Cloud Top Entrainment Instability and Cloud Top Distributions

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The classical cloud top entrainment instability condition is commonly formulated as

$$g\ \Delta\theta_e + h\ \Delta q_t < 0 \qquad (1)$$

where g, h are slowly varying parameters taken to be constant, $\Delta\theta_e$, Δq_t the jumps in equivalent potential temperature and total water mixing ratio at the cloud top.

Plotted in θ_e, q_t coordinates the entrainment instability criterion (EI) for a collection of different atmospheric conditions appears as a highly elliptical point cluster as a result of the strong correlation between θ_e and q_t. More independent coordinates are θ_ℓ and q_t, where θ_ℓ is the liquid water potential temperature or saturation point potential temperature. In this paper we examine EI in θ_ℓ, q_t coordinates. Rather than focussing on jump conditions we will emphasize the gradients $\delta\theta_\ell/\delta q_t$. EI occurs when

$$\delta\theta_\ell/\delta q_t < (\delta\theta_\ell/\delta q_t)_{crit} \qquad (2)$$

where the critical value represents the gradient along the moist virtual adiabat. (2) is entirely consistent with (1), however the manner in which EI is shown by (2) reveals that additional parameters need to be considered in the entrainment of negatively buoyant parcels. For simplicity we neglect radiation.

Figure 1 shows a θ_ℓ, q_t plot typical of 6/29/87 (C130-data). M corresponds to the in-cloud parcel, T to the parcel above the cloud. Cloud top appears at 870 hPa. Mixtures of T and M fall along the straight line TM. This diagram can be interpreted in pressure (p) and saturation point pressure (p_*) coordinates. The p=870 hPa line shows the mixing level and whether M, or T or both are cloudy or clear. Other mixing levels can be arbitrarily defined by simply shifting the p-line up or down. If a point, say M falls below that p-line it means that at that pressure level a parcel with the coordinates M is cloudy. If it falls above that line, it is clear. The p_*-lines indicate how much lifting or lowering is needed to make M or T just saturated. For example if the cloudy boundary layer would be well mixed, it would be

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represented by M and the cloud thickness would be (937-870)= 67 hPa. In Figure 1 the moist virtual adiabat falls to the right of the mixing line MT. This means that all cloudy mixtures (below the level p=870 hPa) are denser than the unmixed cloudy parcel M.

Additional information about the mixing process is available if we draw dry virtual adiabats on this plot as in Figure 2. The moist virtual adiabat through M kinks at the p=870 hPa line at point c and then follows the dry adiabat of θ_{vu}=291.72 K. T lies on the dry virtual adiabat marked as 299.27 K. Therefore there is a difference of $\Delta\theta_v$ = 7.55 K between the dry air and the cloud top so that the cloud is statically very stable. In order to make the cloud statically unstable it would have to be raised several hundred hPa's until point c would be above the dry virtual adiabat through T. Entrainment at pressure level p=870 hPa takes parcels with p_*=566 hPa at T and mixes them with parcels at p_* = 937 hPa. During the mixing process the saturation point coordinates of the mixture slide from T along the mixing line MT towards M. Entrainment will increase p_* of the mixture until it gains the same buoyancy as the cloudy parcel M at point a, where the dry virtual adiabat θ_{vu}=291.72 K intersects the mixing line. At a the parcel is still clear. Subsequent mixing from a to b decreases the mixture buoyancy below that of the cloudy parcel. At b where $p_{*mixture}$=$p_{mixing\ level}$ the mixture becomes cloudy. Below b on the mixing line, lines of equal buoyancy are represented by lines parallel to the moist virtual adiabat. In continuing the mixing process from b to M the buoyancy will increase again until it becomes the same as that of the cloudy parcel M.

Four points can be immediately recognized from this plot: First, the densesst mixture is just cloudy (point b). Although this is well known from the literature, the actual buoyancy difference can be immediately read from this graph by drawing the dry virtual adiabat through point b (θ_{vu}=291.45 K) and computing the difference with θ_{vu} through a and c (0.27 K); Second, the fraction of dense buoyant mixtures to all possible mixtures is given by (a-T)/(T-M); Third, the buoyancy difference of cloudy mixtures with M can be increased if the unmixed dry parcel would be cooler or dryer than represented by point T; Fourth, the fraction of buoyant mixtures can be increased if the mixing level is raised (pressure lowered). Lidar cloud top data shows that over a 30 km flight leg cloud top variability of several hundred meters (several tens of hPa's) is not uncommon. The two dashed lines parallel to the p=870 hPa line indicate how variations in the mixing level changes the fraction of potentially positively buoyant parcels.

Neglecting radiation we can derive the virtual potential temperature flux from this diagram as follows: Let F_{MT} be the convective flux in (θ_e, θ_ℓ, q_t) and ω_e be the entrainment

velocity, then

$$F_{MT} = (T - M) \omega_e = (T - b) \omega_e + (b - M) \omega_e \quad (3)$$

where $T = (\theta_e, \theta_\ell, q_t)_T$
$M = (\theta_e, \theta_\ell, q_t)_M$

then the virtual potential temperature flux is defined as

$$F_{\theta v} = g_u (\theta_{\ell T} - \theta_{\ell b}) \omega_e + h_u (q_{tT} - q_{tb}) \omega_e +$$

$$g_c (\theta_{\ell b} - \theta_{\ell M}) \omega_e + h_c (q_{tb} - q_{tM}) \omega_e \quad (4)$$

where g_u, h_u, g_c, h_c are the appropriate factors for clear and cloudy conditions. From (4) it is immediately clear that although a portion of the buoyancy flux (in cloud) is indeed positive by virtue of the slope of the mixing line MT, this flux is very small in comparison to the flux necessary to create the cloudy mixture in the first place. We believe that this is a point that is commonly overlooked in EI-studies. The energy necessary to create the cloudy mixture is very much greater (proportional to the difference (299.27-291.45 K) than the energy created by mixing inside the cloud (proportional to the difference 291.72-291.45 K). It is therefore not surprising that recent studies have shown that in many cases where condition (1) or (2) was satisfied, the clouds appeared to be stable.

So far we only considered mixing at pressure level p. However a dry parcel from above the cloud is drawn into the cloud and acquires a downward speed representative of the in-cloud circulation. Let this speed be indicated by ω_T. ω_T is responsible for lowering the mixing level of the parcel, increasing the pressure level. On the other hand ω_e, the entrainment velocity is responsible for increasing the saturation point pressure of the mixture. The process of entrainment and vertical movement of the parcel is schematically represented in Figure 3. Assume that the highest cloud tops are at point b, and that at that level the dry unmixed parcels have their saturation level at T (566 hPa). As the parcel is drawn into the layer the mixing pressure level is increased from point b to say point a. If point a corresponds to the lowest cloud top we know that at point a all mixtures are cloudy. This means that in moving an unmixed parcel at point b to point a the saturation level of the mixture has increased from its unmixed value at T to the actual pressure level at a. Simple geometry on Figure 3 reveals the following constraint:

$$f = \omega_T / \omega_e < (p_a - p_b) / (p_a - p_T) \quad (5)$$

For typical cloud top variations of 15 hPa we find f < 0.05
It means that the entrainment velocity needs to be very much

higher than the translation velocity in order to get dense sinking cloud parcels. This statement means that cloud top entrainment instability is unlikely to break up a cloud in case the circulation speed near the cloud top is large; in such cases the strength of the circulation merely draws unmixed parcels into the layer that do not have the chance to become cloudy in their downward transport. Below the mean cloud base those parcels will always be less dense than the mixed layer environment.

In conclusion we have shown that a saturation point diagram can be used to investigate the details of mixing in cases where the cloud top entrainment instability criterion is satisfied. We find that for typical situations found during FIRE, where the EI condition is satisfied clouds are likely to be stable because energy required to create a cloudy mixture is much greater than the energy which is released once the parcel has become cloudy. The mixing level is crucially important in determining the fraction of cool cloudy mixtures. The vertical levels at which cloud tops can be found (derived from cloud top lidar data) puts a constraint on the entrainment velocity and the transport velocity of the mixture in such as way that the entrainment velocity needs to be an order of magnitude higher than the transport velocity in order to get any cloudy sinking mixtures. If the transport velocity is too large, clouds will break up, however mixtures will remain less dense than their environment. Radiation was neglected in this analysis, but is likeky to enhance the instability, as it cools the parcels once they become cloudy. A complete analysis then involves another velocity scale representing the speed at which a mixture moves along lines of equal θ_ℓ. The mixing process will deviate from the simple mixing line structure as shown in this paper and is the subject of further research.

FIGURE 1

FIGURE 2

FIGURE 3