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Micro-Mechanical Analysis of Damage Growth and Fracture in Discontinuous Fiber Reinforced Metal Matrix Composites

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by

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ABSTRACT

The near-crack-tip stresses in any planar coupon of arbitrary geometry subjected to mode I loading may be equated to those in an infinite center-cracked panel subjected to the appropriate equivalent remote biaxial stresses (ERBS). Since this process can be done for all such mode I coupons, researchers may focus their attention on the behavior of the equivalent infinite cracked panel. To calculate the ERBS, the constant term in the series expansion of the crack-tip stress must be retained. It is proposed that the ERBS may be used quantitatively to explain different fracture phenomena such as crack branching.

Equivalent Remote Biaxial Stresses (ERBS) for Planar Cracked Body Analyses

Introduction

In general, the fracture behavior of arbitrary cracked bodies subjected to various loadings is dependent on the near-crack-tip stress state. Exact solutions, however, can be very complicated, and in many cases almost impossible to obtain, even though the general form of the stress state is known. For this reason numerical methods (finite element, boundary element, etc.) have been used to determine local stress states. One of the few exact solutions available is that of an infinitely large center-cracked panel. Authors such as Inglis [1], Griffith [2], Muskhelishvili [3], Lekhnitskii [4], Savin [5], and others have considered this problem for both isotropic and orthotropic materials. In this paper we will show how the solution for the infinite cracked plate may be used in analyzing the stress states in arbitrary cracked coupons subjected to mode I loading.

To start, consider the near-crack-tip stress state. Using a form similar to that of Savin [5], it can be shown that the series expansions of the stresses around the crack tip in an arbitrary anisotropic body subjected to mode I loading (see Figure 1a) are:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} Re\{\frac{S_1 S_2}{(S_1 - S_2)}[\frac{S_2}{\sqrt{C_2}} - \frac{S_1}{\sqrt{C_1}}]\} + A_I + O(\sqrt{r}) \, , \qquad (1)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} Re\{\frac{1}{(S_1 - S_2)}[\frac{S_1}{\sqrt{C_2}} - SHxub\frac{2}{\sqrt{C_1}}]\} + O(\sqrt{r}) \, , \quad and \qquad (2)$$

$$\tau_{xy}=\frac{K_I}{\sqrt{2\pi r}}Re\{\frac{S_1S_2}{(S_1-S_2)}[\frac{1}{\sqrt{C_1}}-\frac{1}{\sqrt{C_2}}]\}+O(\sqrt{r})\,. \tag{3}$$

where K_I and A_I are constants. In an infinite center-cracked panel of crack length c^* (see Figure 1b),

$$K_I=\sigma_y^\infty\sqrt{\pi c^*}\,,\quad and \quad A_I=\sigma_y^\infty Re\{S_1S_2\}+\sigma_x^\infty,$$

giving:

$$\sigma_x=\sigma_y^\infty\frac{\sqrt{\pi c^*}}{\sqrt{2\pi r}}Re\{\frac{S_1S_2}{(S_1-S_2)}[\frac{S_2}{\sqrt{C_2}}-\frac{S_1}{\sqrt{C_1}}]\}+\sigma_y^\infty Re\{S_1S_2\}+\sigma_x^\infty+O(\sqrt{r})\,, \tag{4}$$

$$\sigma_y=\sigma_y^\infty\frac{\sqrt{\pi c^*}}{\sqrt{2\pi r}}Re\{\frac{1}{(S_1-S_2)}[\frac{S_1}{\sqrt{C_2}}-\frac{S_2}{\sqrt{C_1}}]\}+O(\sqrt{r})\,,\quad and \tag{5}$$

$$\tau_{xy}=\sigma_y^\infty\frac{\sqrt{\pi c^*}}{\sqrt{2\pi r}}Re\{\frac{S_1S_2}{(S_1-S_2)}[\frac{1}{\sqrt{C_1}}-\frac{1}{\sqrt{C_2}}]\}+O(\sqrt{r})\,. \tag{6}$$

where C_1 and C_2 are defined by,

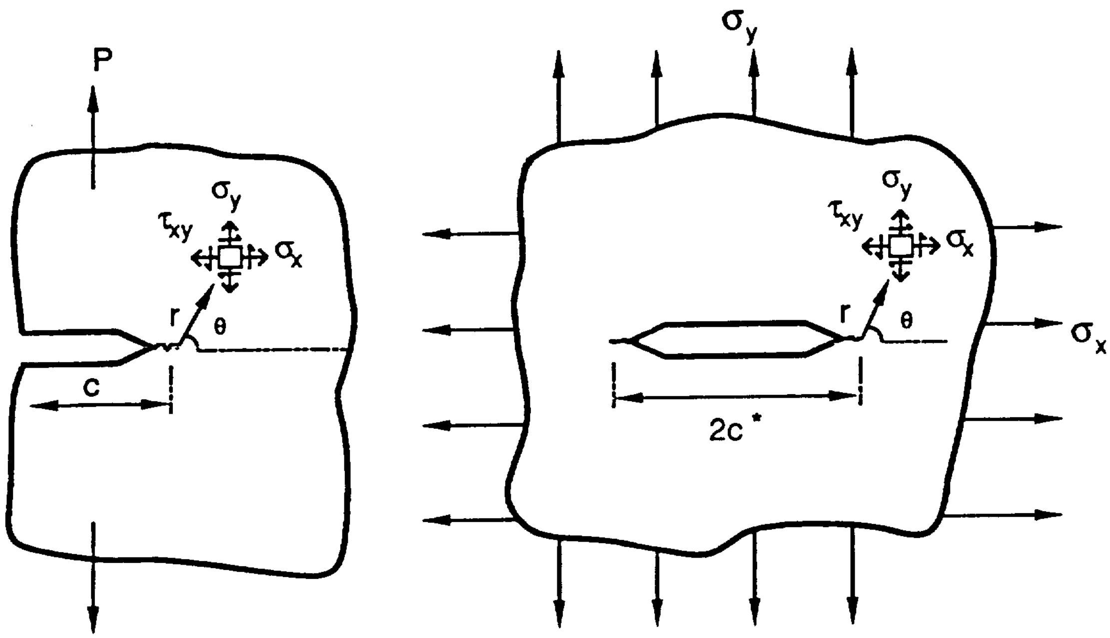
$$C_i=\cos(\theta)+S_i\sin(\theta)\,,$$

and S_1 and S_2 are the appropriate complex roots of the polynomial,

$$A_{11}S^4-2(A_{12}+A_{66}S^2)-2A_{26}S+A_{22}=0\,. \tag{7}$$

The coefficients A_{ij} are the classical anisotropic elastic compliances.

Fig. 1



(a) (b)

Equivalent Remote Biaxial Stresses

The near-crack-tip stresses (retaining only the singular and constant terms) of any arbitrary coupon subjected to mode I loading can be equated to those in the infinite biaxially loaded center-cracked panel (see Figure 1). This is done by equating the transverse stresses of equations (1) and (4), and the longitudinal stresses series of equations (2) and (5) and determining the **Equivalent Remote Biaxial Stresses** (ERBS i.e. σ_x^∞, σ_y^∞) which, when applied to the infinite cracked panel, make the near-crack-tip stresses equal to those in the arbitrary body. Because the stress distributions in both coupons have the same r and θ dependence, ERBS are independent of r and θ. To be specific the infinite cracked plate can be assumed to have a fixed crack length (c^*=constant). The ERBS are then as follows:

$$\sigma_x^\infty = \frac{-K_I}{\sqrt{\pi c^*}} Re\{S_1 S_2\} + A_I, \quad and \tag{8}$$

$$\sigma_y^\infty = \frac{K_I}{\sqrt{\pi c^*}} . \tag{9}$$

From these equations, the ERBS ratio (B) is defined as:

$$B = \frac{\sigma_x^\infty}{\sigma_y^\infty} = -Re\{S_1 S_2\} + \frac{A_I}{K_I}\sqrt{\pi c^*} . \tag{10}$$

For an orthotropic material, equation (7) reduces to:

$$A_{11}S^4 + (2A_{12}+A_{66})\,S^2 + A_{22} = 0, \tag{11}$$

where,

$$A_{11}=\frac{1}{E_{11}}, \qquad A_{22}=\frac{1}{E_{22}}, \qquad A_{12}=-\frac{\nu_{12}}{E_{11}}, \qquad \text{and} \quad A_{66}=\frac{1}{G_{12}}. \tag{12}$$

In order to find S_1 and S_2 a simplification may be made,

$$G_{12}=\frac{E_1 E_2}{E_1+E_2+2\nu_{12}E_2}$$

(An approximation suggested in [6]). Using this relation along with equations (12), equation (11) reduces to:

$$S^4+\left(1+\frac{E_1}{E_2}\right)S^2+\frac{E_1}{E_2}=0.$$

The roots of this equation are:

$$S=\pm i, \quad \text{and} \quad \pm i\sqrt{\frac{E_1}{E_2}}.$$

Of these roots, only the two positive roots are the appropriate roots to be used in equations (8-10), giving:

$$\sigma_x^{\infty}=\sqrt{\frac{K_I}{\pi c^*}}\cdot\sqrt{\frac{E_1}{E_2}}+A_I, \tag{13}$$

$$\sigma_y^{\infty}=\frac{K_I}{\sqrt{\pi c^*}}, \quad \text{and} \tag{14}$$

$$B=\frac{A_I}{K_I}\sqrt{\pi c^*}+\sqrt{\frac{E_1}{E_2}}. \tag{15}$$

To calculate the ERBS it is necessary to determine the near-crack-tip stresses in the arbitrary coupon (either analytically or numerically) to find K_I and A_I, and to choose the fixed crack length c^*. Since these expressions (8-10, and 13-15) for calculating ERBS are independent of r and θ, the stress state at any point near the crack-tip in the arbitrary coupon is sufficient to be able to calculate the constants K_I and A_I. In most cases ERBS will be used to analyze the near-crack-tip stresses in the infinite panel, therefore c^* must be chosen to be much larger than the radius at which the analysis will be made ($c^* >> r$). This restriction will ensure that the first two terms in the series expansion of the stresses around the crack-tip in the infinite panel dominate the stress field. Possible choices for c^* would be 1.0 in., or 25 mm..

Some important observations can be noted from the expressions (8-10, or 13-15). First, if the near-crack-tip stress state is the same, then it can be expected that the materials response to loading would be similar in both the arbitrary coupon and the infinite panel. For example, this implies that both configurations would respond in a similar manner under fatigue loading. Under quasi-static loading both coupons would have the same fracture characteristics. Thus, if crack branching occurs either under fatigue or during fracture in the infinite panel, one could expect branching to occur in the arbitrary coupon and visa versa. Because of this expected loading response similarity, various near-crack-tip phenomena (such as crack branching) may be modeled mathematically by studying the biaxially loaded infinite center-cracked panel (with the fixed crack length of $2c^*$), and then relating coupons of arbitrary geometries to the infinite solution. We are currently studying the use of ERBS in predicting fracture in advanced materials (such as composites) and in non-plane strain coupons [7-8]. Inconsistencies in both values for fracture toughness and crack growth direction (during both fracture and fatigue) have been noted in these materials.

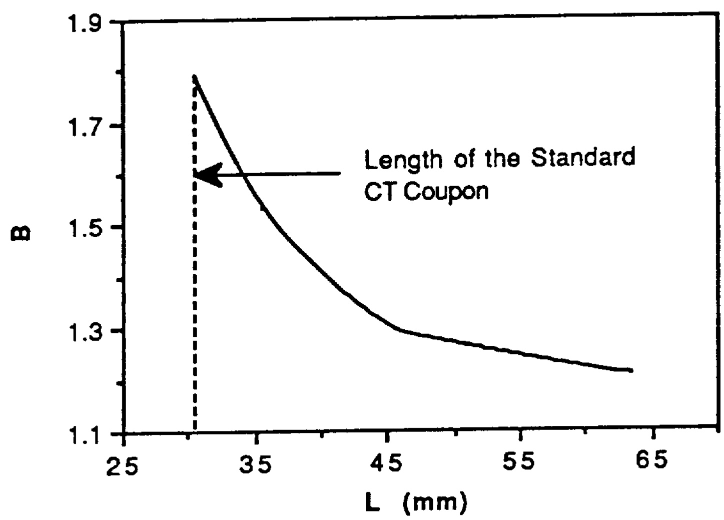
It is important to note that the constant term in the series expansions of the transverse stress is not neglected in the formulation, and the elements of expressions (8-10, and 13-15) which come from the constant term are not insignificant when compared with the other terms in the equations. The constant term has been shown by many researchers [9-12] to be significant when considering fracture behavior of materials.

Another interesting feature of the model is that the ERBS show considerable sensitivity to changes in geometry and crack length. This can be seen in Figures 2 and 3, which are numerical results for an edge-notched coupon (geometry seen in Figure 4 compared with that of a standard compact tension coupon) of a material with $E_1/E_2=0.8$. Figures 2 depicts the change in the ERBS ratio, B, with change in the length of the edge-notched coupon (c=19.05mm). Figure 3 gives the change in B with crack length in an edge-notched coupon (L=30.5mm). The numerical work was done using a finite element alternating method developed by Raju and Fichter [13].

Reedy [15] noted that branching occurred in compact tension (CT) coupons, but did not occur in center-cracked (CCT) panels of continuous boron fiber reinforced aluminum. Qualitatively, this can be explained by considering the pattern depicted in figure 3. For a material with $E_1/E_2=0.8$, the ERBS ratio (B) in a CCT coupon would be nearly zero and in a CT coupon B would be close to two. This would lead one to expect a higher tendency for crack branching in a CT coupon. The effect of the biaxial stress ratio on crack branching in infinite center-cracked panels subjected is discussed in [16-17]. The occurrence of splitting cannot be explained using the stress intensity factor approach of LEFM. As indicated earlier, the authors are currently investigating the use of ERBS to quanitativly explain this branching behavior.

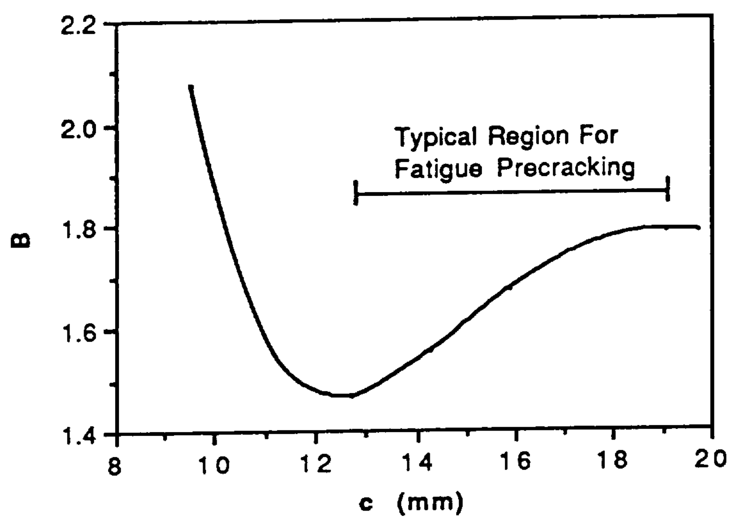
The purpose of this paper has been to present in detail the mathematical derivation of ERBS, and to discuss qualitatively the uses of ERBS. In future papers we will present the results of a detailed experimental program to investigate the use of ERBS in predicting the fracture behavior of cracked bodies of various geometries.

Fig. 2



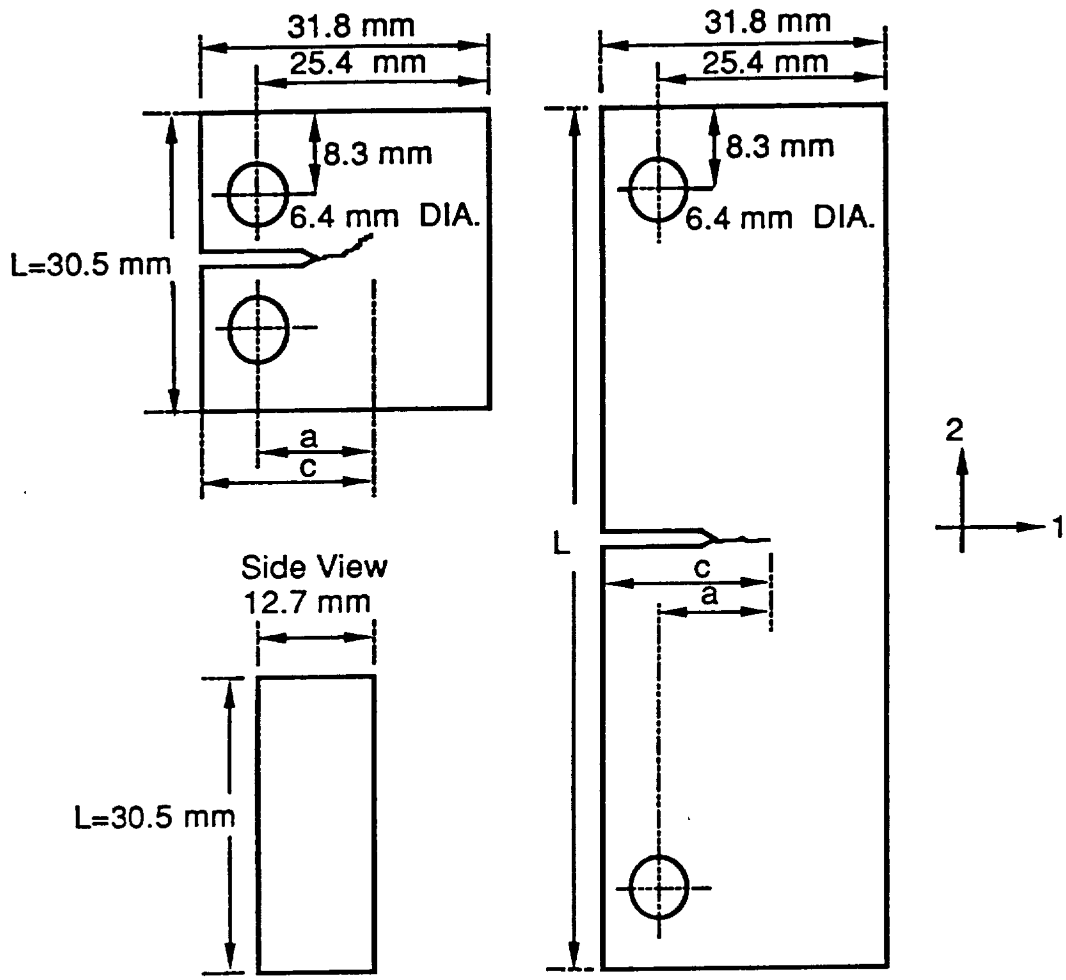
1.9
1.7
1.5
1.3
1.1
B
Length of the Standard
CT Coupon
25
35
45
55
65
L (mm)

Fig. 3



2.2
2.0
1.8
1.6
1.4
B
8
10
12
14
16
18
20
c (mm)
Typical Region For
Fatigue Precracking

Fig. 4


31.8 mm
25.4 mm
8.3 mm
6.4 mm DIA.
L=30.5 mm
a
c
Side View
12.7 mm
L=30.5 mm
31.8 mm
25.4 mm
8.3 mm
6.4 mm DIA.
L
c
a
2
1

(a) (b)

CONCLUSIONS

1. The near-crack-tip stress state in an arbitrary planar coupon subjected to mode I loading may be equated to those in an infinite cracked panel of the same material subjected to the appropriate **equivalent remote biaxial stresses (ERBS)**.

2. We suggest that researchers focus their mathematical analyses on biaxially loaded infinite cracked panels and relate arbitrary coupon geometries to such studies through the use of ERBS.

3. ERBS may be used to qualitatively explain crack branching seen by Reedy [15].

4. The constant term in the series expansion of the near-crack-tip stresses is neither neglected, nor insignificant in the development of ERBS.

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List of Captions

Figure 1: Comparison of: a) an edge-notched coupon of arbitrary geometry with that of b) an infinite center-cracked panel.

Figure 2: B as a function of coupon length (L) for an edge-notched coupon with crack length c=19.05mm.

Figure 3: B as a function of crack length (c) for an edge-notched coupon with coupon length L=30.5mm.

Figure 4: Comparison of: a) the standard compact tension coupon geometry with that of b) the edge-notched coupon used in the numerical analysis.