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AN IMPROVED ADAPTIVE CONTROL FOR REPETITIVE MOTION OF ROBOTS

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Abstract

An adaptive control algorithm is proposed for a class of nonlinear systems, such as robotic manipulators, which is capable of improving its performance in repetitive motions. When the task is repeated, the error between the desired trajectory and that of the system is guaranteed to decrease. The design is based on the combination of a direct adaptive control and a learning process. This method does not require any knowledge of the dynamic parameters of the system.

1. Introduction

The position servo control is an important and basic problem in the successful operation of robot manipulators. Many methodologies regarding the solution of this problem have appeared in the literature. Recently, the interest in adaptive control of robot manipulators has been growing noticeably [1,3]. This growth is mainly due to the fact that, unlike the non-adaptive control methods, the adaptive control strategies do not require the explicit knowledge of robot dynamics parameters.

On the other hand, recently some works have been reported for the generation of the controlling input for the repetitive motion of dynamical systems [4,6]. These are called learning controllers because the control input generated this way is improved through repeated trials. The method proposed in [4] requires the derivative of the error function in the learning process to guarantee the uniform convergence. Moreover, the conditions on the system's transfer function is very restrictive, and it also requires that the system's inverse dynamics be proper and stable. In [5], the concept of a dual system is used for the recursive generation of the input. This result is mainly applied to linear, time-invariant systems and the conditions for it's convergence are not restrictive. However, the construction of the dual system requires the knowledge of the original system, and it is also not practical. In [6], an adaptive learning controller is designed which can be applied to robotic manipulators and can be easily implemented.

In this paper the concept of learning control is applied to the problem of model reference adaptive control of manipulators. Our objective is to design an adaptive controller capable of learning to improve its performance in repetitive motions. The approach is similar to that used in [6], with a slight modification where the concept of inner-loop/outer-loop control is used. The proposed adaptive controller can be applied to a robot manipulator in non-repetitive motion, in which case it performs as a standard model reference adaptive controller. But when it is commanded to perform a task repeatedly, the learning controller improves its performance in subsequent motions.

We first develop a model reference adaptive control strategy [1,2] to the robot manipulator so that the resulting closed-loop system is equivalent to the preselected reference model. The model reference adaptive controller designed in [1] is shown to force the robot dynamics to follow those of a predetermined linear time-invariant reference model very closely.

Then we develop a new learning controller for a linear time-invariant system L, with guaranteed convergence under very mild conditions. This is similar to the learning controller proposed in [6], which is based on the existance of an auxiliary system L* such that its composition with the original system (i.e., LL*), is positive real. It was shown in [6] that such an auxiliary system can always be found if the original system is stable.

Finally, we apply our learning strategy, as an outer-loop control, to the linearized inner-loop system resulted from applying the model reference adaptive controller to the robot. The proposed control system is shown in Figure 1. The overall closed-loop system is shown to be asymptotically stable, and does not require any knowledge about the dynamic parameters of the robot. The error between the desired response and the actual robot response is guaranteed to approach zero after executing the desired task repeatedly.

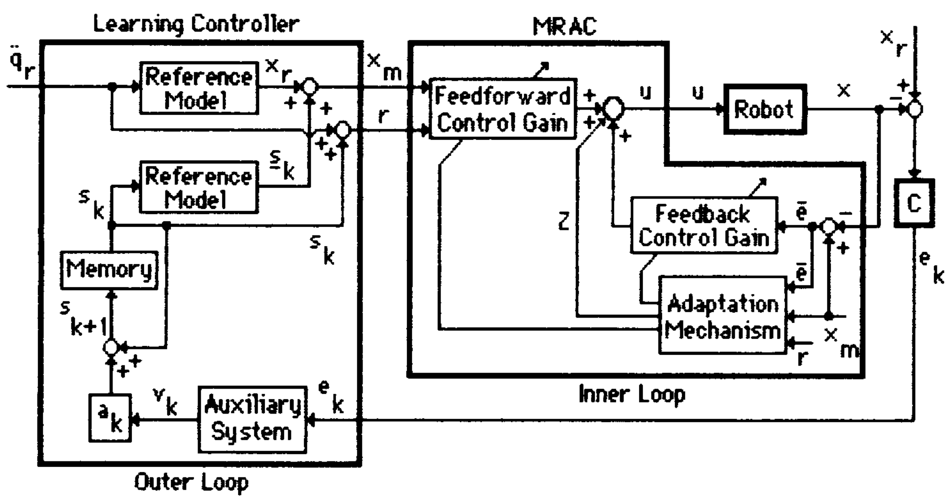

Figure 1

2. Robot Dynamics

The dynamic equation of a robot manipulator is highly nonlinear and is given by

$$M(q)\, q'' + H(q,q')\, q' + g(q) = u \tag{1}$$

where q(t) is the nx1 vector of joint angles, M(q) is the nxn symmetric, positive definite inertia matrix, H(q,q')q' is the nx1 Coriolis and centrifugal force vector, g(q) is the nx1 gravitational force vector, and u is the nx1 applied torque vector for the joint actuators. It can easily be shown that the above equation can also be written as

$$x' = A(x)\,x + B(x)\,u \qquad (2)$$
$$y = C\,x$$

where

$$x(t) = \begin{bmatrix} q \\ q' \end{bmatrix}$$

$$A(x) = \begin{bmatrix} 0 & I \\ A_0(x) & A_1(x) \end{bmatrix}, \quad B(x) = \begin{bmatrix} 0 \\ B_0(x) \end{bmatrix}$$

$$C = [C_1, C_2]$$

for some C_1, C_2, such that x(t) is 2nx1 state vector, u(t) is the nx1 input torque vector and

$A_0 = M^{-1}G$, $A_1 = ^{-1}H$, $B_0 = M^{-1}$, and $g = Gq$.

3. Inner-Loop Adaptive Control

Consider the robotic system, given by equation (2). Let the dynamic equation of a reference model be given by a minimal, 2n-th order, stable linear system

$$x'_m = A_m\,x_m + B_m\,r \qquad (3)$$
$$y_m = C_m\,x_m$$

where x_m is the 2nx1 model state vector, u is the nx1 reference model's input vector,

$$A_m = \begin{bmatrix} 0 & I \\ A_{m0} & A_{m1} \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ B_{m0} \end{bmatrix}$$

$$C_m = C = [C_1, C_2].$$

Now let us define the state error to be

$$\underline{e} = x_m - x. \qquad (4)$$

Then the dynamic equation of the state error is given by

$$\underline{e}' = A\,\underline{e} + (A_m - A)\,x_m + B_m\,r - h - B\,u \qquad (5)$$
$$e = C\,\underline{e}$$

where h is assumed to be any unmodeled disturbances not already considered in the robot dynamics, and $e = y_m - y$ is the output error.

The problem is to design an adaptive controller so that error equation (5) is asymptotically stable, where the state error $\underline{e}$, and hence the output error e, approach zero as time increases.

Result 1

Consider the robot manipulator given by the dynamic equation (2), and the linear, time-invariant, stable, and controllable reference model, given by equation (3). Let the adaptive controller be given by

$$u = K_e \underline{e} + K_x x_m + K_r r + z \tag{6}$$

with the adaptation law

$$\begin{aligned} K'_e &= \alpha_1 E P \underline{e}\,\underline{e}^T + \alpha_2 d/dt(E P \underline{e}\,\underline{e}^T) \\ K'_x &= \beta_1 E P \underline{e}\, x_m^T + \beta_2 d/dt(E P \underline{e}\, x_m^T) \\ K'_r &= \gamma_1 E P \underline{e}\, r^T + \gamma_2 d/dt(E P \underline{e}\, r^T) \\ z' &= \eta_1 E P \underline{e} + \eta_2 d/dt(E P \underline{e}) \end{aligned} \tag{7}$$

where $\alpha_1,\ \beta_1,\ \gamma_1,\ \eta_1 > 0$, $E = [0,I]$, and P and Q are symmetric positive definite matrices such that for some stable matrix D of the designer's choice, we have

$$PD + D^T P = -Q\,. \tag{8}$$

The above adaptive controller results in an asymptotically stable system such that the dynamics of the closed-loop robotic system follow that of the linear reference model. That is, the state error $\underline{e} = x_m - x$, and hence the output error $e = y_m - y$, approach zero as time increases.

Proof

The proof uses Liapunov's direct method for stability, and can be found in [1]. □

The above result indicates that using the proposed adaptive control, the dynamics of the nonlinear robot in the closed-loop matches that of the predetermined, stable, linear, time-invariant reference model. Therefore, the inner-loop control in Figure 1 can be assumed to approximately have the same dynamics as the linear reference model.

4. Outer-Loop Learning Control

Let us consider a linear, stable, time-invariant dynamical system, given by

$$
\begin{aligned}
x' &= A\,x + B\,u \\
y &= C\,x
\end{aligned}
\tag{9}
$$

where x is the n-dimensional state vector, u is the m-dimensional input vector, and y is the m-dimensional output vector. The above system can also be denoted by the linear operator L, where $y = Lu$.

Now let $Y_d(t)$ be the desired output function (trajectory) of the system over the interval [0,T], with the initial state $x(0) = x_0$. Assume that $u_k(t)$ and $y_k(t)$ are the corresponding input and output functions of system (9) over the time interval [0,T] in trial k, with the initial state x_0. Then the learning control strategy is the updating rule that generates the input function $u_{k+1}(t)$ for the interval [0,T] from the knowledge of $u_k(t)$ and the error $e_k(t) = y_d(t) - y_k(t)$. This is then applied to system (9) with the same initial state x_0 at trial k+1 to drive the error function $e_k(t)$ to zero.

Let us define the norm and the inner-product to be given by

$$
\begin{aligned}
\langle x_1 , x_2 \rangle &= \int_0^T x_1^T(t)\, x_2(t)\, dt \\
\|x\|^2 &= \int_0^T x^T(t)\, x(t)\, dt
\end{aligned}
$$

Then the auxiliary system is defined to be a linear, stable, time-invariant system, given by

$$
\begin{aligned}
\underline{z}' &= F\,\underline{z} + G\,e \\
v &= H\,\underline{z} + \underline{E}\,e
\end{aligned}
\tag{10}
$$

or equivalently by the operator L*, as

$$v = L^*\,e$$

such that LL* is a positive real operator. That is

$$\langle x , LL^*\,x \rangle > 0, \text{ for all } x . \tag{11}$$

Now let the dynamics of a reference trajectory be given by the linear operator L such that

$$y_r = L\;\; s \tag{12a}$$

or equivalently

$$
\begin{aligned}
x'_r &= A_r\,x_r + B_r\,s \\
y_r &= C_r\,x_r
\end{aligned}
\tag{12b}
$$

where $s=q''_r$, and

$$x_r = \begin{bmatrix} q_r \\ q'_r \end{bmatrix}$$

$$A(x) = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B(x) = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$C_r = C = [\, C_1 \quad C_2 \,].$$

Utilizing the definition of auxiliary system (10), we can show the following result.

Result 2

Consider the linear, time-invariant, minimal, and stable reference system (12), denoted by operator L. Suppose that a linear auxiliary system, denoted by operator L*, is such that the operator LL* is positive real. Let the learning control strategy for trial k+1 be given by

$$s_{k+1}(t) = s_k(t) + a_k\, v_k(t) \tag{13}$$

with $s_0(t)=q''_r(t)$, where

$$a_k = \langle e_k(t), LL^*\, e_k(t)\rangle \,/\, \|LL^*\, e_k(t)\|^2$$

$$e_k(t) = y_r(t) - y_k(t)$$

and $v_k(t)$ is the output of the linear operator L*, i.e., $v_k = L^*\, e_k$, with zero initial values. The above learning controller is convergent in the sense that $e_k(t)$ vanishes over the interval [0,T] as the number of trials k increases.

Proof

Consider the linear system (12) with input (13). Now let a discrete Liapunov candidate be given by

$$J_k = \|e_k(t)\|$$

where $e_k(t) = y_r(t) - y_k(t)$ is the error at trial k. Then we have

$$
\begin{aligned}
\Delta J_k &= J_{k+1} - J_k \\
&= \|e_{k+1}(t)\|^2 - \|e_k(t)\|^2 \\
&= \|e_k(t) - a_k L v_k(t)\|^2 - \|e_k(t)\|^2 \\
&= a_k^2 \|L v_k(t)\|^2 - 2 a_k \langle e_k(t), L v_k(t)\rangle \\
&= a_k^2 \|L L^* e_k(t)\|^2 - 2 a_k \|e_k(t), L L^* e_k(t)\|^2 .
\end{aligned}
$$

Now taking $a_k = \langle e_k, L L^* e_k\rangle / \|L L^* e_k\|^2$, it is easy to check that ΔJ is minimized with respect to a_k, and hence $\Delta J_k < 0$. Therefore, from the discrete Liapunov method, the learning controller (13) guarantees that the error function $e_k(t)$ approaches zero as the number of trials k increases. □

5. Learning Adaptive Control

Combining Results 1 and 2, it is possible to design a learning adaptive controller for the robotic system. It is desired now that the robot output $y = C x$ follow $y_r = C_r x_r$ in repeated trials. From Figure 1, it can be seen that $x_m = x_r + \underline{s}$, where $\underline{s}_k = [s_k^T, s'_k{}^T]^T$ and $r = q''_r + s_k$. That is the dynamic equation of the reference model can be written as

$$
\begin{aligned}
x''_m &= A_r x_m + B_r r \\
y_m &= C_r x_m .
\end{aligned}
\tag{14}
$$

Now, we have the following result.

Result 3

Consider the robot manipulator given by the dynamic equation (2), and the linear, time-invariant, stable, and controllable reference model given by the operator L as in equation (14). Let the learning adaptive controller be given by

$$
u = K_e \underline{e} + K_x x_m + K_r r + z \tag{15}
$$

with the adaptation law

$$
\begin{aligned}
K'_e &= \alpha_1 E P \underline{e}\,\underline{e}^T + \alpha_2 d/dt(E P \underline{e}\,\underline{e}^T) \\
K'_x &= \beta_1 E P \underline{e}\, x_m^T + \beta_2 d/dt(E P \underline{e}\, x_m^T) \\
K'_r &= \delta_1 E P \underline{e}\, r^T + \delta_2 d/dt(E P \underline{e}\, r^T) \\
z' &= \eta_1 E P \underline{e} + \eta_2 d/dt(E P \underline{e})
\end{aligned}
$$

where the state error $\underline{e} = x_m - x$ is the error between the system and the model states. Also

$$s_{k+1}(t) = s_k(t) + a_k v_k(t) \tag{16}$$

with $s_0(t)=q''_r(t)$ which is generated by reference dynamics equation (12), and

$$a_k = \langle e_k(t), LL^* e_k(t)\rangle / \|LL^* e_k(t)\|^2$$

and also

$$v(k) = L^* e_k(t)$$

with zero initial values, such that LL^* is positive real, $C_r=C=[C_1,C_2]$, and $e_k(t) = y_r(t) - y_k(t)$ is the error function between the desired output trajectory y_r and the robot's output trajectory y in the k-th trial. The above learning adaptive controller is convergent in the sense that starting from the same initial state x_0, the output error $e_k(t)$, and equivalently e(t) and e'(t), , approach zero over the interval [0,T] as the number of trials increases.

Proof

The proof is merely a combination of the proofs of Results 1 and 2, and is found in [6]. □

6. Conclusions

A new adaptive control is proposed, that is capable of improving its tracking performance in repetitive motions. The design can be applied to a class of nonlinear systems that includes robotic manipulators. The controller guarantees that the error between a desired trajectory and the robot's trajectory approaches zero as the number of trials increases. The proposed controller does not require any knowledge of the dynamic parameters of the robot and can be easily implemented.

7. References

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