GLOBAL OPTIMIZATION METHODS FOR

ENGINEERING DESIGN

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PROBLEM DEFINITION

The problem is to find a global minimum for the Problem P. Necessary and sufficient conditions are available for *local optimality*. However, global solution can be assured only under the assumption of convexity of the problem. If the constraint set S is compact and the cost function is continuous on it, existence of a global minimum is guaranteed. However, in view of the fact that *no global optimality conditions* are available, a global solution can be found only by an *exhaustive search* to satisfy Inequality (5). The exhaustive search can be organized in such a way that the entire design space need not be searched for the solution. This way the computational burden is reduced somewhat.

Problem P: Find a design variable vector $\mathbf{x}$ to minimize a cost function

$$f(\mathbf{x}) \text{ for } \mathbf{x} \, \varepsilon \, S \subset R^n \tag{1}$$

where S is the constraint set defined as

$$S = \{ \mathbf{x} \mid g_i(\mathbf{x}) = 0, \ i = 1 \text{ to } p; \ g_i(\mathbf{x}) < 0, \ i = (p+1) \text{ to } m \} \tag{2}$$

Local Minimum $\mathbf{x}^*$

$$f(\mathbf{x}^*) \leq f(\mathbf{x}) \text{ for all } \mathbf{x} \in N(\mathbf{x}^*,\delta) \cap S \tag{3}$$

$$N(\mathbf{x}^*,\delta) = \{ \mathbf{x} \mid \|\mathbf{x}\text{-}\mathbf{x}\| < \delta \} \tag{4}$$

Global Minimum $\mathbf{x}^*$

$$f(\mathbf{x}^*) \leq f(\mathbf{x}) \text{ for all } \mathbf{x} \in S \tag{5}$$

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GLOBAL OPTIMIZATION ALGORITHMS

Most global optimization methods developed in the literature are for the unconstrained problems. It is generally assumed that the constraints can be handled by adding a penalty term to the cost function. Therefore, unconstrained algorithms can be useful. Some of the methods date as far back as 1960s. In the following, we outline some of the algorithms that have been presented in the literature.

The Tunneling Method

The tunneling method was initially developed for unconstrained problems and then extended to the constrained problems [1]. The basic idea of the method is to first find a local minimum x^* for the function $f(x)$. Any reliable and efficient method can be used in this step. Once this has been done another starting point is found that is different from x^* but has cost function value as $f(x^*)$. This can be expressed as a problem of finding root of the nonlinear equation

$$f(x) = f(x^*) \tag{6}$$

that is different from x^*. Again, any reliable and efficient algorithm for finding roots of nonlinear equations, such as the stabilized Newton's method can be used. Once the root of Eq. (6) has been obtained, the method to determine local minimum of $f(x)$ is used to determine the new solution point. The process is repeated until there is no other root of Eq. (6) except $x = x^*$. The nonlinear function defined in Eq. (6) or its modification is called the tunneling function. The phase where root of Eq. (6) is sought is called the *tunneling phase*.
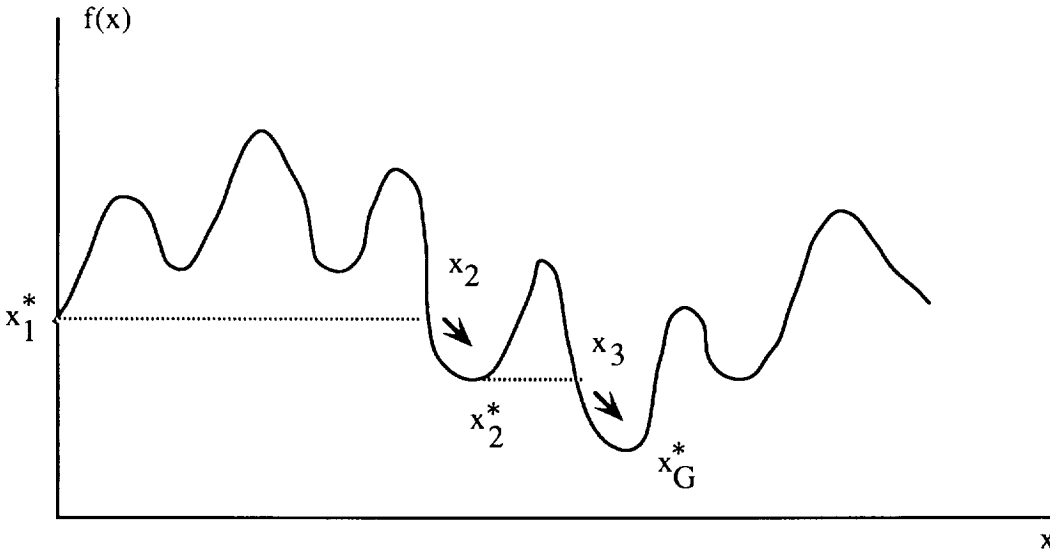

Figure   Basic Concept of Tunneling Algorithm.
The Algorithm tunnels below irrelevant
minima and approaches the global
minimum in an orderly manner [1].

GLOBAL OPTIMIZATION ALGORITHMS (Cont'd)

Stochastic Methods

These methods are based on statistical concepts [2,3].

Random Search
    Global-Local Phase
    Multistart
    Region of Attraction
    Clustering Method

The Annealing Algorithm

This algorithm, also based on probabilistic concepts, can be used to find global optimum solution [4].

Can be used for discrete variables
Analogy between Combinatorial Optimization and Annealing Process
Concept of Statistical Mechanical System
$H(x)$: Hamiltonian (total energy)

Boltzmann-Gibbs Distribution:

$$p(x) = \frac{1}{Z} \exp\{-H(x)/T\} \tag{7}$$

where T is a temperature and Z is a normalization constant (statistical sum).

Let x^* be the equilibrium configuration of the system, i.e.,

$$H(x^*) = \min_{x \in S} H(x) \tag{8}$$

Then the probability of the equilibrium state is maximal, i.e.,

$$p(x^*) = \max_{x \in S} p(x) \tag{9}$$

The Genetic Algorithm

This method is also in the category of stochastic search method, such as the simulated annealing [5,6], in that both methods have their basis in *natural processes*.

Suitable for Discrete Variable Optimization

Three Operators:
1. Reproduction
2. Crossover
3. Mutation

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ZOOMING ALGORITHM FOR GLOBAL MINIMUM SOLUTION

This new global minimization algorithm combines a local minimization algorithm with successive refinements of the feasible region to eliminate regions of local minimum points to "zoom-in" on the global solution. The basic idea is to initiate the search for a local minimum from any point - feasible or infeasible point. Once a local minimum point has been found, the problem is redefined such that the current solution is eliminated from any further search. The search process is reinitiated and a new minimum point is found. The process is continued until no other minimum point can be found.

Once a local minimum point has been obtained, the problem is redefined by adding an additional constraint as follows:

minimize $f(\mathbf{x})$

subject to

$$g_i(\mathbf{x}) = 0, \quad i = 1 \text{ to } p$$

$$g_i(\mathbf{x}) \le 0, \quad i = (p+1) \text{ to } m$$

$$f(\mathbf{x}) \le \gamma f(\mathbf{x}^*) \tag{10}$$

where $f(\mathbf{x}^*)$ is the cost function value at the current minimum point and γ is any number   between 0 and 1 if $f(\mathbf{x}^*) > 0$, and $\gamma > 1$ if $f(\mathbf{x}^*) < 0$. Constraint of Eq. (10) can be written differently as follows:

$$f(\mathbf{x}) \le c \tag{11}$$

$$f(\mathbf{x}) \le f(\mathbf{x}^*) - r \, |f(\mathbf{x}^*)| \tag{12}$$

where $c < f(\mathbf{x}^*)$ and $0 < r < 1$.

EXAMPLE ILLUSTRATING THE CONCEPT

Minimize $f(x) = -(x_1 - 1.5)^2 - (x_2 - 1.5)^2$

subject to

$$x_1 + x_2 - 2 \leq 0$$
$$-x_1 \leq 0, \quad -x_2 \leq 0$$

There are three local minimum points:

1. $(0,2)$, $f = -2.5$
2. $(2,0)$, $f = -2.5$
3. $(0,0)$, $f = -4.5$

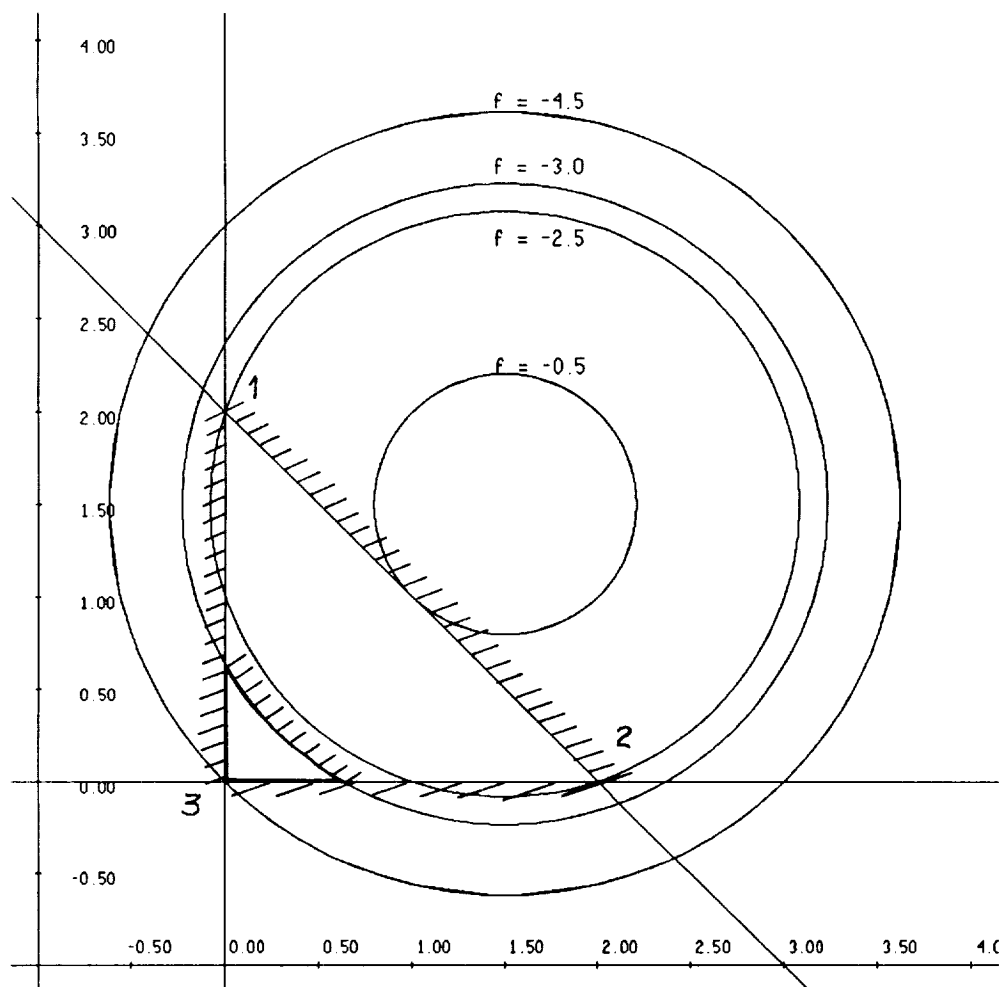The figure illustrates the basic concept of zooming algorithm.

Figure: Graphical Solution for the Example

NUMERICAL EXAMPLE

Minimize $f(x) = 9x_1^2 + 18x_1x_2 + 13x_2^2$

subject to

$$x_1^2 + x_2^2 + 2x_1 = 16$$

This problem has two local minimum points:

1. (2.5945,-2.0198), f = 19.291
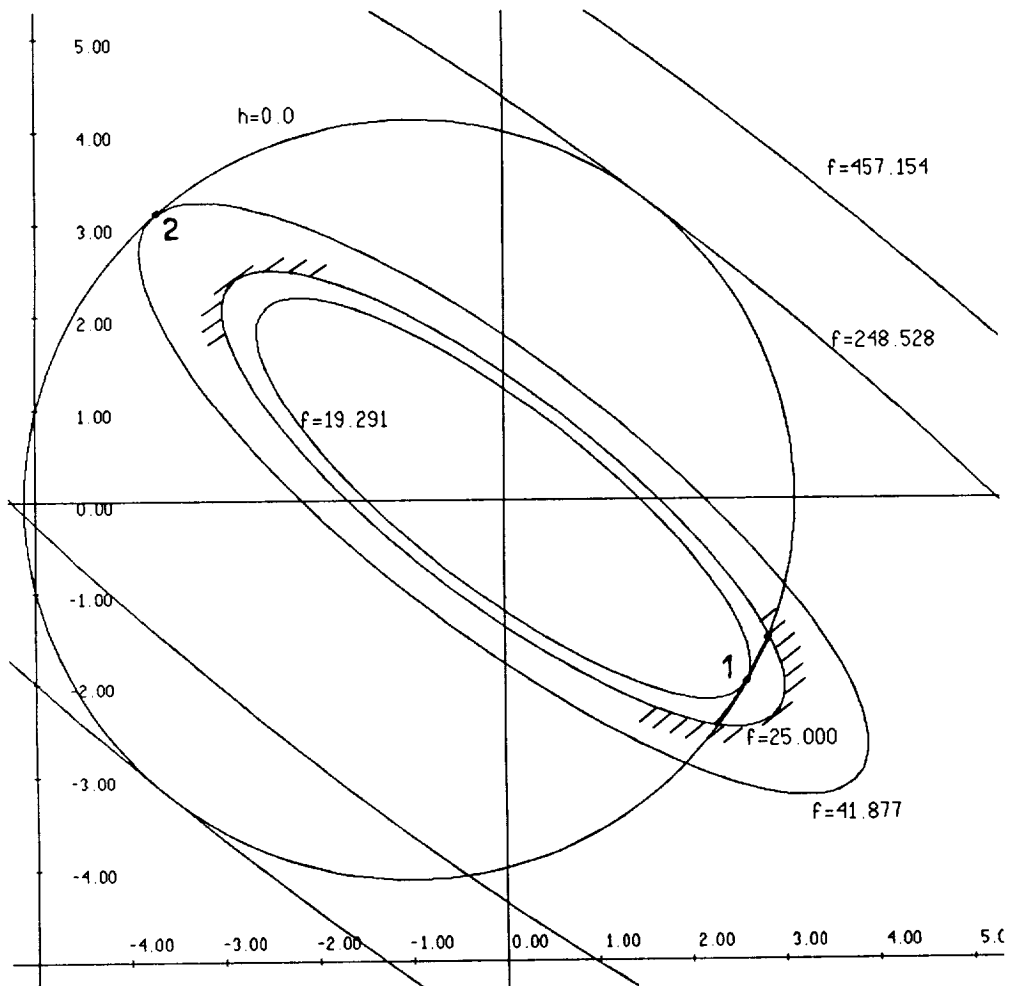2. (-3.7322,3.0879), f = 41.877

Figure: Graphical Solution for the Example

NUMERICAL EXAMPLE

Minimize $\quad f = 2x_1 + 3x_2 - x_1^3 - 2x_2^2$

subject to

$$x_1 + 3x_2 \leq 6$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

This problem has four local minimum points:

1. $(0,0)$, $\quad f = 0.0$
2. $(2,0)$, $\quad f = -4.0$
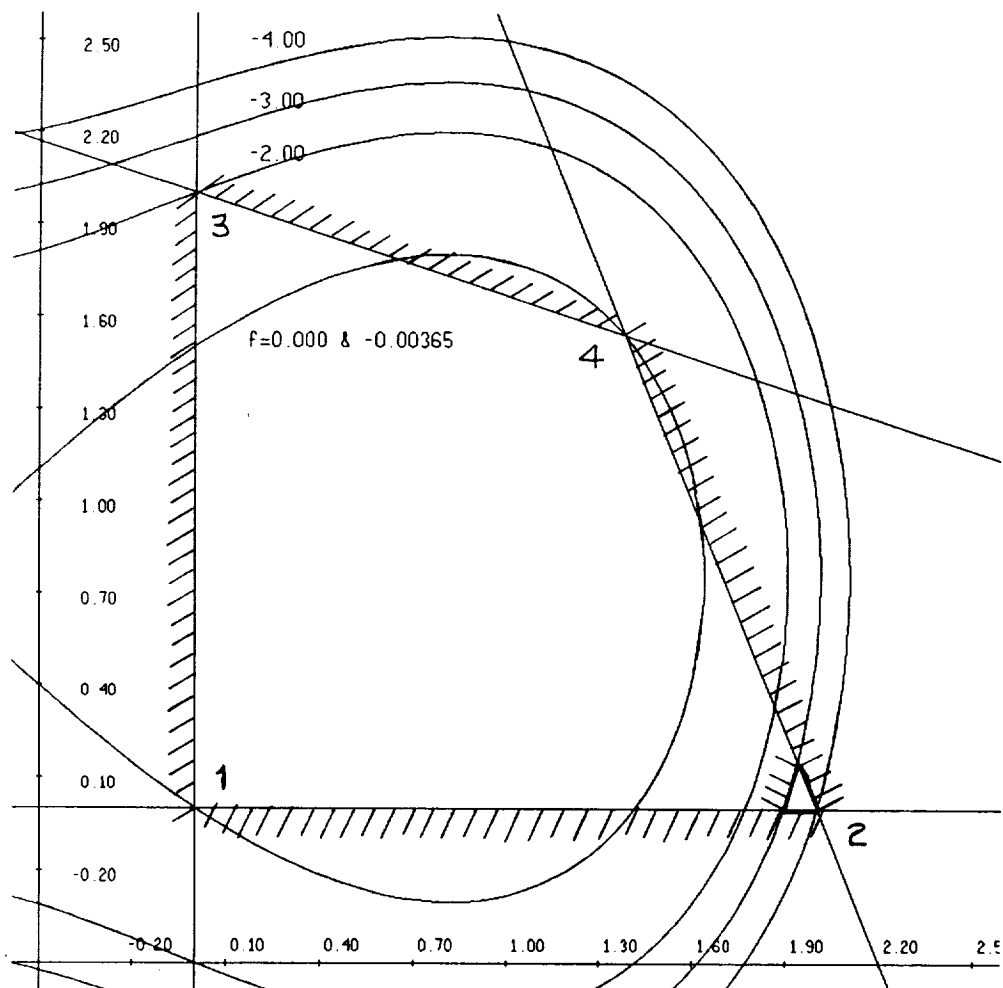3. $(0,2)$, $\quad f = -2.0$
4. $(1.38462, 1.53846)$, $f = -0.003654$

Figure: Graphical Solution for the Example

SIMULTANEOUS CONTROL AND DESIGN OF STRUCTURES

Problem Formulation [7,8]

State Equation: $\dot{x} = Ax + Bf$

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -\omega^2 & -2\zeta\omega \end{array}\right], \quad B = \left[\begin{array}{c} 0 \\ \hline \phi^T D \end{array}\right]$$

Performance Index:

$$PI = \int_0^t [(x,Qx) + (f,Rf)]dt$$

State Feedback Control Law:

$$f = -Gx, \quad G = R^{-1}B^T P$$

$$A^T P^T - PBR^{-1}B^T P + PA + Q = 0$$

Close-Loop System: $\dot{x} = \bar{A}x$

$$\bar{A} = A - BG$$

Complex Eigenvalues and Damping of Close-Loop System

$$\lambda_i = \tilde{\sigma}_i \pm j\,\tilde{\omega}_i$$

$$\xi_i = -\tilde{\sigma}_i/(\tilde{\sigma}_i^2 + \tilde{\omega}_i^2)^{1/2}$$

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EXAMPLE: ACOSS-IV Model

Minimize weight, $W = \Sigma \, \rho_i A_i L_i$

subject to $\quad \xi_j^* - \xi_j \leq 0, \qquad j = 1,2,..$

$\qquad\qquad \omega_j^* - \tilde{\omega}_j \leq 0, \qquad j = 1,2,..$

$\qquad\qquad A_{i\ell} - A_i \leq 0,$

$\tilde{\omega}_1 = 1.341, \qquad \tilde{\omega}_2 \geq 1.5, \qquad \xi_i = 0.1093, \qquad i = 1 \text{ to } 4$
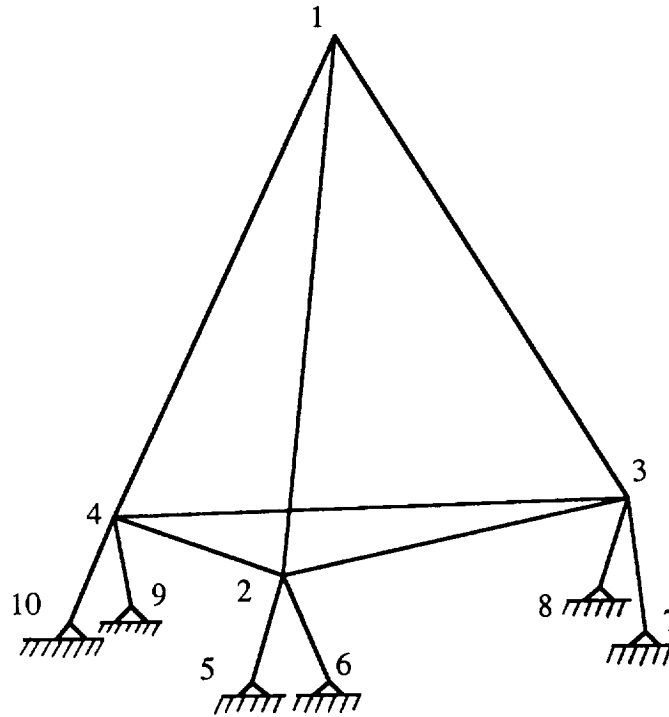
For Global Solution: $\quad W \leq W^*$

Figure: ACOSS-IV Model

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RESULTS FOR 12-BAR ACOSS-IV MODEL

Problem No. $\rightarrow$	1	2	3	4	5
Cost Constraint (W*)	100.0	28.00	24.00	20.75	19.00
Optimum Weight	31.25	28.00	23.29	20.75	No sol.
No. of Iterations	35	26	36	28	35

Starting Point for all problems:

$A_i = 1000$ for $i = 1,2,5,6$; $A_i = 100$ for others

$A_{i\ell} = 10$, $i = 1$ to 12

Convergence criteria:

Constraint Feasibility $\leq 0.1\%$

$\|$Search Direction$\| \leq 0.01$

CONCLUSIONS

1. Zooming algorithm for global optimizations appears to be a good alternative to stochastic methods. More testing is needed.

2. A general, robust, and efficient local minimizer is required. IDESIGN [9] was used in all numerical calculations which is based on a sequential quadratic programming algorithm.

3. Since feasible set keeps on shrinking, a good algorithm to find an initial feasible point is required. Such algorithms need to be developed and evaluated.

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