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CONSERVATIVE BUFFERING OF APPROXIMATE NONLINEAR CONSTRAINTS+

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INTRODUCTION

In engineering design practice behavior is usually predicted based on some known nominal design. However, when the design is fabricated it will differ from the nominal design because of manufacturing tolerances. In order to generate nominal designs that will still satisfy behavior constraints in the presence of manufacturing tolerances, engineers resort to the use of safety factors, over and above those introduced to account for other uncertainties (e.g. in load conditions, material properties, analysis modeling). The accurate selection of the values of these manufacturing tolerances safety factors is dependent on the capability of the engineer to determine the sensitivity of the critical constraints to changes in the design variables. This process usually leads to overly conservative designs.

The task of choosing safety factors is much more difficult in structural synthesis because: 1) it is not known which constraints will be active at the final design, 2) as the design changes during the synthesis process the sensitivities of the constraints with respect to the design variables also change, and 3) the imposition of the safety factors themselves may change the set of critical constraints. These difficulties can be overcome with the approximation concepts approach to structural synthesis by buffering the approximate constraints with quantities that are related to the design variable tolerances and the accurate sensitivities of the constraints with respect to the design variables. Designs generated by this approach tend to be feasible but not overly conservative.

Problems:

- Design variable tolerances lead to analysis errors.
- Choice of accurate safety factors is dependent on engineers intuition.

Difficulties in Structural Synthesis:

- Critical constraints in the final design are not known.
- Sensitivities of constraints with respect to design variables change during synthesis process.
- Safety factors may change the set of critical constraints.

Solution:

- Use the approximation concepts approach to structural synthesis with the constraints buffered by values that are related to the constraint sensitivities and design variable tolerances.

Figure 1

MATHEMATICAL PROBLEM STATEMENT

The structural synthesis problem is stated as: Minimize the weight of a structure (W) that is a function of the design variables ($\mathbf{Y}$) subject to m constraints (displacements, stresses, and frequencies) ($g_j(\mathbf{Y})$). The n design variables are member cross sectional dimensions and nonstructural masses. The design variables are constrained to be in some specified range ($Y_i^L \leq Y_i \leq Y_i^U$).

The design variables have tolerances $\pm\Delta Y_i$. These tolerances may be a percentage (k_i) of the current design variable values.

$$\text{Minimize} \quad \mathrm{W}(\mathbf{Y})$$

$$\text{subject to} \quad \begin{array}{ll} g_j(\mathbf{Y}) \leq 0 & j = 1, 2, 3, \ldots m \\ Y_i^L \leq Y_i \leq Y_i^i & i = 1, 2, 3, \ldots n \end{array}$$

with design variable tolerances $\pm \Delta Y_i$

which may be a percentage of the current design variable value:

$\pm \Delta Y_i = \pm k_i Y_i$

Figure 2

APPROXIMATION CONCEPTS APPROACH TO STRUCTURAL SYNTHESIS

In the approximation concepts approach to structural synthesis an approximate optimization problem is constructed and solved at each design iteration. The use of approximations that better capture the behavior of the actual problem will, in general lead to faster design convergence. Linear Taylor Series Approximations were first used to form approximate problems (Ref. 1). It was observed that displacements and stresses were functions of the reciprocals of the design variables in statically determinate structures. This led to the useof approximations with respect to the inverse of the design variables (Ref. 2). The use of a mixture of linear and reciprocal (hybrid) approximations, based on the sign of the partial derivatives, was found to be a more conservative approximation (Ref. 3) and led to a convex design space (Ref. 4).

More complex and accurate nonlinear approximations, which capture certain explicit nonlinearities of the problem, can be constructed if approximations are formed with respect to intermediate design variables (Ref. 2) such as beam section properties (Ref. 5), and if intermediate response quantities (Ref. 2) such as member forces for stress constraints (Ref. 6) and modal energies for frequency constraints (Ref. 7) are approximated.

Linear Approximation:

$$\tilde{g}_L(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} \frac{\partial g(\mathbf{Y})}{\partial Y_i}(Y_i - Y_{oi}) \qquad (1)$$

Reciprocal Approximation:

$$\tilde{g}_R(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} (-Y_{oi}^2)\frac{\partial g(\mathbf{Y})}{\partial Y_i}\left(\frac{1}{Y_i} - \frac{1}{Y_{oi}}\right) \qquad (2)$$

Hybrid Approximation: Mixture of Linear and Reciprocal Approximations based on the sign of $\frac{\partial g(\mathbf{Y})}{\partial Y_i}$ (assuming $Y_i > 0$)

Intermediate Variables:

(Linear Approximation)

$$\tilde{g}_{IV}(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{j} \frac{\partial g(\mathbf{X})}{\partial X_j}[X_j(\mathbf{Y}) - X_j(\mathbf{Y}_o)] \qquad (3)$$

For Beam Bending:

$$\tilde{g}_{IV}(h,b) = g(h_o,b_o) + \sum_{j} \frac{\partial g(h,b)}{\partial I_j}[I_j(h,b) - I_{oj}] \qquad (4)$$

Intermediate Response Quantities

(Forces in Beam Bending)

$$\tilde{g}_{IR}(\mathbf{Y}) = \frac{\tilde{\sigma}(Y)}{\sigma_a} - 1.0 = \frac{\tilde{M}(\mathbf{Y})c}{I\sigma_a} - 1.0 \qquad (5)$$

where

$$\tilde{M}(\mathbf{Y}) = M_o(\mathbf{Y}) + \sum_{i=1}^{n} \frac{\partial M(\mathbf{Y})}{\partial Y_i}(Y_i - Y_{oi}) \qquad (6)$$

Figure 3

FIRST ORDER CONSTRAINT BUFFERING

One approach used to buffer the constraints, introduced in Ref. 8, is to add a padding term to the constraint function that is equal to the sum of the absolute value of the tolerance on each of the design variables multiplied by the sensitivity of the constraint with respect to the design variable (Eq. 7). This approach has the advantage that it gives good results when the constraints are nearly linear in the design variables. The drawback to this approach is that when the constraint function is nonlinear, due to the use of intermediate design variable or intermediate response quantity concepts, the padding term is still a linear function of the tolerances on the design variables. This can lead to designs that are not conservative enough.

Another drawback to this approach is that the first order derivatives of the constraint functions may contain second order quantities if intermediate design variables and response quantities are used. These second order terms cannot be neglected since they can be larger than the first order terms. Calculation of the second order terms can be quite difficult, since the analytical expression can be very complex. The finite difference technique can be used to calculate the second order terms; however the error associated with this technique may become large, especially if the first order derivatives where generated by finite difference. The second order terms could be approximated by using an approximate Hessian matrix (see Ref. 9), but there are also errors associated with this technique.

$$\tilde{g}^P(\mathbf{Y}) = \tilde{g}(\mathbf{Y}) + \sum_{i=1}^{n} \left| \Delta Y_i \frac{\partial g(\mathbf{Y})}{\partial Y_i} \right| \qquad (7)$$

$$\tilde{g}_L^P(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} \frac{\partial g(\mathbf{Y})}{\partial Y_i}(Y_i - Y_{oi}) + \sum_{i=1}^{n} \left| \Delta Y_i \frac{\partial g(\mathbf{Y})}{\partial Y_i} \right| \qquad (8)$$

$$\tilde{g}_R^P(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} (-Y_{oi}^2) \frac{\partial g(Y)}{\partial Y_i}(1/Y_i - 1/Y_{oi}) + \sum_{i=1}^{n} \left| \Delta(1/Y_i) \left(\frac{Y_{oi}}{Y_i} \right)^2 \frac{\partial g(\mathbf{Y})}{\partial Y_i} \right| \qquad (9)$$

Figure 4

BUFFERING OF NONLINEAR CONSTRAINTS

A more accurate buffered constraint, which captures the explicit nonlinearity in high quality approximations, can be constructed by using the values of the design variables at their upper or lower tolerance values, depending on the sign of the derivative of the constraint with respect to each design variable, in the constraint function. For example, in a structure with displacement constraints the values of the member cross sectional dimensions at their lower tolerances would be used in the constraint function. The lower tolerance value is used, as opposed to the upper tolerance, because the lower values lead to larger displacements (the sign of the derivative of the displacement with respect to the design variable is negative). Since the tolerance is included in the constraint function, all of the nonlinearity that is captured by the constraint function is also present in the buffered constraint. Note that the accurate calculation of the first derivatives of the constraints with respect to the design variables is the same and as simple as the method that is used for unbuffered constraints. The only difference is that the value of the design variable is replaced by its upper or lower tolerance value.

If constraints are formed using intermediate design variables, then the values of the design variables at their upper or lower tolerance values are used to calculate the buffered value of the intermediate design variables. Note that in some cases, such as frequency constraints, some of the design variables associated with an intermediate design variable may be at their upper tolerance values while the others are at their lower tolerance values.

$$\tilde{g}^B(\mathbf{Y}) = \tilde{g}(\mathbf{Y}^B) \tag{10}$$

$$\text{where } Y_i^B = \begin{cases} Y_i + \Delta Y_i & \text{if } \dfrac{\partial g(\mathbf{Y})}{\partial Y_i} \geq 0 \\ Y_i - \Delta Y_i & \text{if } \dfrac{\partial g(\mathbf{Y})}{\partial Y_i} < 0 \end{cases}$$

$$\tilde{g}_L^B(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} \frac{\partial g(\mathbf{Y})}{\partial Y_i}(Y_i^B - Y_{oi}) \tag{11}$$

$$\tilde{g}_R^B(\mathbf{Y}) = g(\mathbf{Y}_o) + \sum_{i=1}^{n} (-Y_{oi}^2)\frac{\partial g(\mathbf{Y})}{\partial Y_i}\left(\frac{1}{Y_i^B} - \frac{1}{Y_{oi}}\right) \tag{12}$$

$$\tilde{g}_{IV}^B(\mathbf{X}(\mathbf{Y})) = \tilde{g}(\mathbf{X}^B(\mathbf{Y}^B)) \tag{13}$$

Figure 5

EXAMPLE

Consider a rectangular cantilevered beam of height *h* and width *b* loaded by a moment *M* at the tip. If the intermediate response quantity approach is used, then the approximate stress is calculated using the approximate moment. In statically determinate problems such as this one, this is trivial since the approximate moment is constant. Hence, the approximate stress is exact.

The approximate stress is calculated using the value of *b* and *h*. The buffered approximate stress is calculated using the buffered values of *b* and *h*. Since the stress is greater when the values of *b* and *h* are smaller, the values of *b* and *h* at their lower tolerances are used in the buffered constraint approximation. Note that the constraint is a nonlinear function of the design variable tolerance (Eq. 18). The buffered value of the stress is also exact. Therefore, when the design is fabricated and the manufacturing tolerances are at their lower values, the stress constraint will not be violated. Equation 19 is the first order form of the buffered constraint. Although this type of buffered constraint is exact for linear approximations, there is some error when it is used with nonlinear approximations because the buffering is only a linear function of the design variable tolerances.

$$\tilde{g} = \frac{\tilde{\sigma}}{\sigma} - 1 \quad ; \quad \tilde{\sigma} = \frac{6\tilde{M}}{bh^2} \tag{14}$$

$$\tilde{g}^B = \frac{\tilde{\sigma}^B}{\sigma_a} - 1 \quad ; \quad \tilde{\sigma}^B = \frac{6\tilde{M}^B}{b^B (h^B)^2} \tag{15}$$

$$\tilde{M}^B = \tilde{M} = M \tag{16}$$

$$b^B = b - \Delta b \quad , \quad h^B = h - \Delta h \tag{17}$$

$$\tilde{\sigma}^B = \frac{6M}{(b - \Delta b)\,(h - \Delta h)^2} \tag{18}$$

$$\tilde{\sigma}^P = \frac{6M}{bh^2} + \left| \Delta b \left(\frac{-6M}{b^2 h^2} \right) \right| + \left| \Delta h \left(\frac{-12M}{bh^3} \right) \right| \tag{19}$$

Figure 6

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