On Discrete Control of Nonlinear Systems
With Applications to Robotics

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Abstract

Much progress has been reported in the areas of modeling and control of nonlinear dynamic systems in a continuous-time framework. From implementation point of view, however, it is essential to study these nonlinear systems directly in a discrete setting that is amenable for interfacing with digital computers. But to develop discrete models and discrete controllers for a nonlinear system such as robot is a nontrivial task. Robot is also inherently a variable-inertia dynamic system involving additional complications. Not only the computer-oriented models of these systems must satisfy the usual requirements for such models, but these must also be compatible with the inherent capabilities of computers and must preserve the fundamental physical characteristics of continuous-time systems such as the conservation of energy and/or momentum.

In this exploratory presentation we will review preliminary issues regarding discrete systems in general and discrete models of a typical industrial robot that is developed with full consideration of the principle of conservation of energy. Subsequently, we will review some research on the pertinent tactile information processing. Finally, we will review system control methods and how to integrate these issues in order to complete the task of discrete control of a robot manipulator.

1. Introduction

The real-time applications of robotics technology in space and/or in hostile environments require sophisticated use of teleoperational equipment that are capable of maneuvering difficult task in such situations. These nonlinear and inertia-variable systems exhibit a number of challenging problems for us that must be carefully analyzed before implementing a particular design. The overall nonlinear control problem can be divided into the following sub-problems:

> Discrete Systems
> Discrete Robot Models
> Tactile Information
> Control Methods
> Integration and Future Research Directions.

In this exploratory paper we will describe some general results on these issues and we will present some preliminary thoughts on the future directions of research in this area.

2. Discrete Systems

The overall dynamic system is, of course, nonlinear and thus it must be looked at first in the most general sense. Certainly, and with the advent of computer-controlled methodologies, it is preferable to study the modeling of a general dynamical system directly in discrete forms, that are easily amenable for interfacing with digital computers. This approach is often superior to that of discretization of a continuous-time process (model) which inevitably yields errors of significant magnitudes specially for a nonlinear system. Although the robot manipulator is a special case of a nonlinear dynamic system, we can not escape some

commonalities that must be reviewed in the context of a general nonlinear discrete dynamic system.

According to the authoritative survey paper of Jury and Tsypkin [24] a discrete automatic system can be subdivided into the following types.

(a) Relay automatic systems wherein the quantization takes place in jumps in both system parameters and maybe system structure resulting in a nonlinear problem.

(b) Pulse (or impulse) automatic systems also known as sampled-data systems. In this case the system is described by a set of finite-difference equations in time. Here we treat the parameters as constants, or in a realistic but much complicated way as time-varying variables. The theory for time-varying situation is not yet fully developed and this is an exciting and promising research area.

(c) Digital automatic system that combines both cases (a) and (b). Here unavoidably digital computers must be used as part of the overall systems.

Analysis of linear pulsed system entails certain classical methods that are extensively developed and documented in the literature. A special characteristic of linear pulsed systems is that they have a finite critical amplification which is independent of the order of the system [24]. Only recently stability robustness under large parameter variations of this class of systems has been established in [12]. The main analytical *tool* to analyze these dynamic systems is theory of difference equations that is well developed in the early textbook of Levy and Lessman [33].

The synthesis of pulsed system also entails properties that are now considered classical, but nevertheless very important. One issue is concerned with pulse correction or compensation that corresponds to introducing a discrete filter that changes (or corrects) the sequence of pulses [24]. An interesting observation that is reported in [24] and is credited to a 1964 Rutman's paper (in Russian) is as follows. When synthesizing an optimal discrete control system, if we combine discrete correction policy that changes the shape of control pulses with some external control action (which is resulted from the sensory information *per se*), then we get the best result. This simple concept has been extensively studied by a number of researchers in a number of different disciplines. In a very interesting paper on nonlinear discrete-time systems Geromel and Cruz [14] show that the additive perturbation (the same as the external input) is much richer than the gain perturbation (that is designed using feedback automatic policies). They also show that a system with high tolerance to gain perturbation may present a low tolerance to additive perturbation and *vice versa*. Therefore a compromise must be made between weighting these two inputs in a given application and this is, in our opinion, a very interesting result. In an earlier work we used the term auxiliary input to emphasize similar situations, for example [36].

Generally speaking, the synthesis of discrete linear system is well developed for constant parameter cases. It is extremely complicated and to the large extent undeveloped for the case in which the parameters are changing. This is a very exciting problem to investigate and no bonafide procedure is documented for its treatment. It seems that the general theory of averaging, which was introduced for continuous-time differential equations, for example in [19], may be the starting point to deal with these types of problems. Recently, Astrom also suggests that this approach is very promising to understand parameter variation behavior of a dynamic system [4]. The extension of this concept to discrete system is not fully developed, although some results are reported in [2].

3. Discrete Robot Models

The discretization of robot dynamics, which is inherently a variable-inertia dynamic system, involves additional complications. Not only the computer-oriented models of these systems must satisfy the usual requirements for such models, but these must also be compatible with the inherent capabilities of computers and must preserve the fundamental physical characteristics of continuous-time systems. Neuman and Tourassis [42] have developed a

discrete-time dynamic model for robot that guarantees conservation of energy and momentum at each sampling time. The principle of conservation of energy plays the central role in discretization of a nonlinear mechanical system. Here we will briefly describe the discrete model which is reported in [42] for illustration.

Consider the generalized state of an n-degree-of-freedom robot as follows.

$$D(x^P)x^a + C(x^P, x^v)x^v + g(x^P) = u(t). \tag{1}$$

Here $x^P \in R^n$ is the generalized position vector, $x^v = \dot{x}^P \triangleq v$ is the velocity vector and $x^a = \dot{x}^v \triangleq a(x^P, x^v, u)$ is the acceleration vector. The symmetric positive-definite matrix $D(x^P)$ represents an inherently variable-inertia matrix of the robot. The Coriolis and centrifugal forces are shown by $C(x^P, x^v)x^v$; and $g(x^P)$ is the vector of gravity. The external or applied input torque (or force) is shown by $u(t)$. Equation (1) can also be written in a general form as follows.

$$D(x^P) a(x^P, x^v, u) = f(x^P, x^v, u), \tag{2}$$

where $f(x^P, x^v, u)$ represents the remaining nonlinear terms and is the *net* generalized force which is acting on the system. Needless to say that a closed-form solution of (2) is very difficult to attain and the final answer is very complicated to compute. There are, however, a number of functions of position and velocity vectors whose values remain constant throughout the motion and depend only on the initial conditions [42]. These functions are called the *integrals-of-the-motion* [29]. For an n-degree-of-freedom (n-DOF) robot there are $(2n - 1)$ independent such functions. To find all of these functions is equivalent to solving for the entire system of differential equations of the dynamic system.

To discretize a nonlinear mechanical system, Greenspan [17] and [18] has defined a new concept for the work which guarantees the conservation of energy. In this analysis Greenspan has assumed that the mechanical system has constant mass/inertia and the acceleration remains constant in each sampling interval. Neither of these assumptions are true in robotics. These constraints have led Neuman and Tourassis [42] to develop a new discrete model for a 3-DOF cylindrical robot which is based on generating the *integrals-of-the-motion* of this system while preserving the principle of the conservation of energy.

To develop the Neuman and Tourassis discrete model we present the following set of notation first. The *work* that is done by a force f (according to the classical mechanics) is equal to $f \cdot dx$, where dx is displacement along the motion direction and *dot* represents the inner product. Also corresponding to (2) at instant k we have $Da(k) = f(k)$, where D is a constant inertia matrix. The principle of the conservation of energy with constant $D(x^P) \equiv D$ states that the work at instant k is

$$E(k+1) - E(k) \triangleq w(k) = \tfrac{1}{2}v^T(k+1)Dv(k+1) - \tfrac{1}{2}v^T(k)Dv(k). \tag{3}$$

Here E(k) is the total energy at the kth-sampling time. The Greenspan discrete mechanics uses a trapezoidal (smoothing) formula for the position x(k) and a forward Euler formula for the velocity v(k). These equations and an expression for the work can be stated as follows.

$$x(k+1) = x(k) + \frac{T}{2}[v(k+1) + v(k)], \tag{4a}$$

$$v(k+1) = v(k) + Ta(k), \tag{4b}$$

$$w(k) = f(k) \cdot [x(k+1) - x(k)]. \tag{4c}$$

In this study the input is updated at each sampling time and is maintained constant in that period. Our task is to use the above schemes which may not satisfy the principle of the conservation of energy. Or to devise a new scheme to discretize the robot nonlinear dynamics.

To illustrate the Neuman and Tourassis procedure a cylindrical robot dynamics is chosen [9]. The coordinate vector of this robot is q = [θ, z, r], where θ is the rotation angle, z is the vertical translation and r is a radial translation. For this arrangement the kinematic equations and the equation-of-the-motion are, respectively, as follows.

$$\dot{\theta} = \omega, \quad \dot{z} = v, \quad \text{and} \quad \dot{r} = v, \tag{5}$$

$$[J + j(r)]\dot{\omega} + \frac{\partial j}{\partial r}\omega v = F_\theta(t), \tag{6}$$

$$M\dot{v} + Mg = F_z(t), \tag{7}$$

$$m\dot{v} - \tfrac{1}{2}\frac{\partial j}{\partial r}\omega^2 = F_r(t). \tag{8}$$

Here J is the constant inertia for vertical column; j(r) is the variable inertia of the radial link (note that: $j(r) = mr^2 - m_p Rr$); m is the mass of the radial link with the mass of payload m_p, that is concentrated at the tip of radial link; M is the vertically translated mass of column and radial link; and F_θ, F_z, F_r are the external forces/torques that drive the θ, z, and r DOF, respectively [42]. The inertial matrix D(r) = diag[J + j(r)Mm] of the cylindrical robot is diagonal and depends explicitly upon the radial displacement. In (6) the term $(\partial j/\partial r)\omega v$ corresponds to the Coriolis torque and in (8) the term $(\partial j/\partial r)\omega^2$ corresponds to the centrifugal force. We note that in this case the vertical motion (7) is decoupled but (6) and (8) are two coupled-nonlinear differential equations.

If we assume that in each sampling period $F_\theta(k)$ remains constant, then a direct differentiation of (9) below shows that (6) can be expressed as follows.

$$\frac{d}{dt}\{[J + j(r)]\omega\} = F_\theta(k). \tag{9}$$

The complete integrability of this equation is a consequence of the fact that the total energy of the cylindrical robot is independent of the rotation θ. Here the *integral-of-the-motion* reinforces the conservation of the *angular* momentum $p_\theta = [J + j(r)]\omega$.

Similarly, the vertical z-coordinate can be integrated to yield

$$\frac{d}{dt}(Mv) = F_z(k) - Mg, \tag{10}$$

that shows the conservation of the *linear* momentum $p_z = Mv$. Finally, with some algebra we can show that from (8) and (6) we get

$$\tfrac{1}{2}\frac{d}{dt}\{[J + j(r)]\omega^2 + mv^2\} = \frac{d}{dt}[F_\theta(k)\theta + F_r(k)r]. \tag{11}$$

This equation guarantees the conservation of energy for the coupled rotational and radial degree of freedom.

In summary, we have three *integrals-of-the-motion* for a 3-DOF cylindrical robot. Integration of these equations will lead directly to the discrete-time robot model which are exact and as follows.

$$\{J + j[r(k+1)]\}\omega(k+1) - \{J + j[r(k)]\}\omega(k) = TF_\theta(k), \tag{12}$$

$$M[v(k+1) - v(k)] = T[F_z(k) - Mg], \tag{13}$$

$$\tfrac{1}{2}\{J + j[r(k+1)]\}\omega^2(k+1) - \tfrac{1}{2}\{J + j[r(k)]\}\omega^2(k) + \tfrac{1}{2}m[v^2(k+1) - v^2(k)] =$$

$$[\theta(k+1) - \theta(k)]F_\theta(k) + [r(k+1) - r(k)]F_r(k). \tag{14}$$

These equations are developed with no constraints upon T.

Finally, the following smoothing formulae are used to simplify (12) to (14) to yield (18) to (20).

$$\theta(k+1) - \theta(k) = \frac{T}{2}[\omega(k+1) + \omega(k)], \tag{15}$$

$$z(k+1) - z(k) = \frac{T}{2}[v(k+1) + v(k)], \tag{16}$$

$$r(k+1) - r(k) = \frac{T}{2}[v(k+1) + v(k)], \tag{17}$$

$$TF_\theta(k) = \{J + j[r(k+1)]\}\omega(k+1) - \{J + j[r(k)]\}\omega(k), \tag{18}$$

$$TF_z(k) = M[v(k+1) - v(k)] + TMg, \tag{19}$$

$$TF_r(k) = m[v(k+1) - v(k)] - \frac{1}{2}T\{\frac{j[r(k+1)] - j[r(k)]}{r(k+1) - r(k)}\}\omega(k)\omega(k+1). \tag{20}$$

These equations are the discrete state-space model of a cylindrical robot which guarantee the conservation of energy and momentum in each sampling period. The main approximation comes from the first-order smoothing equations. From algorithmic point of view the Neuman and Tourassis discrete model at each sampling time requires that a system of 2n linear and nonlinear coupled equations be computed and/or analyzed. At each sampling instant the above formulation lends itself to a nested two-loop iterative algorithm consisting of an outer loop (which sets the system of 2n equations) and an inner loop (which solves the system of 2n equations). This approach eliminates the indeterminacies that may occur in (20) when r(k+1) becomes r(k). The inner loop corresponding to the kth iteration of the outer loop starts with the current values of all variables. Then from (15) to (20) the next set of values of variables at the (k+1)th-sampling time is generated. This procedure is followed accordingly and when this algorithm identifies that r(k+1) = r(k), it automatically replaces c(k+1) by c(k), where c(k+1) = $\frac{1}{2}\{j[r(k+1)] - j[r(k)]\}$ / [r(k+1) − r(k)] is the centrifugal coefficient in (20). This procedure thus eliminates the indeterminacies by using the last computed value of centrifugal coefficient. The algorithm then repeats. Recently this result of Neuman and Tourassis [42] has been revisited by Lee and Tsay [32], who have introduced a shift transformation matrix based on general discrete orthogonal polynomials. Lee and Tsay have shown that using these polynomials we can transform the difference equations (15) to (20) into a set of algebraic equations resulting in computational simplicity. In other words, instead of using a nested two-loop iterative algorithm we can solve these equations simultaneously and we can reduce the computational burden significantly.

The above model can also be used in a number of inverse problems in order to compute, for example, the desired forces/torques for a desired trajectory. This approach can be also used to compute a sampling time if a desired trajectory is specified. Although in general there are other issues regarding the choice of sampling strategy which must be considered as we will briefly mention them.

Finally, in order to assess the fidelity of a given discrete model we need to compare its response to a known input with the response which will be resulted from other discretization methods. To this effect Neuman and Tourassis [42] have suggested to look at the step response of (18) to (20) with that resulted from discretizing the robot model based on a number of other discretization methods. The initial condition is chosen zero and T = 10 ms with a 3 seconds total simulation time. In particular, Neuman and Tourassis have suggested to select $|\Delta E(k)|/|E(k)|$ and/or $|\Delta p_\theta(k)|/|p_\theta(k)|$ as an appropriate criterion for evaluating the performance of each algorithm. The rationale behind the residual terms is that, in general, we can not compute the exact value of responses analytically. In this study, it has been

shown that the discrete dynamic robot model outperforms constant fixed-sampling-period algorithms including the forth-order-Runge-Kutta algorithm. The Neuman and Tourassis result compares well with the fifth and sixth-order Runge-Kutta-Verner algorithm [42].

Recently, Tumeh has also looked at robot dynamic discretization by using discentralized equivalent joint model that each is developed in a discrete-time form [53]. This work has some potential use but it is based on the assumption of constant inertia in each sampling time which is not desirable.

The research in this area must be continued in order to generate a more versatile discrete model which are applicable to a number of industrial robot manipulators.

4. Tactile Information

Recent experimental studies by Sharpe [52] show that the most exacting tasks are those which can be performed through the feel and not visually. Sharpe has concluded from his experimental studies that for any dexterous skilled operation there must be a normal positional forward bandwidth (20 to 30 Hz) and a high frequency bandwidth force reflection (5 to 10 KHz). To achieve this task a new class of bilateral control system that uses force feedback manipulator called covariant bilateral control (CBC) is then introduced. This method enables us to design various sub-systems independently, and to control a nonlinear and resonant system more effectively than before. The use of *feel* enables us to adapt the level of damping of the overall system by altering the response to the sensing information and it also enables us to sense gravity and inertia effects and to respond to these effectively.

The inherent time delays that are introduced in a bilateral servomechanism causes many concerns regarding stability, fidelity and usefulness of the force reflection. Sharpe's experimental work has shown that the application of CBC in position controlling a resonant load with adjustable delays in the forward and return paths, enables a manipulator operating in earth orbit be conceivably controlled with force reflection from an earth station. This task can be done using a high frequency force feedback information in an adaptive manner. The sub-systems rely on the local control loops and therefore their stability are not disturbed by the transmission delays. The stability of these sub-systems must be studied separately. The experimental work has shown that up to a delay of 6.0 ms the full force feedback loop-gain could be used. For higher delays the acceptable gain setting reduced as delay was increased. With a delay of 120 ms the forces tend to fall below an acceptable threshold [52]. In general, the conclusion is that the CBC manipulator has many interesting properties and is capable of providing a good *feel* even with sub-systems that are having nonlinearity, backlash and delay. Clearly the force fidelity is a function of the quality of the proper sub-system. With force feedback the operator can control the behavior of a resonant system while this control is not possible even when the operator is capable of viewing what is happening to the system in a conventional sense [52]. Also we note that the force reflected onto the operator is measured in terms of various electrical quantities associated with each sub-systems. The effects of delay is very clear to the operator, but the amount of the delay is very difficult to quantify. This fact implies that the operator's response is instinctive. The very fast force feedback response (500Hz) allows the human operator to adapt to needs in controlling an otherwise uncontrollable higher-order resonant system.

5. Control Methods

In an ideal situation, in which the manipulator model and the workspace are known precisely and we have sufficient sensory information, simple controllers will enable us to perform most of our difficult tasks. In a real-world environment, however, the manipulator model and/or parameters are not accurately known and we do not have precise sensory information. Thus we need to devise sophisticated feedback controllers and we need to put together coherently a number of issues which are complementing each other in order to perform a nontrivial telerobotic task.

To study the discretization of a general nonlinear discrete control problem we must choose: first, a proper (or *optimal*) sampling strategy; and secondly a method to deal with the parameter variations. We believe to develop an optimal sampling strategy we should set a constrained optimization scheme in which we define a performance index as a function of sampling time and subsequently that function, subject to the physical (or hardware) realization constraints, will be minimized. A simple version of this problem is reported in [39]. We define a sampling time *optimal* if is physically (or hardware) realizable and it also gives enough times for various data acquisition or computational tasks to be completed. This sampling strategy must also give enough times for controllers to execute their functions.

To account for parameter variations we should use the discrete model reference adaptive control to lay out a foundation for various controller designs. Although this choice may conflict with our sampling method. Subsequently, we must look at parameter variations using the averaging concepts to study its impact on the overall system stability. There are a number of unanswered questions in discrete adaptive control which must be looked at simultaneously and in that sense this is an interesting research area.

Therefore in short the control method must entail certain constrained optimization methods to develop the *optimal* sampling strategy and discrete model reference adaptive control with averaging concepts to design controllers. These general methodologies are well established in their own rights. But a number of important questions regarding discrete non-linear control problems must now be answered before we can successfully complete the task of real-time implementing these control algorithms and before we can successfully assemble a number of perhaps conflicting issues together.

6. Integration and Future Research Directions

Using the methodologies that we have cited before, analytical methods to yield the *optimal* sampling strategies for real-time implementation, specially in conjunction with the force control; and the robustness of system performance regarding the large parameter changes must all be developed. Subsequently, synthesis methods that incorporate this information to correct or to compensate for discretization errors as well as all the other usual disturbances that exit in a complicated dynamic system such as a space teleoperator must be established.

The utilization of discrete averaging theory to deal with parameter variations will advance the state of knowledge in design of discrete controllers for a general nonlinear system. This new result will enable us to address many uncertainties that we usually left out due to the lack of information regarding: friction; unprecise inertia; and unmodeled dynamics or other external uncertainties in our system.

7. References

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