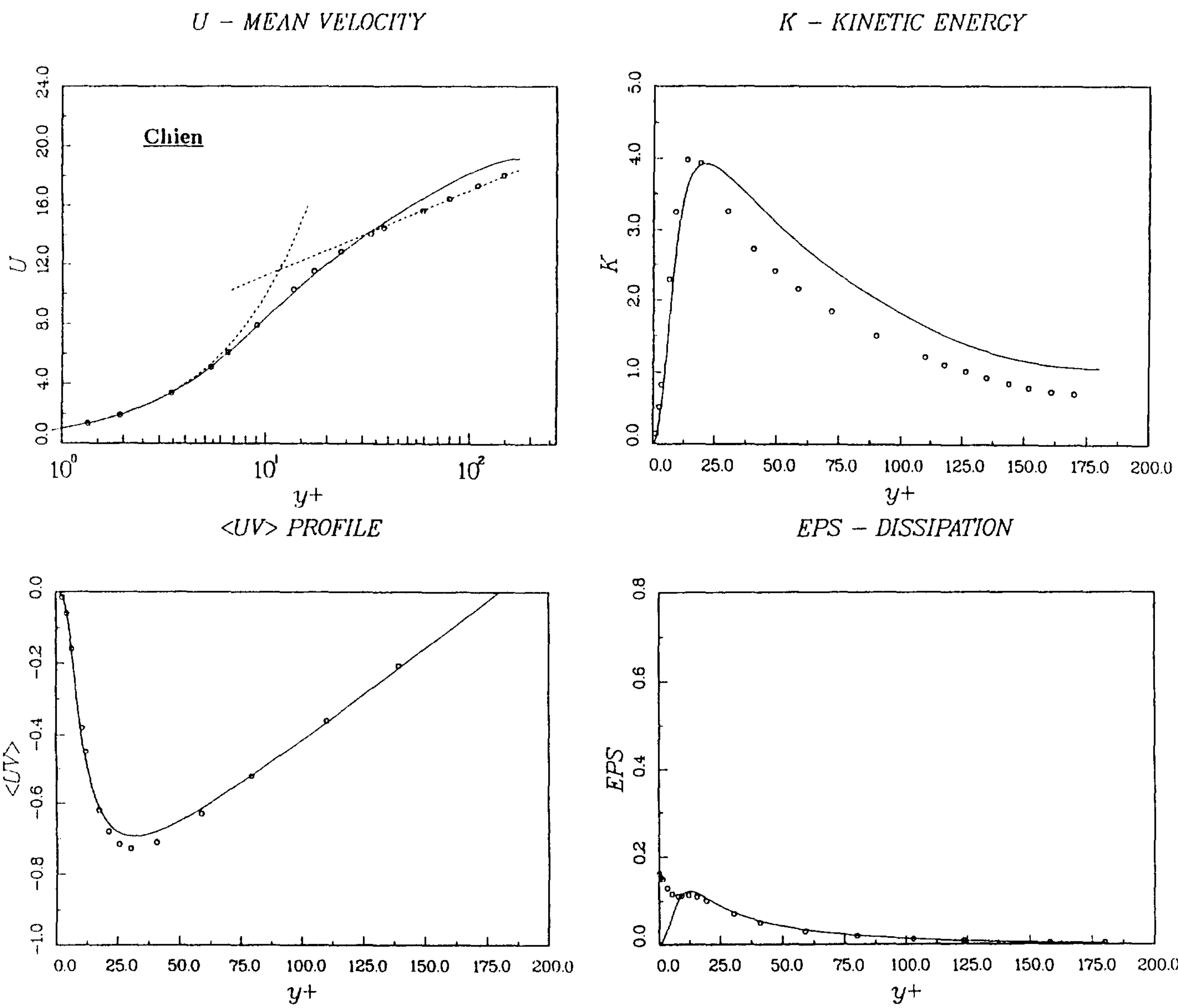


FIGURE 2. - COMPARISON OF k-ε MODEL WITH DIRECT NUMERICAL SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW. —— CHIEN; O DNS, KIM ET AL.

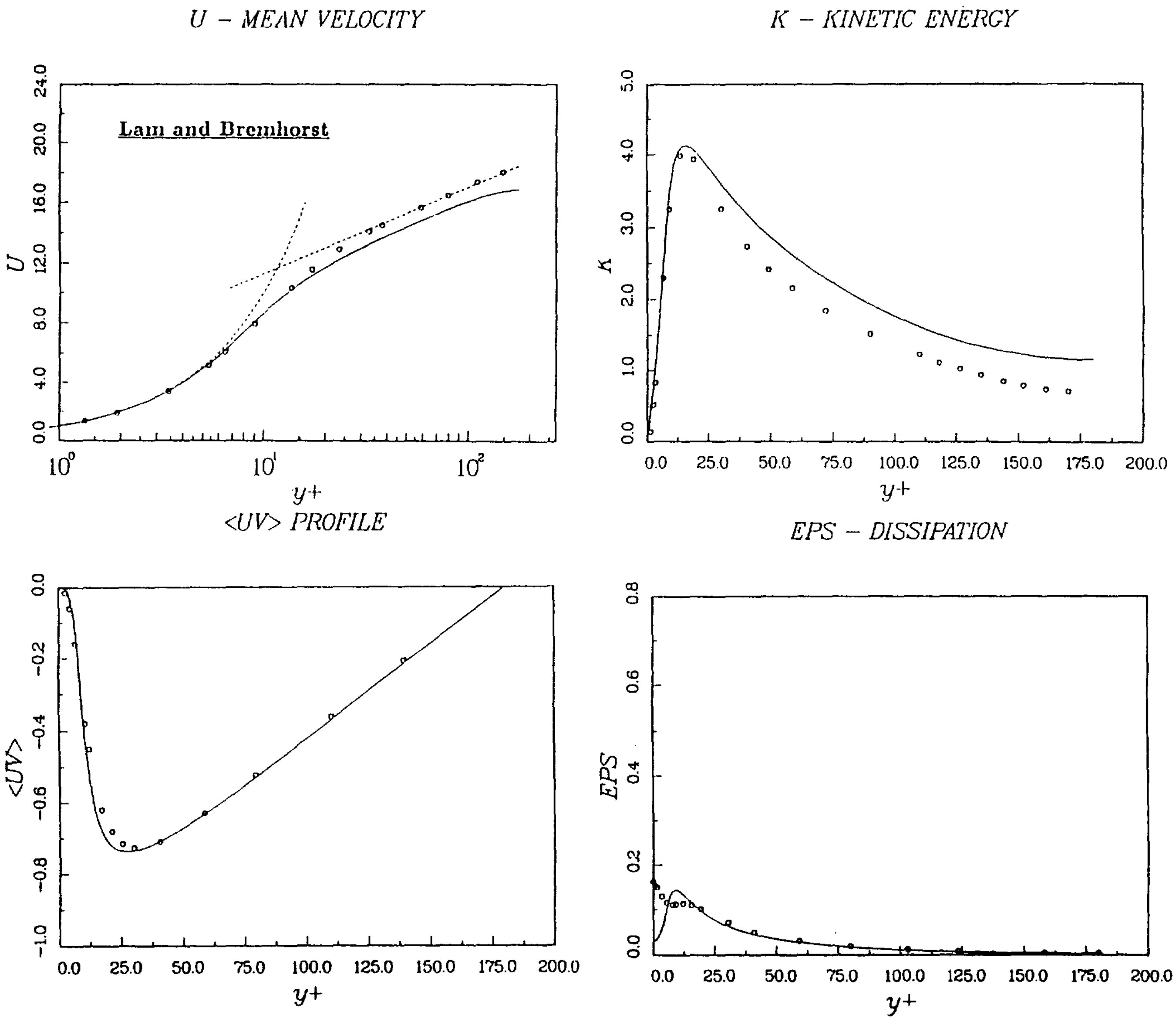


FIGURE 3. - COMPARISON OF k-ε MODEL WITH DIRECT NUMERICAL SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW. —— LAM AND BREMHORST; O DNS, KIM ET AL.

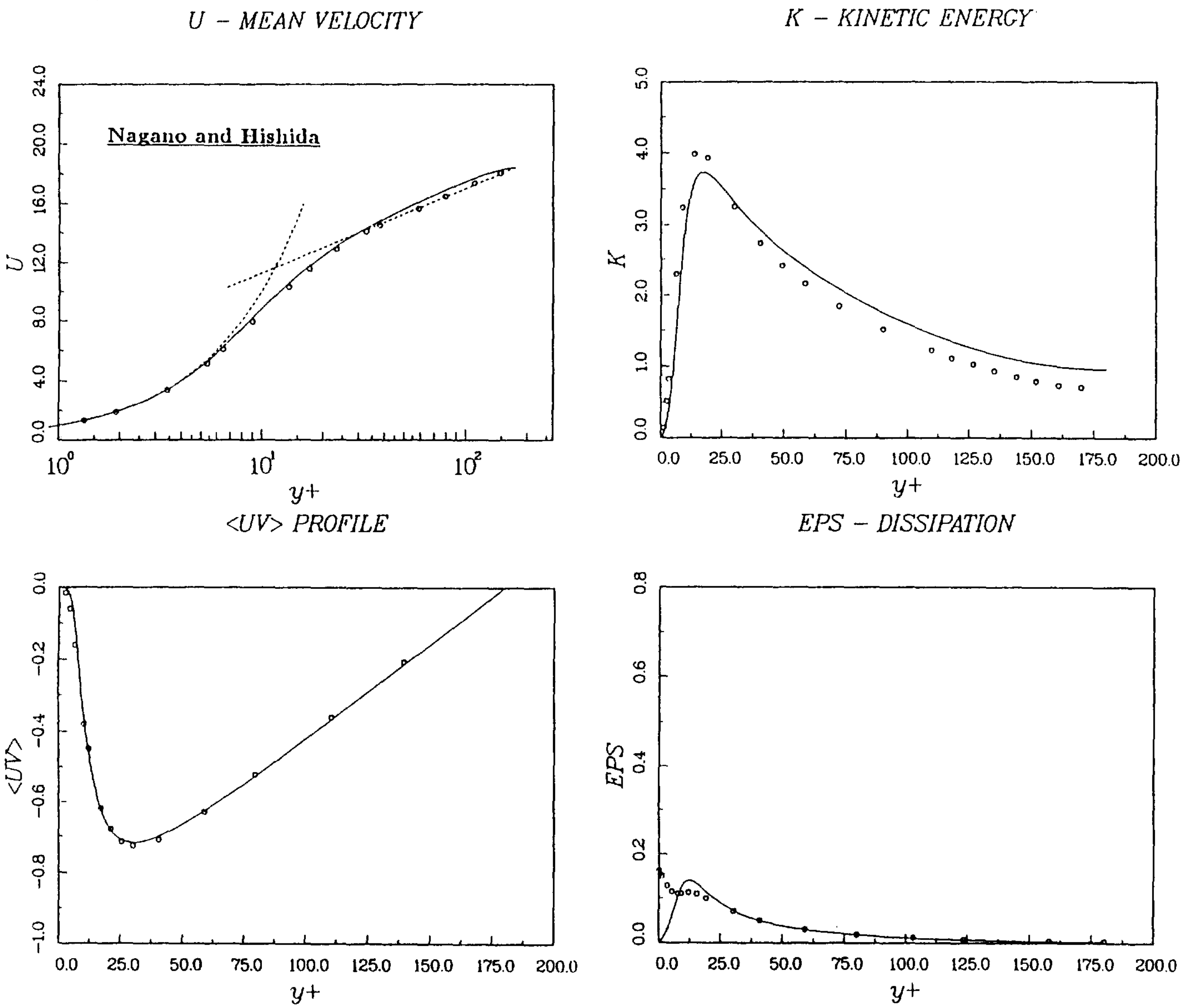


FIGURE 4. - COMPARISON OF k-ε MODEL WITH DIRECT NUMERICAL SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW. —— NAGANO AND HISHIDA; O DNS, KIM ET AL.

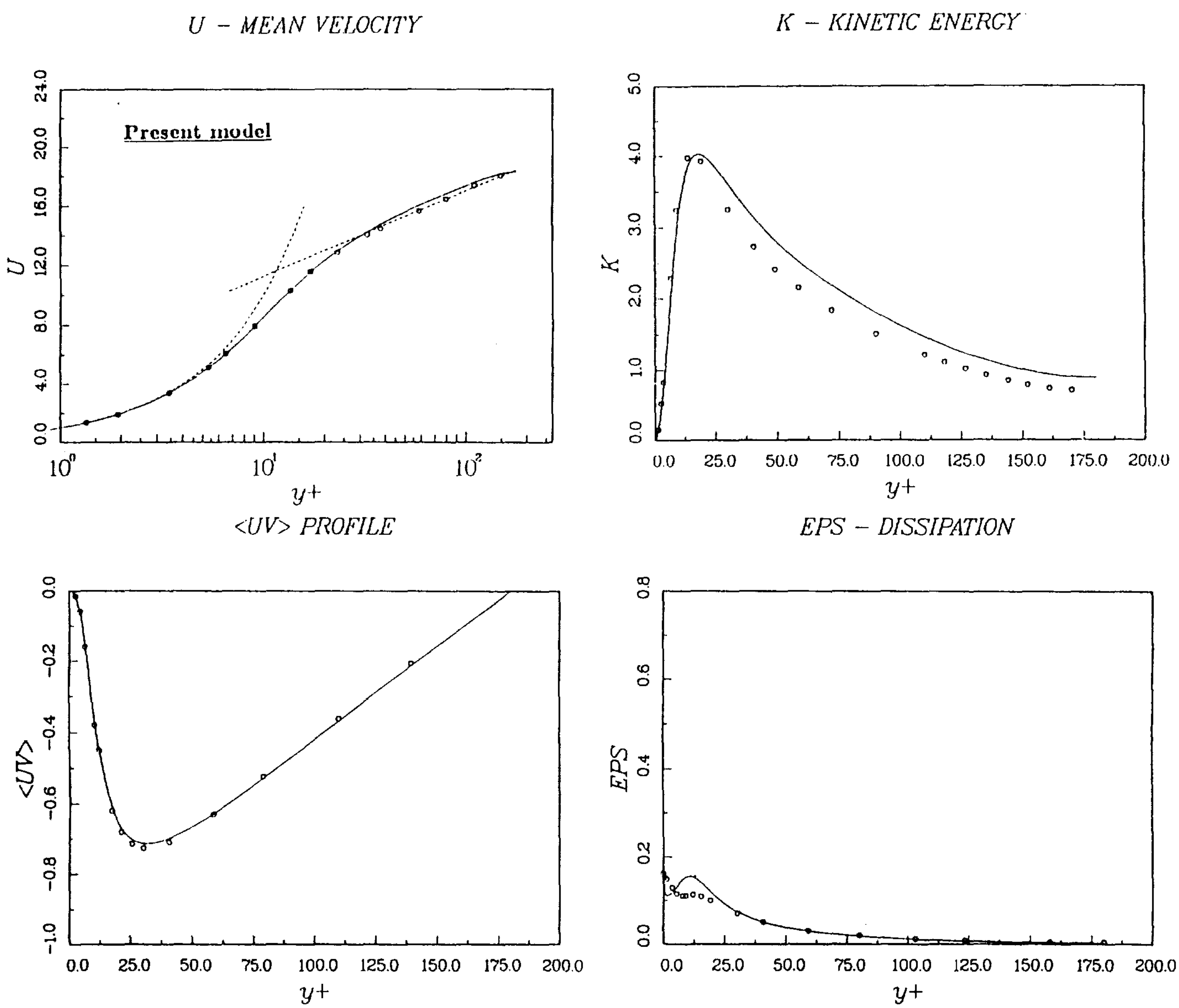


FIGURE 5. - COMPARISON OF k-ε MODEL WITH DIRECT NUMERICAL SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW. —— PRESENT MODEL; ○ DNS, KIM ET AL.

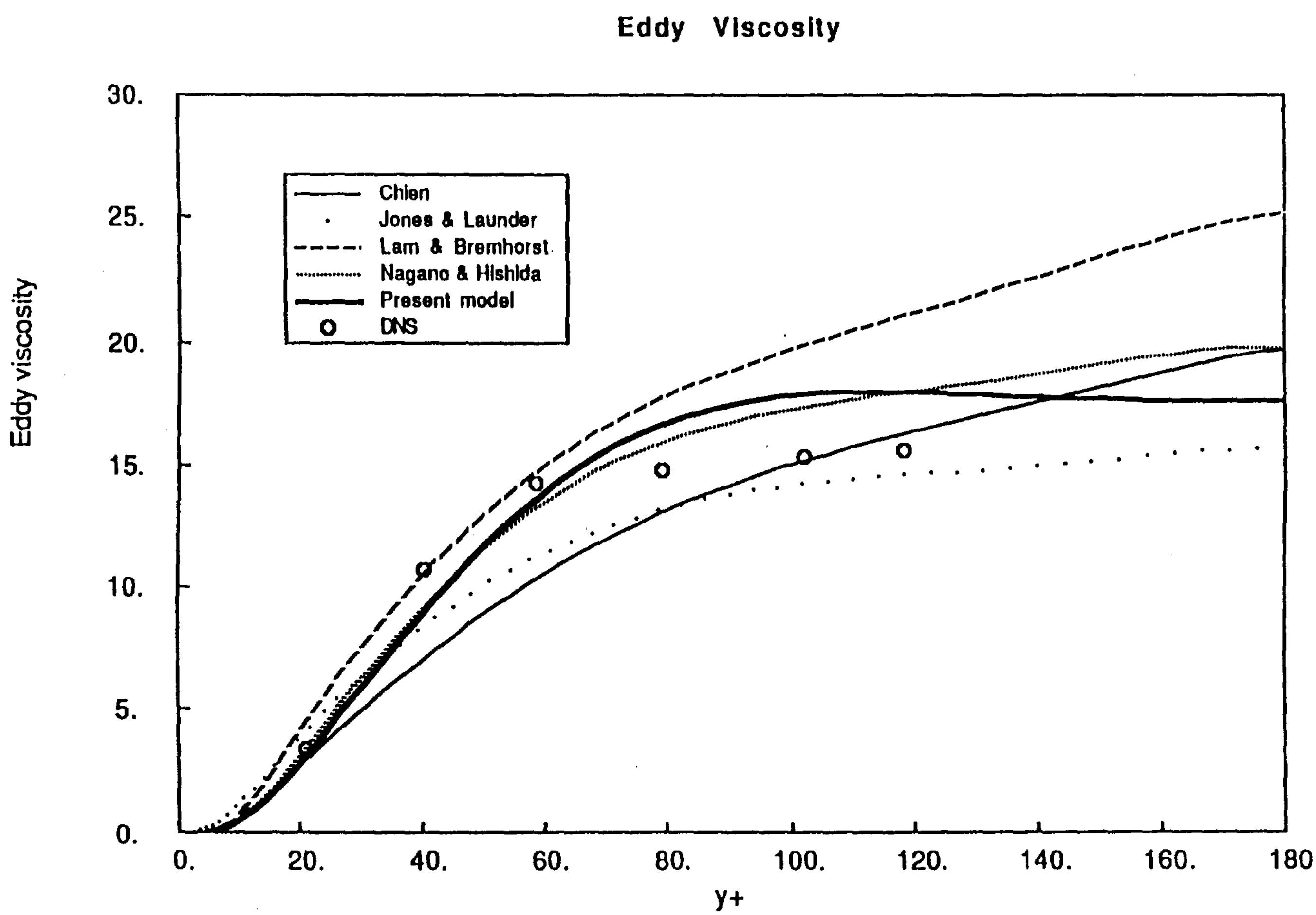


FIGURE 6. - COMPARISON OF EDDY VISCOSITIES WITH DIRECT SIMULATION IN A FULLY DEVELOPED CHANNEL FLOW.

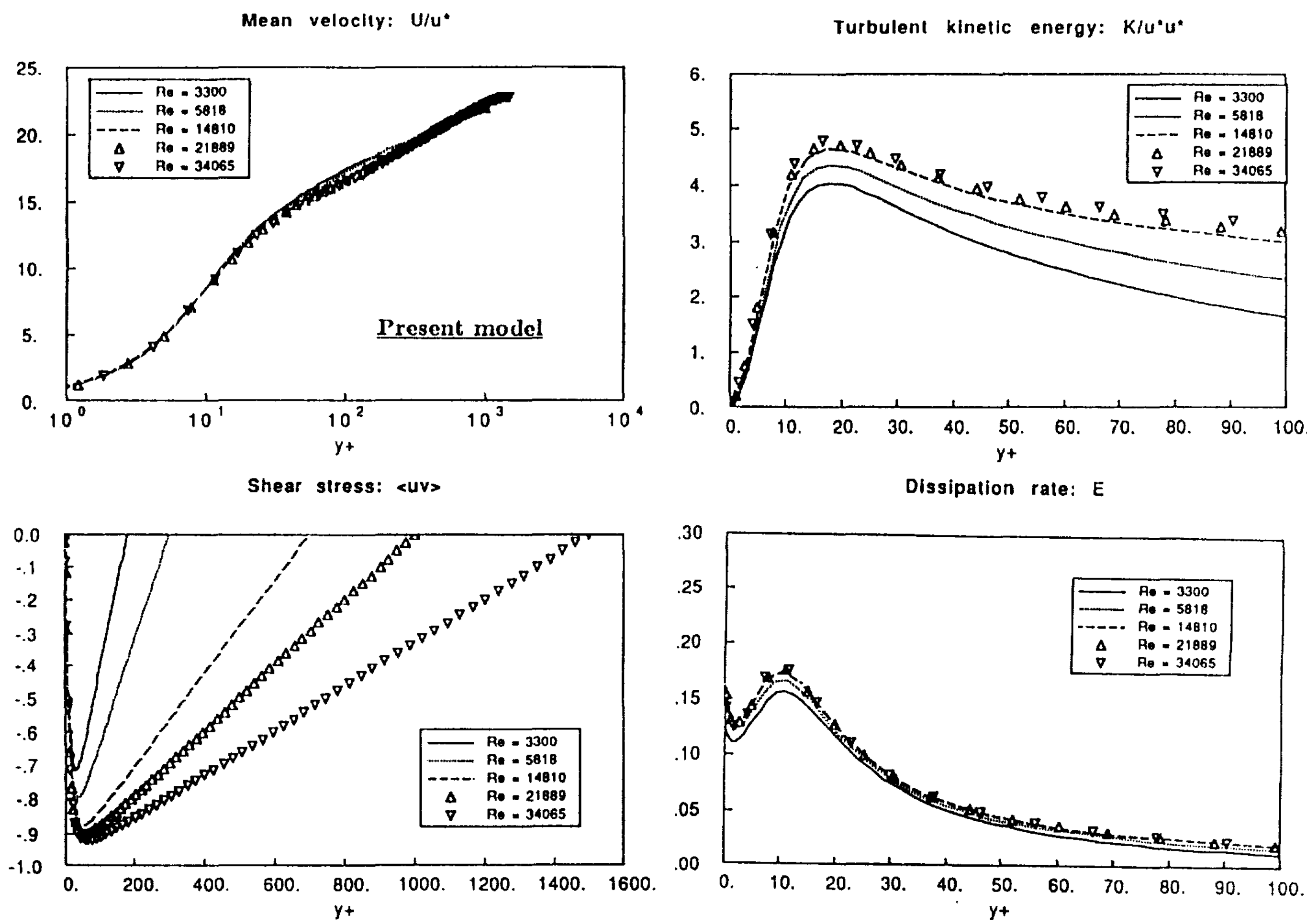


FIGURE 7. - CALCULATIONS FROM THE PRESENT k-ε MODEL FOR SOME HIGH REYNOLDS NUMBER CHANNEL FLOWS.

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16. Abstract

This paper presents an improved $\kappa$-$\epsilon$ model for low Reynolds number turbulence near a wall. The near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the turbulent kinetic energy equation is analyzed. Based on this analysis, a modified eddy viscosity model, having correct near-wall behavior, is suggested, and a model for the pressure transport term in the $\kappa$-equation is proposed. In addition, a modeled dissipation rate equation is reformulated. We use fully developed channel flows for model testing. The calculations using various $\kappa$-$\epsilon$ models are compared with direct numerical simulations. The results show that the present $\kappa$-$\epsilon$ model performs well in predicting the behavior of near-wall turbulence. Significant improvement over previous $\kappa$-$\epsilon$ models is obtained.

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