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Dynamical Modeling of Serial Manipulators with Flexible Links and Joints Using the Method of Kinematic Influence

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ABSTRACT:

A method of formulating the dynamical equations of a flexible, serial manipulator is presented, using the Method of Kinematic Influence. The resulting equations account for rigid body motion, structural motion due to link and joint flexibilities, and the coupling between these two motions. Nonlinear inertial loads are included in the equations. A finite order mode summation method is used to model flexibilities. The structural data may be obtained from experimental, finite element, or analytical methods. Nonlinear flexibilities may be included in the model.

INTRODUCTION:

Link and joint flexibility often have significant effects on the performance of robotic manipulators. Simulations which include the dynamical effects of flexibility should include the structural dynamics coupled with the dynamics due to the gross motion of the links. A method of formulating such a dynamical model is presented. It extends the Method of Kinematic Influence to include a finite order mode summation model of structural dynamics.

The Method of Kinematic Influence is used to obtain a geometric and kinematic description of the robotic device, which includes the effects of flexibility. The kinematic description is then used to obtain a dynamical model which includes structural motions, gross motion of the links and base, and the coupling terms between the structural and gross motions. Nonlinear inertial forces are included. The operations used in obtaining this model are simple transformations of the inertias of each link, and first order transformations of forces and torques. A computer program, called V-Sim, has been written which uses this method to automatically generate the dynamical model for simulations.

REVIEW OF PREVIOUS WORK:

Various models of structural dynamics in robots have been presented in the literature. Many of these models use a finite order modal representation of the distributed mass and stiffness of each link. [6-8,11,14-19,21,23-25] Some lumped parameter models, [1,5,13,20,22] and some finite element models [2,9,17] have been presented. Linearized and quasi-static models have also been analyzed to determine how they differ from the full non-linear dynamical model. [16,17,20]

The method used to derive the dynamical equations should be chosen because it is easy to understand, or because it meets some other desirable criteria. Lagrangian derivations are common in flexible body dynamical modeling because they use the kinetic and potential energies, which may be easily obtained for a system of flexible bodies. [6,13,23-25] Hamilton's Principle has also been used because it provides similar advantages. [17] Other derivations have used Newton's Laws. [7,20]

Many computational algorithms have been presented. Great variations exist in the order of the calculations, and in how the algorithms collect common operations and common terms. Recursive methods of computation have been popular because of their tractability and efficiency. [11] Other more general methods which do not depend upon a specific recursive algorithm, have been presented. [3,4,10,12,19,23] Most methods may also be used to obtain the system inertia matrix and the non-linear torques which are necessary in control algorithms involving dynamical compensation.

The assumptions of the structural model have great effects on the validity of the resulting dynamical model. For example, mode summation models often assume modes to be geometrically decoupled at the local link level, and may ignore certain dynamical stiffening effects which may occur. [15,19] If inertial variations due to flexibility are included, these models will predict extremely large deflections and become unstable at high rotation rates. When such a model is used, the assumptions which restrict its application should be examined. In general, these models are valid only for small link deflections.

The Method of Kinematic Influence was developed first for rigid body, open loop (serial) mechanisms. [3,4,10] The method was extended to closed loop (parallel) mechanisms [12], and then to mechanisms with flexible joints [13,22], and flexible links [23]. This method is an extremely powerful and simple way of obtaining the dynamical model of a complex mechanism. Its organized structure yields information which is useful in mechanical design and analysis. It may also be used to calculate information about the system inertia matrix and non-linear forces and torques which are required in many advanced control algorithms.

THE GEOMETRY OF A FLEXIBLE SERIAL MECHANISM:

The geometry of a serial mechanism can be represented by a series of links connected by translational or rotational joints. A local coordinate frame is attached at the proximal end of each link. The z-axis coincides with the proximal joint axis. The x-axis is perpendicular to both the proximal joint and distal joint of the undeflected link. The undeflected link is represented by the vectors $\mathbf{a}_i$ and s_{i+1}. The joint angle is denoted by ϕ_i, and the link angle by α_i, as show in figure 1. If the proximal joint is rotational, ϕ_i will be a variable, but if it is translational, s_i will be a variable. Link deflections are represented by a displacement vector, $\mathbf{d}$, and a small rotation vector, $\boldsymbol{\Theta}$.

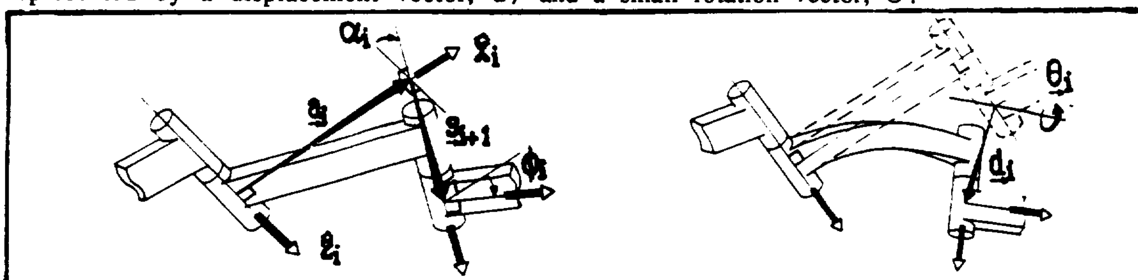

Figure 1. Geometric Parameters

A finite order modal representation is used to describe the structural deflections of each link. The deflection of a point on the link is a function of the magnitude of the modes of the mechanism, $\mathbf{q}$.

$$\begin{Bmatrix} \vec{d} \\ \vec{\theta} \end{Bmatrix}_p = \vec{\psi}_p(\vec{q}) = \begin{Bmatrix} \psi_{tp}(\vec{q}) \\ \psi_{rp}(\vec{q}) \end{Bmatrix} \tag{1}$$

Often, the modes are assumed to be geometrically decoupled at the local link level, and the deflections can be written in modal matrix form.

$$\left\{\begin{matrix} \underset{\sim}{d} \\ \underset{\sim}{\theta} \end{matrix}\right\}_P = [\dot{\psi}_P]\, \underset{\sim}{q} = \begin{bmatrix} [\dot{\psi}_{tP}] \\ [\dot{\psi}_{rP}] \end{bmatrix} \underset{\sim}{q} \tag{2}$$

The rotational coordinate transformation between sequential local coordinate frames can be represented in a 3x3 matrix form, $[{}^{i}T_{i+1}]$. The rotational deflection is represented by the skew-symmetric form of the small rotation vector, θ_i, added to the identity matrix. This matrix is post-multiplied by a matrix representing the angle α_i about the x-axis, and then by a matrix representing the angle ϕ_i about the distal joint (z-axis).

$$[{}^{i}T_{i+1}] = \begin{bmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Notice that the rotational transformation matrices between other frames may be found by concatenating these matrices, and that the inverse of a rotational transformation matrix may be approximated as the transpose, since the determinant is very close to one.

$$[{}^{h}T_i] = \prod_{j=h}^{i-1} [{}^{j}T_{j+1}] = [{}^{h}T_{h+1}]\,[{}^{h+1}T_{h+2}] \cdots [{}^{i-1}T_i] \tag{4}$$

$$[{}^{h}T_i]^{-1} = [{}^{h}T_i]^{T} = [{}^{i}T_h] \tag{5}$$

The position vector of a point on link P is:

$$\underset{\sim}{R}_P = \sum_{j=h}^{p-1} \{\, \underset{\sim}{a}_j + \underset{\sim}{d}_j + \underset{\sim}{s}_{j+1} \} + \underset{\sim}{x}_P + \underset{\sim}{d}_P \tag{6}$$

where x_P is the undeflected position of the point. All vectors are referenced to a common coordinate frame.

THE METHOD OF KINEMATIC INFLUENCE:

The Method of Kinematic Influence allows the cartesian velocities of any point on the mechanism to be expressed in terms of the positions and speeds of the joints, modes, and the base. This expression may be organized in the form of a Jacobian matrix, [J]. The translational and rotational cartesian velocities of the point, *P*, are described by a 6x1 vector, such that:

$$\left\{\begin{matrix} \underset{\sim}{v} \\ \underset{\sim}{\omega} \end{matrix}\right\}_P = [J_P] \left\{\begin{matrix} \dot{\underset{\sim}{\phi}} \\ \dot{\underset{\sim}{q}} \end{matrix}\right\} = \begin{bmatrix} [J_{tP}] \\ [J_{rP}] \end{bmatrix} \left\{\begin{matrix} \dot{\underset{\sim}{\phi}} \\ \dot{\underset{\sim}{q}} \end{matrix}\right\} \tag{7}$$

It is convenient to combine the joint variables with the modal variables in one vector. This hybrid combination of joint space and modal space will be called *j-m space*. Base motion is modeled as three rotational and three translational joints at the origin of the base link.

The columns of the Jacobian matrix are called the Kinematic Influence Coefficients, **g**, and are functions of the mechanism geometry, the joint positions, and the deflections. For a rotational joint or base motion which contributes to the motion of point, P , the Kinematic Influence Coefficient is defined as:

$$\underset{\to}{g}_i = \begin{Bmatrix} \hat{\underset{\to}{S}}_i \times \underset{\to}{R}_{i/p} \\ \hat{\underset{\to}{S}}_i \end{Bmatrix} \tag{8}$$

For a translational joint or base motion which contributes to the motion of point, P , the coefficient is defined as:

$$\underset{\to}{g}_i = \begin{Bmatrix} \hat{\underset{\to}{S}}_i \\ \underset{\to}{0} \end{Bmatrix} \tag{9}$$

For a mode q_i which contributes to the motion of point P , the coefficient is defined as:

$$\underset{\to}{g}_i = \begin{Bmatrix} \left\{ \dfrac{\partial \underset{\to}{\psi}_{rp}}{\partial q_i} \right\} \times \underset{\to}{R}_{i/p} + \left\{ \dfrac{\partial \underset{\to}{\psi}_{tp}}{\partial q_i} \right\} \\ \left\{ \dfrac{\partial \underset{\to}{\psi}_{rp}}{\partial q_i} \right\} \end{Bmatrix} \tag{10}$$

But, if the modes are geometrically decoupled, the column of the modal matrix which is associated with this mode may be substituted for the partial derivatives.

$$\underset{\to}{g}_i^P = \begin{Bmatrix} \left\{ [\dot{\psi}_{rp}]_{:i} \times \underset{\to}{R} \right\} + [\dot{\psi}_{tp}]_{:i} \\ [\dot{\psi}_{rp}]_{:i} \end{Bmatrix} \tag{11}$$

For a joint or mode which does <u>not</u> contribute to the motion of point, P , the coefficient is defined as:

$$\underset{\to}{g}_i^P = \underset{\to}{0} \tag{12}$$

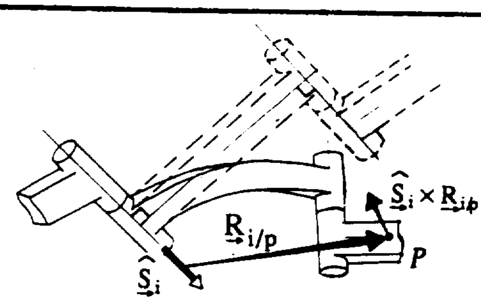

Figure 2. The Geometric Influence Coefficient for Joint i

The S and **R** vectors can be found from our knowledge of the geometry of the mechanism. **R** is the vector position from the joint or deflection to the point, P, and S is the vector of direction cosines which describe the line-of-action of the joint or deflection, as shown in Figure 2. All of the vectors which are used in these formulas must be expressed in a common frame of reference.

KINEMATIC INFLUENCE AND EQUIVALENT FORCES:

The Jacobian matrix also serves as a relationship between cartesian force/torques and the equivalent j-m loads:

$$[J_P]^T \begin{Bmatrix} \underset{\rightarrow}{F} \\ \underset{\rightarrow}{M} \end{Bmatrix}_P = \begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix} \tag{13}$$

where $\{\tau\ Q\}$ is the vector of equivalent loads in j-m space, $\{F\ M\}$ is the 6x1 vector of cartesian forces and torques, and $[J_P]^T$ is the transpose of the Jacobian Matrix for the point where $\{F\ M\}$ is applied. This relationship is a result of the duality of forces and velocities, and may be proven by showing that the virtual work performed by the cartesian force/torque is equal to the virtual work done by the equivalent j-m loads.

$$\begin{Bmatrix} \underset{\rightarrow}{F} \\ \underset{\rightarrow}{M} \end{Bmatrix}_P \cdot \begin{Bmatrix} \delta \underset{\rightarrow}{x} \\ \delta \underset{\rightarrow}{\theta} \end{Bmatrix}_P = \begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix} \cdot \begin{Bmatrix} \delta \underset{\rightarrow}{\phi} \\ \delta \underset{\rightarrow}{q} \end{Bmatrix} \tag{14}$$

THE DYNAMICAL EQUATIONS:

Consider a differential element of mass in one of the links. The kinetic energy of this mass element is:

$$KE_p = \frac{1}{2} \begin{Bmatrix} \underset{\rightarrow}{v}_P \\ \underset{\rightarrow}{\omega}_P \end{Bmatrix}^T \begin{bmatrix} \delta m_p[I] & [0] \\ [0] & [\delta \Pi_p] \end{bmatrix} \begin{Bmatrix} \underset{\rightarrow}{v}_P \\ \underset{\rightarrow}{\omega}_P \end{Bmatrix} \tag{15}$$

The velocities are referenced to a local coordinate frame fixed in the element. Potential Energy is defined as the integral of an elastic force and moment, from a reference position where there is no Potential Energy, to the current position, allowing for nonlinear stiffnesses in the system.

$$PE = \int_{\{\underset{\rightarrow}{x},\, \underset{\rightarrow}{\theta}\}_{Reference}}^{\{\underset{\rightarrow}{x},\, \underset{\rightarrow}{\theta}\}} \begin{Bmatrix} \underset{\rightarrow}{F}_P \\ \underset{\rightarrow}{M}_P \end{Bmatrix}_{Elastic} d\{\underset{\rightarrow}{x}, \underset{\rightarrow}{\theta}\} \tag{16}$$

Lagrange's Equation provides a convenient starting point for deriving the final form of the dynamical equations of the mass element.

$$\tau_j = \frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{x}_j}\right) - \frac{\partial KE}{\partial x_j} + \frac{\partial PE}{\partial x_j} \tag{17}$$

By chosing **x** to be the cartesian coordinates fixed in the mass element, the resulting equation is a familiar form.

$$\begin{Bmatrix} \underset{\rightarrow}{F}_P \\ \underset{\rightarrow}{M}_P \end{Bmatrix}_{External} = \begin{bmatrix} \delta m_p[I] & [0] \\ [0] & [\delta \Pi_p] \end{bmatrix} \begin{Bmatrix} \underset{\rightarrow}{a}_P \\ \underset{\rightarrow}{\alpha}_P \end{Bmatrix} + \begin{Bmatrix} \underset{\rightarrow}{0} \\ \underset{\rightarrow}{\omega}_P \times [\delta \Pi_p]\, \underset{\rightarrow}{\omega}_P \end{Bmatrix} + \begin{Bmatrix} \underset{\rightarrow}{F}_P \\ \underset{\rightarrow}{M}_P \end{Bmatrix}_{Elastic} \tag{18}$$

But this form of equation is not suitable for simulation. The equations must be converted to j-m space, and all of the other mass elements in the mechanism must be considered.

To express these dynamical equations in terms of j-m space, the Jacobian transpose relationship is used, where $[J_P]$ is the Jacobian for the mass element.

$$\begin{Bmatrix} \underset{\to}{T_P} \\ \underset{\to}{Q_P} \end{Bmatrix}_{External} = [J_P]^T \left\{ \begin{bmatrix} \delta m_P[I] & [0] \\ [0] & [\delta II_P] \end{bmatrix} \begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix} + \begin{Bmatrix} \underset{\to}{0} \\ \underset{\to}{\omega_P} \times [\delta II_P]\, \underset{\to}{\omega_P} \end{Bmatrix} \right\} + \begin{Bmatrix} \underset{\to}{T_P} \\ \underset{\to}{Q_P} \end{Bmatrix}_{Elastic} \tag{19}$$

Next, the dynamical equations for each mass element in the mechanism must be combined to form the dynamical equations for the system. This can be accomplished by adding all of the equations together, and can be written as a volumetric integral throughout the mechanism, assuming the mass elements are very small. Notice that the forces between the particles cancel because they are equal and opposite, <u>except</u> at the joints, and "at" the modes.

$$\begin{Bmatrix} \underset{\to}{T} \\ \underset{\to}{Q} \end{Bmatrix}_{External} = \int_V [J_P]^T \left\{ \begin{bmatrix} \rho_P[I] & [0] \\ [0] & \left[\frac{II}{V_P}\right] \end{bmatrix} \begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix} + \begin{Bmatrix} \underset{\to}{0} \\ \underset{\to}{\omega_P} \times \left[\frac{II}{V_P}\right] \underset{\to}{\omega_P} \end{Bmatrix} \right\} dV + \begin{Bmatrix} \underset{\to}{T} \\ \underset{\to}{Q} \end{Bmatrix}_{Elastic} \tag{20}$$

where ρ is the mass density and II/V is the inertia "density". For practical purposes, the integral term must be simplified. Let the acceleration of the particle can be expressed as the sum of a linear function of the j-m accelerations, and a nonlinear function of the j-m velocities.

$$\begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix} = [J_P] \begin{Bmatrix} \ddot{\phi} \\ \dot{q} \end{Bmatrix} + \begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix}_{Nonlinear} \tag{21}$$

Substituting this formula into the integral,

$$\int_V [J_P]^T \left\{ \begin{bmatrix} \rho_P[I] & [0] \\ [0] & \left[\frac{II}{V_P}\right] \end{bmatrix} \begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix} + \begin{Bmatrix} \underset{\to}{0} \\ \underset{\to}{\omega_P} \times \left[\frac{II}{V_P}\right] \underset{\to}{\omega_P} \end{Bmatrix} \right\} dV$$

$$= \int_V [J_P]^T \left\{ \begin{bmatrix} \rho_P[I] & [0] \\ [0] & \left[\frac{II}{V_P}\right] \end{bmatrix} [J_P] \begin{Bmatrix} \ddot{\phi} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} \rho_P[I] & [0] \\ [0] & \left[\frac{II}{V_P}\right] \end{bmatrix} \begin{Bmatrix} \underset{\to}{a_P} \\ \underset{\to}{\alpha_P} \end{Bmatrix}_{Nonlinear} + \begin{Bmatrix} \underset{\to}{0} \\ \underset{\to}{\omega_P} \times \left[\frac{II}{V_P}\right] \underset{\to}{\omega_P} \end{Bmatrix} \right\} dV$$

$$= \int_V \left\{ [J_P]^T \begin{bmatrix} \rho_P[I] & [0] \\ [0] & \left[\frac{II}{V_P}\right] \end{bmatrix} [J_P] \right\} dV \begin{Bmatrix} \ddot{\phi} \\ \dot{q} \end{Bmatrix} + \int_V \begin{Bmatrix} \underset{\to}{T_P} \\ \underset{\to}{Q_P} \end{Bmatrix}_{Nonlinear} dV \tag{22}$$

The nonlinear inertial forces are lumped in one term for notational brevity. Notice that the linear inertial integral involves a similarity transformation. This will yield the system inertia matrix. It is most convenient to perform the integration over each link, and then sum the results. To do this, it is necessary to express $[J_p]$ in terms of the Jacobian of the link coordinate frame, and a Jacobian expressing the motion of the mass element with respect to the link coordinate frame.

$$[J_p] = \begin{bmatrix} [I] & \left[[\tilde{\vec{x}}+\tilde{\vec{d}}_p]^T\right] & [\psi_{t_{p/i}}] \\ [0] & [I] & [\psi_{r_{p/i}}] \end{bmatrix} \begin{bmatrix} \begin{bmatrix} [J_{t_i}] \\ [J_{r_i}] \end{bmatrix} \\ [0][I] \end{bmatrix} = [J_{p/i}] \begin{bmatrix} [J_i] \\ [0][I] \end{bmatrix} \tag{23}$$

The nxn zero matrix and mxm identity matrix in this formula are used to make the matrix multiplication conformable (n equals the number of joints, and m equals the number of modes of the mechanism). The matrix $[\psi_{p/i}]$ is equivalent to $[\psi_p]$ - $[\psi_i]$. The linear inertial integral then becomes the definition of the system inertia matrix:

$$\int_V \left\{ [J_p]^T \begin{bmatrix} \rho_p[I] & [0] \\ [0] & \left[\frac{II}{V_p}\right] \end{bmatrix} [J_p] \right\} dV$$

$$= \sum_i \left\{ \begin{bmatrix} [J_i] \\ [0][I] \end{bmatrix}^T \left[\int_{V_i} [J_{p/i}]^T \begin{bmatrix} \rho_p[I] & [0] \\ [0] & \left[\frac{II}{V_p}\right] \end{bmatrix} [J_{p/i}]\, dV \right] \begin{bmatrix} [J_i] \\ [0][I] \end{bmatrix} \right\}$$

$$= \sum_i \left\{ \begin{bmatrix} [J_i] \\ [0][I] \end{bmatrix}^T [II_i] \begin{bmatrix} [J_i] \\ [0][I] \end{bmatrix} \right\} = \sum_i [II_i^*] = [II^*] \tag{24}$$

If this integration is performed in the local link coordinate frame, and the modes are assumed to be decoupled at the local link level, and variations due to the flexibility are not considered, the elements can be defined as:

$$[{}^iII_i] = \begin{bmatrix} [II_{xx}] & [II_{x\theta}] & [II_{xq}] \\ [II_{x\theta}]^T & [II_{\theta\theta}] & [II_{\theta q}] \\ [II_{xq}]^T & [II_{\theta q}]^T & [II_{qq}] \end{bmatrix}_i \tag{25}$$

where:

$$[II_{xx}] = m_i\,[I] \tag{25.a}$$

$$[II_{x\theta}] = m_i [\widetilde{\vec{cm}}_i]^T = m_i \begin{bmatrix} 0 & cm_z & -cm_y \\ -cm_z & 0 & cm_x \\ cm_y & -cm_x & 0 \end{bmatrix} \tag{25.b}$$

$$[II_{xq}] = \int_V \rho(\underline{x}) \begin{bmatrix} [\psi_i(\underline{x})] \\ [0] \end{bmatrix}^T dV \tag{25.c}$$

$$[II_{\theta\theta}] = [II_{cm_i}] + [\underline{cm}^T \underline{cm}\, [I] - \underline{cm}\, \underline{cm}^T]\, m_i \tag{25.d}$$

$$[\underline{cm}^T \underline{cm}\, [I] - \underline{cm}\, \underline{cm}^T] = \begin{bmatrix} cm_y^2+cm_z^2 & -cm_x cm_y & -cm_x cm_z \\ -cm_x cm_y & cm_x^2+cm_z^2 & -cm_y cm_z \\ -cm_x cm_z & -cm_y cm_z & cm_x^2+cm_y^2 \end{bmatrix} \tag{25.e}$$

$$[II_{\theta q}] = \int_V \begin{bmatrix} \rho(\underline{x})\, [\tilde{\underline{x}}_i]\, [\psi_i(\underline{x})] \\ \frac{I(\underline{x})}{V} [\psi_{r_i}(\underline{x})] \end{bmatrix} dV \tag{25.f}$$

and,

$$[II_{qq}] = \int_V \begin{bmatrix} \rho(\underline{x}) [\psi_{t_i}(\underline{x})]^T [\psi_{t_i}(\underline{x})] & [0] \\ [0] & \frac{I(\underline{x})}{V} [\psi_{r_i}(\underline{x})]^T [\psi_{r_i}(\underline{x})] \end{bmatrix} dV \tag{25.g}$$

such that $[II_{cm}]$ is the rigid body rotational inertia at the center of mass, m_i is the total link mass, and **cm** is the undeflected position of the center of mass in the link coordinate frame. In practical situations, these inertial parameters may be estimated via finite element analysis, or some appropriate experimental technique. To transform the inertial parameters back into a common frame (which is necessary), the matrix is pre- and post-multiplied by a transformation matrix, where the mxm identity matrix is used to make the matrix multiplication conformable.

$$[II_i] = \begin{bmatrix} [^hT_i] & [0] & [0] \\ [0] & [^hT_i] & [0] \\ [0] & [0] & [I] \end{bmatrix} [^iII_i] \begin{bmatrix} [^hT_i] & [0] & [0] \\ [0] & [^hT_i] & [0] \\ [0] & [0] & [I] \end{bmatrix}^T \tag{26}$$

Often the nonlinear inertial terms are presented as Christoffel Symbols of the inertia matrix, which are multiplied by the appropriate joint velocities to obtain the nonlinear loads. The computation involved in computing the Christoffel Symbols is overwhelming for a mechanism with many flexible modes, and the mathematical operations involved are not easy to understand. The number of computations can be minimized by collecting common operations. To do so, the nonlinear acceleration of each link is computed using an iterative algorithm, then the nonlinear loads on each link are computed, and finally the loads are transformed back into j-m space using the $[J^T]$ relationship. The nonlinear loads will be computed from the accelerations and angular velocities of the center of mass of the link. The nonlinear accelerations may be computed in an iterative fashion:

$$\begin{Bmatrix} \underline{a} \\ \underline{\alpha} \end{Bmatrix}_{nonlinear_b} = \begin{Bmatrix} \underline{a} \\ \underline{\alpha} \end{Bmatrix}_{nonlinear_a} + \begin{Bmatrix} \underline{\omega}_a \times \underline{v}_{b/a} \\ \underline{\omega}_a \times \underline{\omega}_b \end{Bmatrix} \tag{27}$$

And the nonlinear forces are approximated by:

$$\begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix}_{Nonlinear} = \sum_i [J_{cm}]^T \left\{ \begin{bmatrix} m_i[I] & [0] \\ [0] & [I_{cg}] \end{bmatrix} \begin{Bmatrix} \underset{\rightarrow}{a} \\ \underset{\rightarrow}{\alpha} \end{Bmatrix}_{cgi,\, Nonlinear} + \begin{Bmatrix} \underset{\rightarrow}{0} \\ \underset{\rightarrow}{\omega}_{cg} \times [I_{cg}]\, \underset{\rightarrow}{\omega}_{cg} \end{Bmatrix} \right\}$$

The final dynamical equations are expressed in j-m space, and in a standard form:

$$\begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix}_{Applied} = [II^*] \begin{Bmatrix} \underset{\rightarrow}{\ddot{\phi}} \\ \underset{\rightarrow}{\ddot{q}} \end{Bmatrix} + \begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix}_{Nonlinear} + \begin{Bmatrix} \underset{\rightarrow}{\tau} \\ \underset{\rightarrow}{Q} \end{Bmatrix}_{Elastic} \tag{28}$$

where $[II^*]$ is the system inertia matrix in j-m space, and the vectors of externally applied, and nonlinear inertial loads are given in j-m space.

V-Sim: A SIMULATOR FOR FLEXIBLE ROBOTICS:

These dynamical equations have been implemented as a computer program named V-Sim. It is currently being used in a variety of applications ranging from simulation of cantilever beams to simulations of the Space Shuttle Remote Manipulator System. The program automatically formulates the dynamical model for an open-loop manipulator. The manipulator may have n joints, which may be translational and rotational, and may have m modes of vibration. The resulting equations of motion may be used in simulations for controls design and analysis, mechanical design and analysis, or operational assessments.

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