

## PROBABILISTIC STRUCTURAL ANALYSIS METHODS DEVELOPMENT FOR SSME

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## ABSTRACT

The development of probabilistic structural analysis methods is a major part of the SSME Structural Durability Program and consists of three program elements: (1) composite load spectra, (2) probabilistic finite element structural analysis, and (3) probabilistic structural analysis applications. Recent progress includes: (1) the effects of the uncertainties of several factors on the HPFP blade temperature pressure and torque, (2) the evaluation of the cumulative distribution function of structural response variables based on assumed uncertainties on primitive structural variables, and (3) evaluation of the failure probability. Collectively, the results obtained demonstrate that the structural durability of critical SSME components can be probabilistically evaluated.

## INTRODUCTION

It is becoming increasingly evident that deterministic structural analysis methods will not be sufficient to properly design critical structural components for upgraded Space Shuttle Main Engines (SSME). Structural components in the SSME are subjected to a variety of complex, severe cyclic and transient loading conditions including high temperatures and high temperature gradients. Most of these are quantifiable only as best engineering estimates. These complex loading conditions subject the material to coupled nonlinear behavior which depends on stress, temperature, and time. Coupled nonlinear material behavior is nonuniform, is very difficult to determine experimentally, and perhaps impossible to describe deterministically. In addition, test data on critical SSME structural components are relatively small. Fabrication tolerances on these components, which in essence are small thickness variations, can have significant effects on the component structural response. Fabrication tolerances by their very nature are statistical. Furthermore, the attachment of the components to the structural system generally differs by some indeterminate degree from that which was assumed for designing the component. In summary, all four fundamental aspects of: (1) loading conditions, (2) material behavior, (3) geometric configuration, and (4) supports - on which structural analyses are based, are of a statistical nature. One direct way to formally account for all these statistical aspects is to develop probabilistic structural analysis methods where all participating variables are described by appropriate probabilistic functions.

NASA Lewis Research Center is currently developing probabilistic structural analysis methods for select SSME structural components under

the SSME Structural Durability Program. Briefly, the deterministic, three-dimensional, inelastic analysis methodology developed under the Hot Section Technology (HOST and R&T Base Programs) is being augmented to accommodate the complex probabilistic loading spectra, the thermo-viscoplastic material behavior, and the material degradation associated with the environment of space propulsion system structural components representative of the SSME, such as turbine blades, transfer duct, and liquid-oxygen posts, Fig. 1.

The development of probabilistic structural analysis methodology consists of the following program elements: (1) composite load spectra, (2) probabilistic structural analysis methods, and (3) probabilistic structural analysis application (ref. 1). The program main goal is to develop the methodology to address the problem depicted schematically in figure 2. Past progress of tasks in specific elements of the program are described in papers presented in conferences (ref. 2, 3, and 4).

Recent activities focused on extending the methodology to include the combined uncertainties in several factors on the structural response. An executive summary of this progress is shown in figure 3. The objective of the present paper is to briefly describe progress in three program elements: composite load spectra, probabilistic finite element structural analysis, and strength degradation. Progress is described in terms of fundamental concepts, computer codes, and representative results.

#### COMPOSITE LOAD SPECTRA

The fundamental assumption for developing composite load spectra is that each individual load condition is the probabilistic time synthesis of four primitive parts: (1) steady state, (2) periodic, (3) random, and (4) spike. Each of these parts, except random, is described by a deterministic portion and a probabilistic perturbation about this deterministic portion as depicted schematically in figure 4. One justification for describing each loading condition in terms of primitive variables is that: experts over the years have developed good judgments of the ranges of the perturbations about nominal (deterministic) conditions. The objective of the Composite Load Spectra program is to formalize the fundamental assumption in a computer code using: (1) available data from various rocket engines, (2) probability theory, and (3) a dedicated expert system.

A schematic diagram of the Composite Load Spectra (CLS) Computer code is shown in figure 5. Representative results obtained for the perturbations of different engine factors on the high pressure turbo-pump blade are shown in figures 6 and 7. Figure 6 is the nominal temperatures, while figure 7 indicates the temperature changes due to hot gas seal geometry and respective perturbations indicated in the figure caption. For example, the greater temperature change due to Gas Seal Geometry (fig. 7) of 0.06 is 53.3 °F. The combined contributions

of this and other factors (not shown here) is 87.6 °F which is in addition to the greatest nominal temperature of 1860 °F. Although at first glance a change of 87.6 °F may seem insignificant, this is not the case because at these high temperature small temperature changes have dramatic effects on the material structural durability and attendant cooling requirements.

Another representative example is shown in figure 8, where the effect of comparable factors on the torque of the High-Pressure-Oxidizer-Turbopump (HPOTP) are plotted as bounds versus time. These bounds are substantial at some times and relatively close at others. Similar plots can be obtained for pressures or any other loading condition. The current CLS code permits the simultaneous perturbation of 47 different factors for each different load condition.

#### PROBABILISTIC FINITE ELEMENT STRUCTURAL ANALYSIS

The fundamental assumption for developing probabilistic finite element structural analysis (PFESAN) is that "the uncertainties in each primitive structural variable can be described by assumed corresponding probabilistic distributions." Primitive structural variables are those which are used to describe a structure such as: (1) stiffness, (2) strength, (3) thickness and tolerance, (4) spatial location, (5) attachment, (6) various nonlinear dependencies (temperature, stress, time, etc.). A schematic of the probabilistic distributions for some primitive variables is shown in figure 9. Subsequently, the uncertainties in the load conditions (described by the composite load spectra) and the uncertainties in the primitive structural variables are computationally simulated by performing multiple finite element structural analysis to determine the probabilistic structural response of a specified SSME structural component. The structural response is generally described in terms of displacement, frequencies, buckling loads, and structural fracture toughness. The integration is illustrated schematically in figure 10.

It is instructive to compare component development by the traditional engineering approach and component evaluation using PFESAN. The parallism is summarized in Table 1. The former approach relies on physical experimental and requires that the material, fabrication process, and test methods are already available. The latter approach is entirely computational and requires the integration of available structural analysis methods with available probability theory. The former approach has the advantage of demonstrating a specific technology while the latter has the advantage of assessing undeveloped but with high payoff potential candidate technologies. In addition the former approach requires a large number of experiments while the latter can be verified with strategically selected few.

PFESAN has been formalized and integrated into a computer code NESSUS (Nonlinear Evaluation of Stochastic Structures Under Stress). NESSUS is driven by a dedicated expert system. A schematic diagram of

NESSUS is shown in figure 11. The user interacts with NESSUS through a dedicated expert system schematically shown in figure 12. Representative results obtained using NESSUS are shown in figure 13. The distributions assumed for the primitive variables listed in the table. Both the individual and the combined effects of the primitive variables on the combined stress (von Mises) are shown in the figure in terms of cumulative distribution functions (CDF). The information generated during the PSESAN can be used to establish bounds on the CDF. A sample result is shown in figure 14 for one blade location.

The curves in figure 14 can be used in a number of ways. Two of them are: (1) all the blades tested in the assumed conditions will have a mean combined stress between 57.9 and 62.1 ksi 90 percent of the time; (2) the mean combined stress in all the blades tested (under the assumed conditions) will range from 42 ksi to 83 ksi. This indicates that a wide scatter in the mean combined stress is probable. Two implications follow; (1) assessing the durability life of the blades using only material uncertainties will not be sufficient, and (2) obtaining wide scatter in measured stress/strain magnitudes does not indicate test procedure difficulties.

It is noted that all the NESSUS results presented herein were obtained using 50 simulations for each case studied. These simulations are relatively small compared to direct Monte Carlo simulation which will normally require 1000 and greater simulations. The reduced but with comparable accuracy simulations is a NESSUS feature which uses the fast probability integration method to select subsequent simulations in a self-adaptive manner.

#### PROBABILISTIC ANALYSIS FOR STRENGTH DEGRADATION

The fundamental assumption for developing probabilistic analysis methods for strength degradation is that "the uncertainties in primitive variables of the strength degradation model can be described by assumed probabilistic distributions." Two different models were selected to demonstrate the concept. The models express the number of mechanical load cycles to failure. One of the models is based on linear elastic fracture mechanics and the other on a strength degradation model recently studied at Lewis. The models with their respective primitive variables are summarized in table 2.

Both of these models were used to predict the number of cycles to failure in a material used in SSME components. The input for the fracture mechanics model is summarized in table 3. The CDF obtained from this input is shown in figure 15. The input for the strength degradation model is summarized in Table 4 and the corresponding CDF is shown in figure 16. Both CDF exhibit wide ranges for the probable number of cycles to failure. The linear fracture mechanics model shows a mean of 10,000 cycles while the strength degradation model shows a mean of 10 million cycles. Based on this comparison the linear fracture mechanics model penalizes the material by three decades. It is

important to note the differences between the two models: (1) the linear fracture mechanics model assumes the existence of a crack-like defect and then evaluates the number of cycles required to grow this defect to a critical size for imminent rapid propagation to fracture. (2) The strength degradation model does not presuppose the existence of defects and, therefore, includes both defect initiation and propagation resulting in greater number of cycles. (3) The linear fracture mechanics model has five primitive variables while the strength degradation has 13. Assuming that the greater the number of primitive variables in the model the more inclusive the representation of the physics in the model, then the strength degradation model will be more accurate. (4) The linear fracture mechanics model requires determination of  $C$ ,  $M$ , and  $a_i$  by specialty and often complex test methods while the strength degradation model uses available room temperature material properties.

Irrespective of the model used, the important conclusion is that the uncertainties in fatigue cycles to failure can be evaluated probabilistically.

#### CONCLUSIONS

The development of probabilistic structural analysis methods for select SSME components continues. Recent progress includes (1) the effects of the uncertainties of several factors on blade temperatures, pressures, and torque, (2) the evaluation of the cumulative distribution function of structural response variables based on assumed uncertainties in the structural primitive variables, (3) evaluation of failure probability, and (4) life assessment in terms of cumulative distribution function using linear fracture mechanics and a strength degradation model. Three different computer codes are being developed in parallel: (1) Composite Load Spectral (CLS) for the probabilistic description of SSME load, (2) NESSUS, for the probabilistic structural analysis of select SSME structural components, and (3) a life durability code for the assessment of the fatigue cycles to failure of structural components in SSME mission environments. Collectively, the results obtained to date demonstrate that the structural durability of SSME critical components can be evaluated using the methodology developed under the SSME Structural Durability Program.

#### REFERENCES

1. C. C. Chamis: Probabilistic Structural Analysis Methods for Space Propulsion System Components. NASA TM 88861, 1986.
2. Anon.: AIAA/ASME/ASCE/AHS 28th Structural Dynamic and Materials Conference, April 6-8, 1987 (papers 87-0764, -0765, -0766).
3. Anon.: Structural Integrity and Durability of Reusable Space Propulsion Systems. NASA CP 2471, 1987, PP. 117-200.
4. Anon.: AIAA/ASME/ASCE/AHS: 29th Structural Dynamics and Materials Conference, April 18-20, 1988 (papers 88-2371, -2372, -2373, -2374, -2376).

TABLE 1. SOME HELPFUL PARALLELS

<u>COMPONENT DEVELOPMENT</u>	<u>PSAM COMPONENT EVALUATION</u>
PARAMETER IDENTIFICATION	PARAMETER IDENTIFICATION
CHARACTERIZATION - DATABASE	PROBABILISTIC DISTRIBUTIONS
COMPONENT FABRICATION	COMPONENT MECHANISTIC MODEL
COMPONENT TESTING	COMPONENT ANALYSIS
COMPONENT TEST RESULTS DATABASE	COMPONENT ANALYSIS RESULTS DATABASE
STATISTICAL INFERENCES	STATISTICAL INFERENCE
RELIABILITY/CONFIDENCE LEVEL	RELIABILITY/CONFIDENCE LEVEL

<u>BOTTOM LINE</u>	
<u>PHYSICAL</u>	<u>COMPUTATIONAL</u>

TABLE 2.

PROBABILISTIC ANALYSIS FOR STRENGTH DEGRADATION MODELS

MODELS SELECTED FOR STUDY:

1. FATIGUE CRACK GROWTH MODEL (PARIS EQUATION)

$$N(a_f) = \frac{1}{C \gamma^m \pi^{m/2} \Delta \sigma^m} \left[ \frac{a_f^{-m/2+1} - a_i^{-m/2+1}}{-m/2+1} \right],$$

WHERE C, M,  $\Delta \sigma$  AND  $a_i$  ARE RANDOM VARIABLES

2. STRENGTH REDUCTION MODEL (CHAMIS EQUATION)

$$N_M(S) = N_{MF} - \left\{ (N_{MF} - N_{M0}) \left[ \frac{S}{S_0 \left[ \frac{T_f - T}{T_f - T_0} \right]^n \left[ \frac{S_f - \sigma}{S_f - \sigma_0} \right]^m} \right] \right\}^{1/q}$$

WHERE  $N_{MF}$ ,  $N_{M0}$ , S,  $S_0$ ,  $T_f$ , T,  $T_0$ ,  $S_f$ ,  $\sigma$ ,  $\sigma_0$ , n, m AND q ARE RANDOM VARIABLES

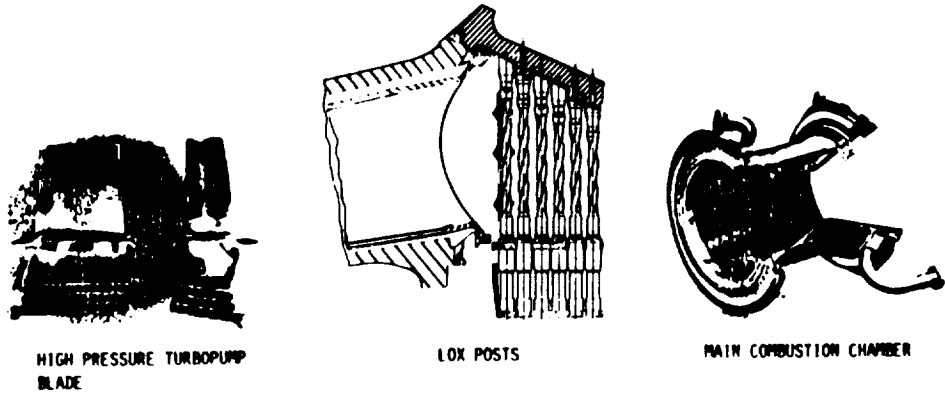
TABLE 3. RANDOM2 INPUT (FATIGUE CRACK GROWTH MODEL)

VARIABLE	VARIABLE TYPE	DISTRIBUTION TYPE	MEAN	STANDARD DEVIATION	
C (MATERIAL PROPERTY)	RANDOM	LOGNORMAL	$8.66 \times 10^{-10}$ IN/CYCLE	$0.866 \times 10^{-10}$	(10%)
M (MATERIAL PROPERTY)	RANDOM	NORMAL	$6.48 \times 10^{-2}$ IN/CYCLE/ KPSI IN <sup>1/2</sup>	$3.24 \times 10^{-3}$	(5%)
$\Delta\sigma$ (ALTERNATING STRESS)	RANDOM	LOGNORMAL	90 KPSI	9	(10%)
A <sub>i</sub> (INITIAL CRACK SIZE)	RANDOM	LOGNORMAL	$118 \times 10^{-4}$ IN	$17.7 \times 10^{-4}$	(15%)
A <sub>f</sub> (FINAL CRACK SIZE)	DETERMINISTIC	N/A	$7.87 \times 10^{-2}$ IN	N/A	
Y (COMPONENT/ CRACK SHAPE PAR.)	DETERMINISTIC	N/A	1.0	N/A	

TABLE 4. RANDOM3 AND RANDOM4 INPUT (STRENGTH REDUCTION MODEL)

VARIABLE	VARIABLE TYPE	DISTRIBUTION TYPE	MEAN	STANDARD DEVIATION	
T <sub>f</sub> (MELTING TEMPERATURE)	RANDOM	NORMAL	2732°F	82.	(3%)
S <sub>f</sub> (ULTIMATE TENSILE STRENGTH)	RANDOM	LOGNORMAL	130 KPSI	6.5	(5%)
N <sub>mf</sub> (LOG OF FINAL CYCLE)	RANDOM	LOGNORMAL	8	0.8	(10%)
T <sub>o</sub> (REFERENCE TEMPERATURE)	RANDOM	NORMAL	68°F	2.0	(3%)
$\sigma_o$ (RESIDUAL COMPRESSIVE STRESS)	RANDOM	LOGNORMAL	-2.9 KPSI	-0.145	(5%)
N <sub>mo</sub> (LOG OF REFERENCE CYCLE)	RANDOM	LOGNORMAL	7	0.7	(10%)
S <sub>o</sub> (REFERENCE FATIGUE STRENGTH)	RANDOM	LOGNORMAL	72.6 KPSI	3.6	(5%)
T (CURRENT TEMPERATURE)	RANDOM	NORMAL	1562°F	46.7	(3%)
$\sigma'$ (CURRENT MEAN STRESS)	RANDOM	LOGNORMAL	21.8 KPSI	1.1	(5%)
S (CURRENT FATIGUE STRENGTH)	RANDOM	LOGNORMAL	36.3 KPSI	1.8	(5%)
n (TEMPERATURE EXPONENT)	RANDOM	NORMAL	0.5	0.15	(3%)
m (STRESS EXPONENT)	RANDOM	NORMAL	0.5	0.15	(3%)
q (CYCLE EXPONENT)	RANDOM	NORMAL	0.5	0.15	(3%)

FIGURE 1. SELECT SSME COMPONENTS



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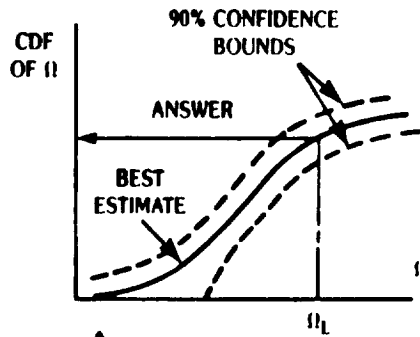
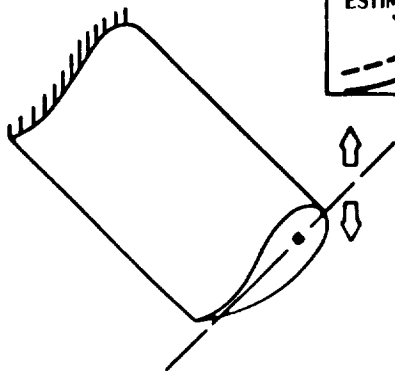
FIGURE 2. PROBABILISTIC STRUCTURAL ANALYSIS DEFINITION

**PROBLEM:**

PROB [ NATURAL FREQUENCY OF BLADE,  $\Omega$       FREQUENCY LIMIT,  $\Omega_L$  ] ?

**VARIABLES:**

- GEOMETRY
- STIFFNESS
- MASS



**CONFIDENCE**

- DIFFICULT ANALYSIS PROBLEM
- APPROXIMATE METHOD DEFINED
- USES NESSUS DATA BASE
- GIVES INTERVAL ESTIMATES
  - PROBABILITY
  - VARIABILITY

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FIGURE 3. **PROBABILISTIC STRUCTURAL ANALYSIS METHODS (PSAM)**

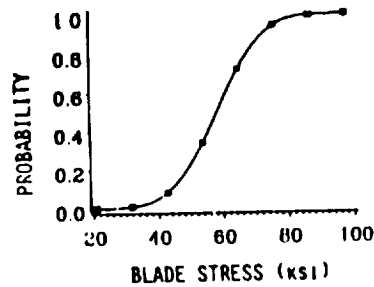
**PLANNED ACCOMPLISHMENT**

DEVELOP PROBABILISTIC STRUCTURAL ANALYSIS METHODS/COMPUTER CODES FOR QUANTIFYING THE INDIVIDUAL AND COMBINED EFFECTS OF UNCERTAINTIES ON THE STRUCTURAL DURABILITY OF SSME COMPONENTS

**STATUS**

- 0 FORMULATED PROBABILISTIC FINITE ELEMENT METHODS
- 0 DEVELOPED SECOND GENERATION COMPUTER CODE - OPERATIONAL AT LEWIS. UNCERTAINTIES INCLUDED:
  - LOADS
  - GEOMETRY
  - MATERIAL
  - SPATIAL POSITION
  - ATTACHMENT

**COMBINED PROBABILITY OF OCCURRENCE**



**SIGNIFICANCE**

THE COMBINED PROBABILITY OF MAGNITUDE OF STRUCTURAL DURABILITY VARIABLES CAN BE DETERMINED

FIGURE 4.

**EACH LOAD CONDITION IS THE PROBABILISTIC TIME SYNTHESIS OF FOUR PRIMITIVE PARTS: COMPOSITE LOAD SPECTRA**

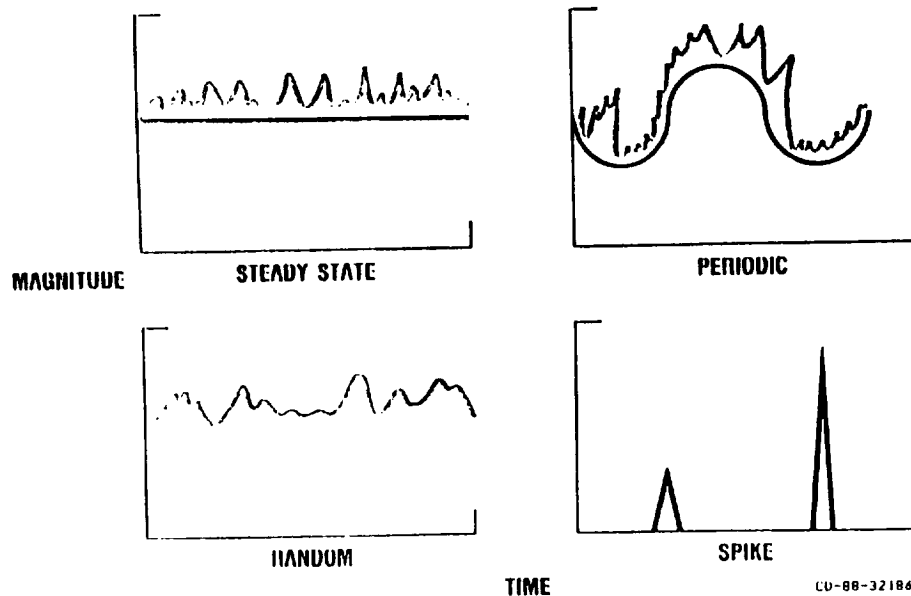
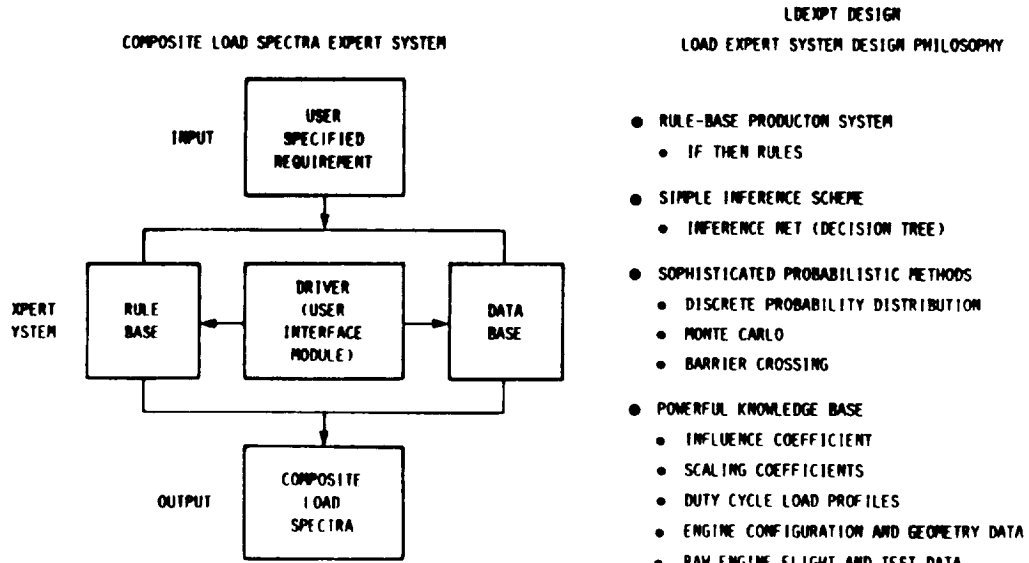
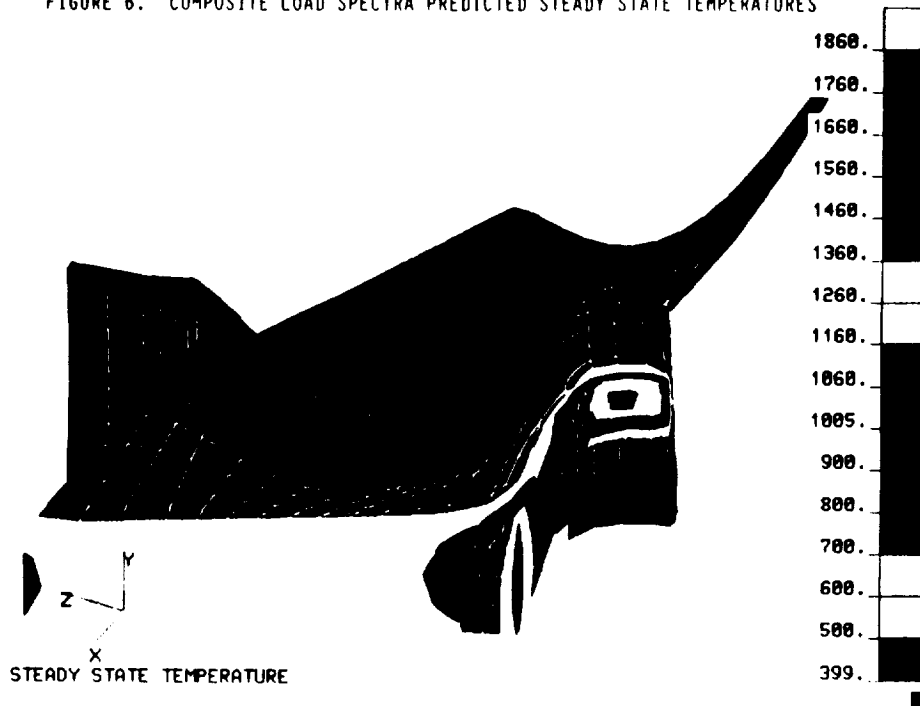


FIGURE 5.  
COMPOSITE LOAD SPECTRA SIMULATION USING EXPERT SYSTEMS



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FIGURE 6. COMPOSITE LOAD SPECTRA PREDICTED STEADY STATE TEMPERATURES



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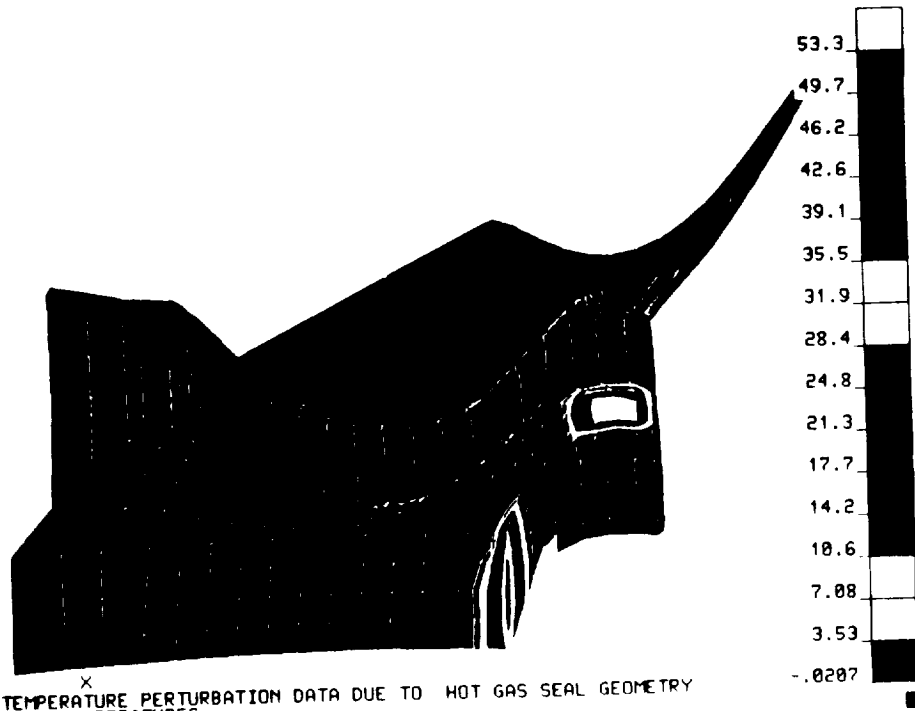


FIGURE 7. <sup>X</sup> TEMPERATURE PERTURBATION DATA DUE TO HOT GAS SEAL GEOMETRY  
 DELTA TEMPERATURES  
 ONE STANDARD DEVIATION OF .060

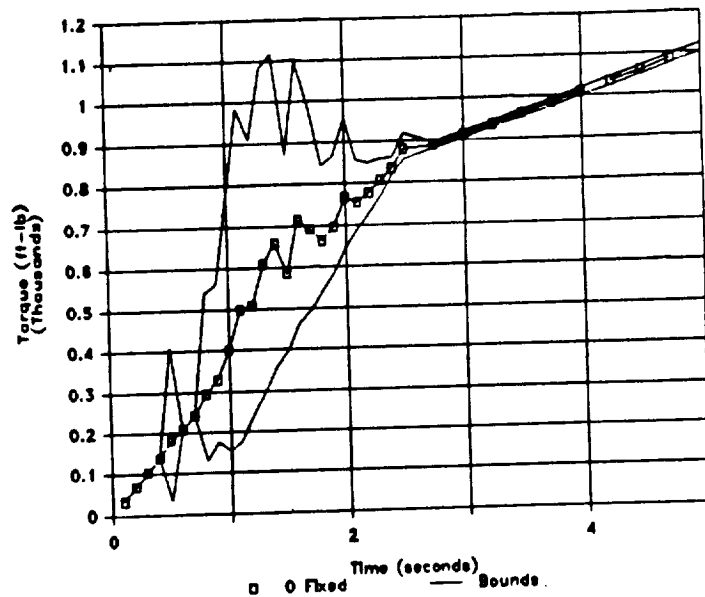
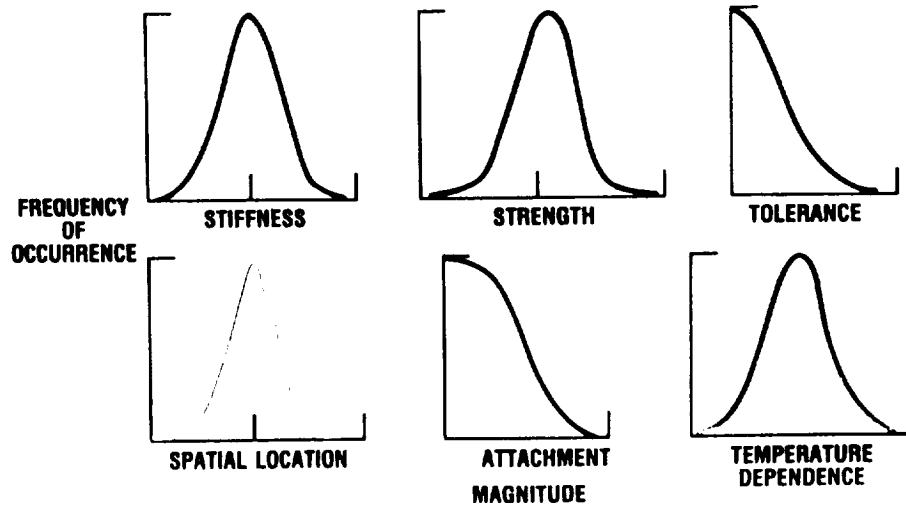


FIGURE 8. TRANSIENT MODEL HPTOTP TORQUE

FIGURE 9.

**PROBABILISTIC STRUCTURAL ANALYSIS: THE UNCERTAINTIES IN EACH STRUCTURAL PRIMITIVE VARIABLE CAN BE DESCRIBED BY ASSUMED CORRESPONDING PROBABILISTIC DISTRIBUTIONS**



AND SIMILARLY FOR UNCERTAINTIES IN OTHER PRIMITIVE VARIABLE

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FIGURE 10.

**THE UNCERTAINTIES IN THE LOAD CONDITIONS AND STRUCTURAL PRIMITIVE VARIABLES ARE INTEGRATED BY PERFORMING MULTIPLE FINITE ELEMENT ANALYSES**

GLOBAL: AT ALL NODAL POINTS



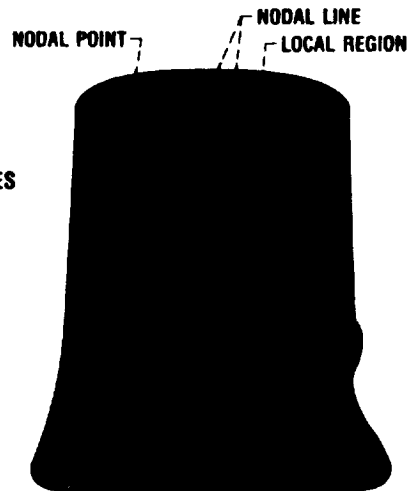
LOCAL: AT EACH REGION BOUNDED BY NODAL LINES

$$\mathbf{K} = \int_v [\mathbf{B}_m]^T [\mathbf{D}] [\mathbf{B}_m] dv$$

$$\mathbf{K} = \int_v [\mathbf{B}_e]^T [\mathbf{f}] [\mathbf{D}] [\mathbf{B}_e] dv$$

$$\mathbf{K} = \int_v [\mathbf{B}_e]^T [\mathbf{D}] [\mathbf{B}_e] dv$$

THE UNCERTAINTIES IN THE STRUCTURAL RESPONSE VARIABLES ( $u, w, \sigma, \epsilon, G$ ) ARE DETERMINED FROM THE RESULTS OF THE MULTIPLE FINITE ELEMENT ANALYSES



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FIGURE 11.  
PROBABILISTIC FINITE ELEMENT COMPUTER CODE NESSUS

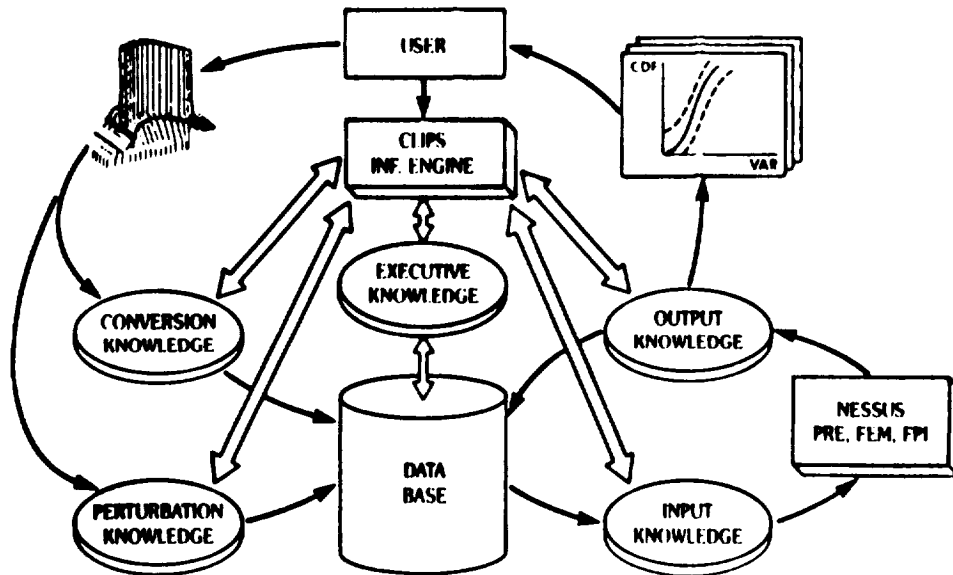
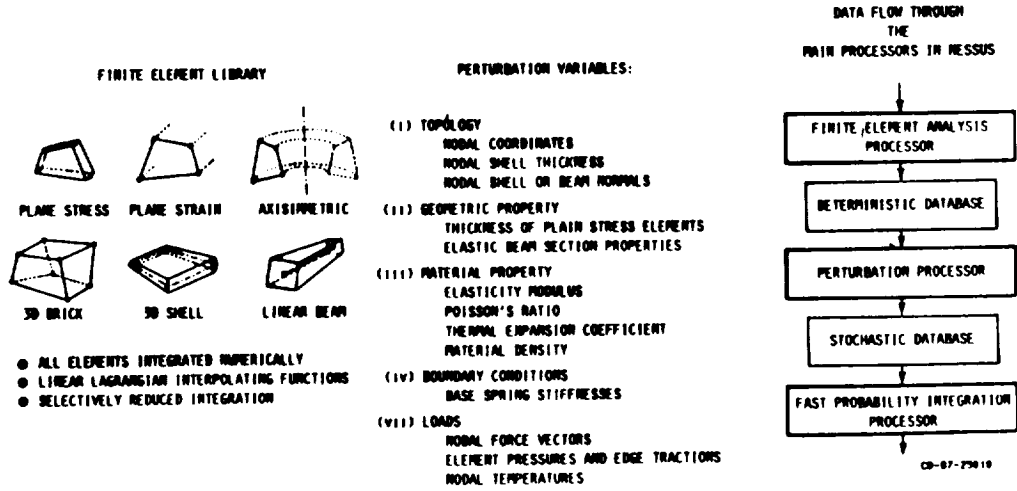
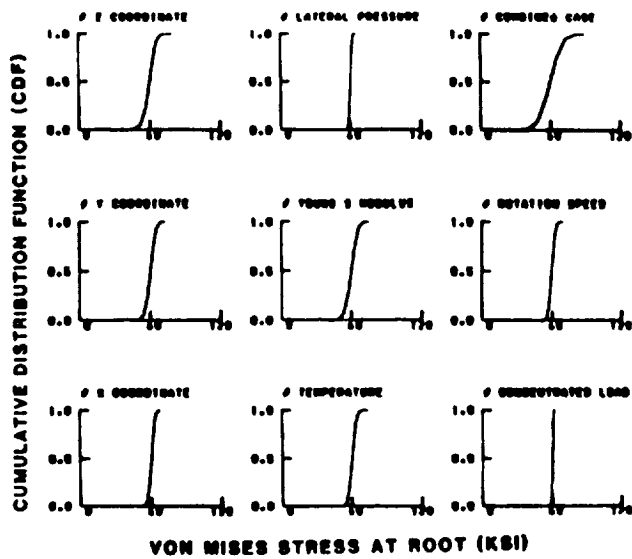


FIGURE 12. NESSUS DEDICATED EXPERT SYSTEM

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CASE	RANDOM VARIABLE (RV)	COV OF RV
1	X-COOR	0.005 (std)
2	Y-COOR	0.005 (std)
3	Z-COOR	0.005 (std)
4	Y-TYP	40° P (std)
5	YOUNG'S MODULUS	0.10
6	LATERAL PRESSURE	0.20
7	CONC. ROTATION	0.20
8	R-SPIN	0.02
9	C-COMBINE	



std : STANDARD DEVIATION  
 COV : COEFFICIENT OF VARIATION  
 (std/MEAN)

( # RANDOM VARIABLE (CONSIDERED IN EACH CASE) )

RANDOM VARIABLE DISTRIBUTION TABLE

CDF OF VON MISES STRESS DUE TO SINGLE/COMBINED RANDOM VARIABLE PERTURBATION

FIGURE 13. PROBABILISTIC STRUCTURAL ANALYSIS USING NESSUS

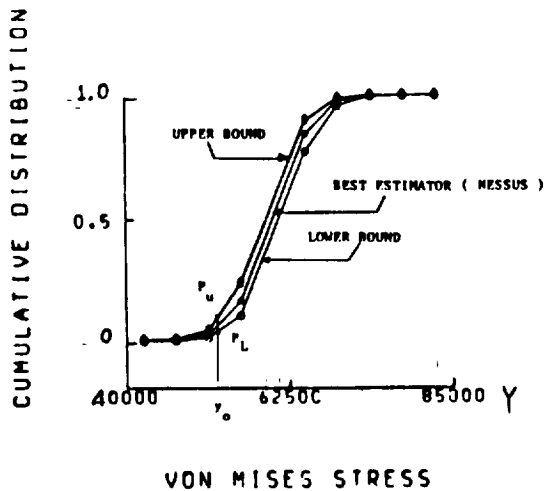


FIGURE 14. 90% CONFIDENCE BOUND RESULT FROM UNCERTAINTY IN THE INPUT PARAMETERS

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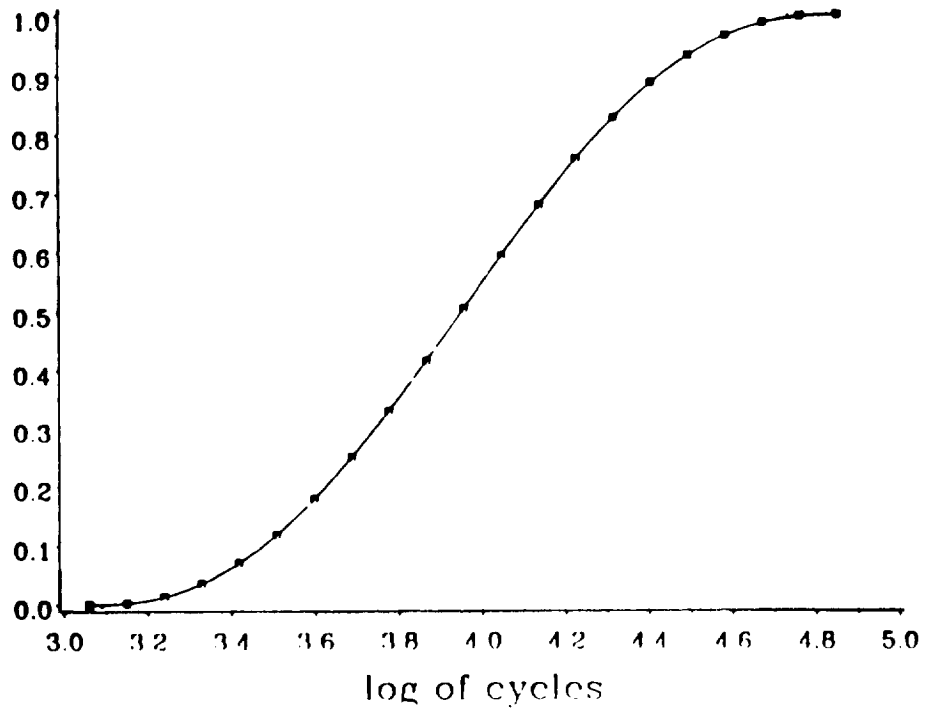


FIGURE 15. FATIGUE CRACK GROWTH MODEL

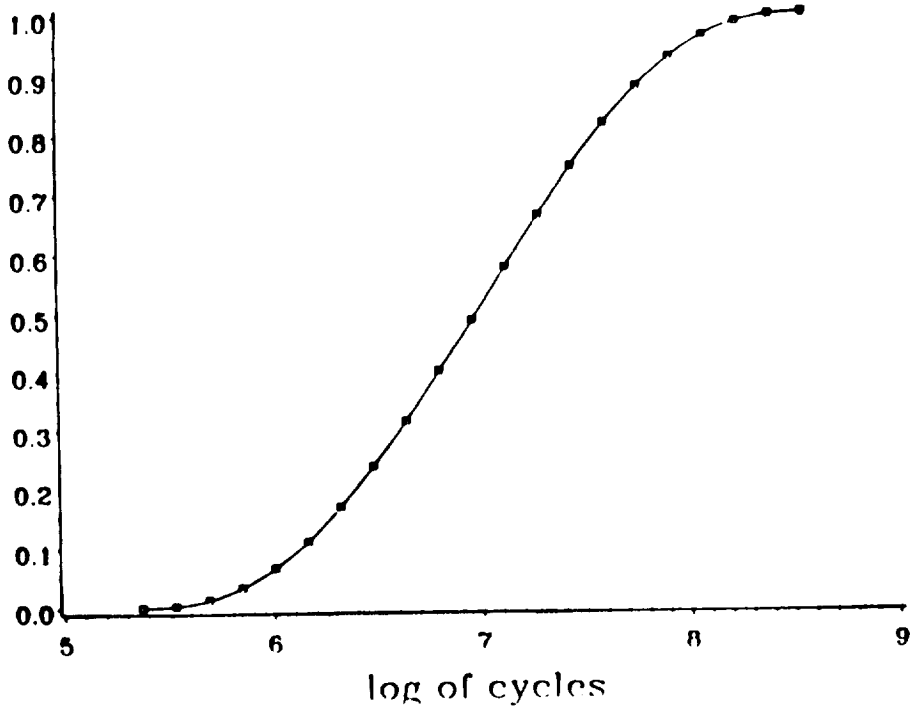


FIGURE 16. STRENGTH REDUCTION MODEL