Combined Design of Structures and Controllers for Optimal Maneuverability

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Abstract

This paper presents approaches to the combined design of structures and controllers for achieving optimal maneuverability. A maneuverability index which directly reflects the minimum time required to perform a given set of maneuvers is introduced. By designing the flexible appendages, the maneuver time of the spacecraft is minimized under the constraints of structural properties, and of the post maneuver spillover being within a specified bound. The spillover reduction is achieved by making use of an appropriate reduced order model. The distributed parameter design problem is approached using assumed shape functions, and finite element analysis with dynamic reduction. Solution procedures have been investigated. Approximate design methods have been developed to overcome the computational difficulties. Some new constraints on the modal frequencies of the spacecraft are introduced in the original optimization problem to facilitate the solution process. It is shown that the global optimal design may be obtained by tuning the natural frequencies to satisfy specific constraints. We quantify the difference between a lower bound to the solution for maneuver time associated with the original problem and the estimate obtained from the modified problem, for a specified application requirement. Numerical examples are presented to demonstrate the capability of this approach.

I. Introduction

Large space structures such as antennas or space stations will be very flexible, not only because of the high cost of transportation of structures from Earth to space, but also because they will be constructed or deployed in orbit and will not need to withstand large launching and gravity loads. However, when a space structure is very flexible, its active control system can excite and otherwise significantly interact with its flexible modes. Thus, the idea arises of achieving the best flexible mode suppression for attitude maneuver of spacecraft. The control problem of time-optimal, rest to rest, slewing of a flexible spacecraft through a large angle has been investigated [1]. In that work, a specific spacecraft is modelled using a reduced order model, and the time-optimal control history of this modelled system is derived. In some time critical applications, it is required that the maneuver be performed as rapidly as possible. As a consequence, structural optimization is considered so as to further minimize the maneuver time. The whole design process, the idea of combined design of controllers and structures for optimal maneuverability, is considered in this work.

Traditionally, the overall design problem for actively controlled space structures is treated via an iterative two-part scheme. Redesign of the structure including sensor and actuator placement is performed in one stage, and then the control law is modified for the resulting system to complete an iteration cycle. Generally different design objectives apply in the separate steps. More recently, the need to integrate the design of a structure and its control system has been recognized. An integrated approach is justified simply on the basis that structural and control purposes are substantially coupled. Bodden and Junkins [2] presented a method for eigenvalue optimization with sequential or simultaneous design of structure and control. Khot, Oz, Venkayya, and Eastep [3-5] considered structural optimization, including constraints on control gain norm and transient behavior of the control system, based on a linear-quadratic model of the controller. Hale, Lisowski, and Dahl's [6,7] treatment of the problem of simultaneous structure and control design for a maneuvering spacecraft resulted in a linear-quadratic optimization problem. Bendsoe, Olhoff, and Taylor [8] presented an algorithm for integrated design of the structure and its control system which includes a constraint to limit the control spillover into the unmodelled modes. Lust and Schmit [9] presented a control-augmented structural synthesis methodology in which the structural member sizes and active control system feedback gains are treated simultaneously as independent design variables. Onoda and Haftka [10] considered the optimization of the total cost of the structure and control system subject to constraints on the magnitude of the response to a given disturbance involving both rigid-body and elastic modes. Lim and Junkins [11] presented an idea for optimizing the robustness of structures and structural controllers, using homotopy and sequential linear programming algorithms. Khot [12] presented algorithms for design of minimum weight structures with the goal of improving system dynamics by use of a closed-loop control system.

Most of the developments on simultaneous design of structures and controllers reported in the literature use simple linear feedback control laws and quadratic performance indices. Practical constraints such as limitation on the amplitude of the control effort generally are not taken into account. The use of such relatively narrow forms of problem statements may have serious implications in terms of the usefulness of the results. It is understood, for example, that the use of performance indices expressed as linear/quadratic functionals is generally inappropriate unless loop transfer recovery techniques [13-16] are incorporated into the formulation. Furthermore, the constraints usually used in literature are on the closed-loop eigenvalue distribution and structural frequencies. These constraints are not as direct to the application problem as constraints on rise time, maximal displacement, or maximal stress. The consideration of performance degradation of the optimal system coming from the control and observer spillover is also generally not included.

In the present work, we examine the problem of fully coupled design for a spacearaft and its associated control. The design of the structural system and control is to be integrated so as to optimize with respect to a single cost function. The objective is chosen to reflect the maneuverability of this structure/control system, i.e. the time required to perform a given maneuver or set of maneuvers. Various forms of Mission Specification can be reflected in the definition of the performance index. Ours includes criteria related to sets of maneuvers with specified probability of occurrence. This performance index is generally more meaningful than the usual LQG index with minimum weight. The 'minimum time' objective is appropriate for application in slewing or other retargeting maneuvers. Furthermore, the problem is formulated in a way to accommodate in explicit form of various practical constraints, such as limits on control action and performance error (control spillover). Also, the formulation is consistent with a nonlinear bang-bang form of optimal control design.

The spacecraft is modeled as a linear, elastic, undamped, nongyroscopic system. The necessary-and-sufficient condition for the time-optimal rest-to-rest control problem can be considered as a mapping from the structural dynamic properties to the optimal maneuver time. The maneuverability is optimized by updating design parameters. Characteristics of the problem and problem solving procedures have been investigated. Approximate design methods have been developed to overcome the computational difficulties. Numerical examples are presented to demonstrate the capability of those approaches.

II. Combined Design of Structures/Controllers - Problem Formulation

Consider the linearized rotational dynamics of a flexible spacecraft where control inputs are used to actively control the rigid body mode and flexible modes. The spacecraft is modeled as a linear, elastic, undamped, nongyroscopic system. There is a rigid central body, as shown in Figure 1, to which N ($N \geq 2$) identical flexible appendages are attached with uniform spacing between them. Along the appendages, there might be some kinds of distributed or concentrated payload masses for practical usage. The spacecraft may be very large and flexible. The spacecraft is to be controlled by a single torque actuator located on the central body and m torquers located at identical locations on each of the N appendages. The amplitude of the torque applied by each torquer is limited. The objective of the control design is to time-optimally slewing the spacecraft through a specified angle θ, and achieve flexible mode suppression at the end of the maneuver. Assume that the appendage displacements, slopes and central body rotation rates remain small and the appendages are inextensible. The appendage displacements are restricted to a plane orthogonal to the central body's axis of rotation.

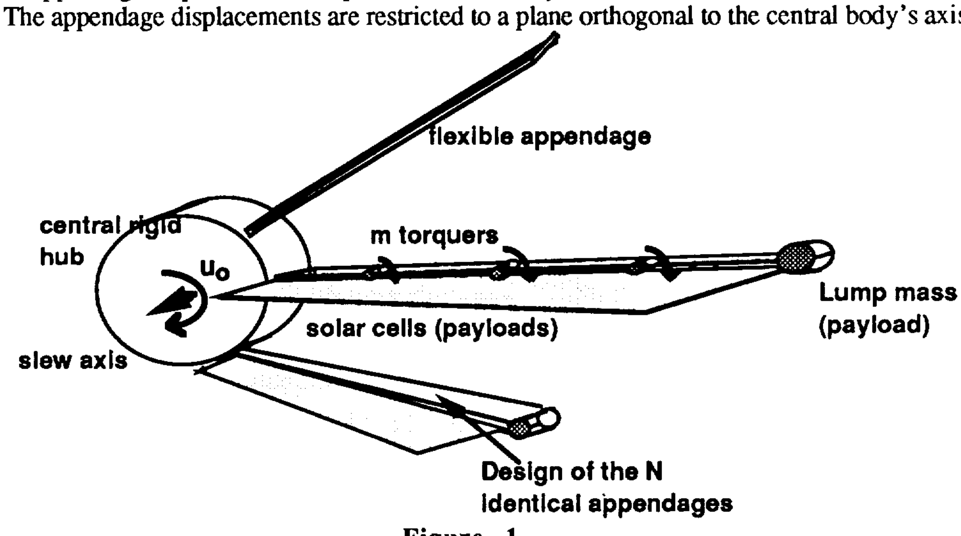


Figure 1

The design parameters of the appendages can be the cross section, stiffness or density of the material, layouts of the composite material or the location of torquer actuators along the appendage. Let the design parameter vector be $\xi \in R^N$, implying that the structural dynamics properties are implicit functions of ξ.

Maneuverability Index

The maneuverability, formulated as a maneuverability index, reflects the cost required to perform a given maneuver or set of maneuvers. The mission profile is specified by giving the probability density function $p(\theta)$ of the required maneuver amplitude θ. Let $t_f^*(\theta)$ be the optimal maneuver time for maneuver θ, and $t_f^*(\theta)$ is a function of the structural design parameter vector ξ. Therefore the maneuverability index is also a function of ξ. We define the maneuverability index as

$$\mu^*(\xi) = \int_{-\infty}^{+\infty} p(\theta) \cdot t_f^*(\theta)\, d\theta, \tag{2.1}$$

For example, let $p(\theta) = \delta(\theta - \theta^*)$ then $\mu^*(\xi) = t_f^*(\theta^*)$. In other words, the maneuverability index represents the expected value of the optimal maneuver time for a given mission profile. The structural design problem is then to optimize $\mu^*(\xi)$ with respect to ξ.

Optimal Design Problems

Assume that the structural design parameter ξ is restricted to belong to a compact set Ξ, which represents feasible designs. Assume that the design of the appendages will not change the characteristic of the torquers along the appendages. In other words, the amplitude limits of the torquers remain the same for all values of the design parameters. Therefore, we can formulate the optimal combined structure/control design problem as :

$$\mu = \min_{\Xi} \mu^*(\xi)$$

where Ξ is the space of structural design variables

subject to two sets of constraints :

I. a. Material resource constraint,

b. Geometric configuration constraints :

such as the max. and min. thickness limits of cross section,

c. Dynamic response constraints :

such as the max. stress and displacement limits,

and, II. The post-maneuver control spillover is within a specified bound. (2.2)

Constraint II takes into account the performance degradation associated with the unmodelled dynamics.

We approach the distributed parameter design of the cross section of the appendage using assumed shape functions. For example, let the design parameter of the cross section be the thickness distribution. We assume that the thickness function, h(x) (x is the location along the appendage) is represented via a linear combination of a set of assumed shape functions. This approach uses the same idea as design variable linkage [17].

The distributed infinite degree-of-freedom system is approximated with finite elements. We discretize the spacecraft into a finite number of elements and then perform modal analysis. There are two kinds of mathematical models for design and analysis. Let subscript $\mathcal{E}$ indicate a quantity derived based on the control evaluation model. The number of modes in this model is the number of degrees of freedom in the finite element analysis (let it be n in this paper). Assuming this model to represent the exact dynamic system, we can evaluate the performance of the controlled system on it. Let subscript $\mathcal{R}$ indicate a quantity derived based on the control design model. The control design model is the model on which we obtain the optimal-time maneuver law. We assume there are r (r << n) vibrational modes retained in this model. The natural frequency and mode shape of the modes in the control design model can be easily obtained from the control evaluation with the dynamic reduction method.

Results of the Linear Time-Optimal Control Problems

Results presented in this section were obtained in the recent paper [1]. The optimal control characterized here is based on a control design model, or so called reduced order model, which has one rigid body mode and r undamped flexible modes. There are 2*(r+1) state variables in this system. The problem of time-optimal rest-to-rest slewing maneuver can be formulated as

Problem $M(\theta)_{\mathcal{R}}$:

$$\min\ t_f(\theta) \tag{2.3}$$

Subject to :

$$\dot{x} = A\,x(t) + B\,u(t)$$

$$|u_j(t)| \le U_j \ ; \ j = 0, 1, \ldots, m$$

where u_0 is related to the control input at the central rigid body and $u_1, u_2, \ldots u_m$ are related to the m torquer actuators along the appendages. U_j ; j = 0, 1, ... , m are the corresponding amplitude limits.

$$\mathbf{x}(0) = (0, 0, ..., 0)^t \text{ , where ( )}^t \text{ denotes transpose.}$$

$$\mathbf{x}(t_f) = (\theta, 0, ..., 0)^t$$

A = Block diag[$\mathbf{A}_i$], **B** = Block col [$\mathbf{B}_i$], where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad ; \; i = 0,$$

$$\begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix} \quad ; \; \omega_i \text{ is the natural frequency, } i = 1, 2, ..., r$$

$$\mathbf{B}_i = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \beta_0^i & \beta_1^i & \dots & \beta_r^i \end{bmatrix} \quad ; \; i = 1, 2, ..., r \tag{2.4}$$

Let the solution for problem $M(\theta)_{\mathcal{R}}$ be $t_f^*(\theta)$.

Theorem 1.1. Let t_f^* be the optimal maneuver time. For all θ, Problem $M(\theta)_{\mathcal{R}}$ has a unique solution t_f^*.

Theorem 1.2. For a given θ, the optimal control law is of bang-bang type, and is symmetric around $t_f^*/2$, i.e.

$$\mathbf{u}(t_f^*/2 - t)^* = -\mathbf{u}(t_f^*/2 + t)^*, \quad 0 \le t \le t_f^*/2 .$$

Reference [1] treats the general multiple control case, where there are m+1 control inputs. However, for simplicity, herein we assume that only one control input is used to control the maneuver, that is, the scalar control case. This assumption means that the N torque actuators on the appendages and the actuator on the rigid central body taken together represent one control input.

Theorem 1.3. Assume there are k switching times between 0 and $t_f^*/2$, and let them be t_i, i = 1,2,... k. Let J* be the total rotational moment of the spacecraft, and $(p^0_0, 0, p^1_0, 0, ..., p^r_0, 0)$ be the costate variable at mid-maneuver time. Then the optimal maneuver time and the switching times satisfy as necessary and sufficient conditions, the following system of nonlinear algebraic equations :

$$(t_f^*)^2 - 2(t_k)^2 + 2(t_{k-1})^2 - \dots + 2(-1)^k (t_1)^2 = \theta J^*/U_0, \tag{2.5}$$

$$\cos(\omega_i t_f^*/2) - 2\cos(\omega_i t_k) + 2\cos(\omega_i t_{k-1}) - \dots + 2(-1)^k \cos(\omega_i t_1) + (-1)^{k+1} = 0$$
$$i = 1, 2, \dots r \tag{2.6}$$

$$\begin{bmatrix} U_0 t_f^*/2 & U_0 \sin(\omega_i t_f^*/2) & \dots\dots & U_0 \sin(\omega_r t_f^*/2) \\ t_k & \sin(\omega_1 t_k) & \dots\dots & \sin(\omega_r t_k) \\ \vdots & \vdots & & \vdots \\ t_1 & \sin(\omega_1 t_1) & \dots\dots & \sin(\omega_r t_1) \end{bmatrix} \begin{Bmatrix} p_0^0 \\ \beta_0^1 p_0^1 \\ \vdots \\ \beta_0^r p_0^r \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{Bmatrix} \tag{2.7}$$

and two inequality equations :

$$t_f^*/2 > t_k > t_{k-1} > \dots > t_2 > t_1 > 0$$

$$p_0^0 t + \sum_{i=1}^{r} \beta_j^i p_0^i Sin(w_i t) \neq 0 \tag{2.8}$$

$$\text{where } 0 \le t \le t_f^*/2 \qquad t <> t_i \quad i = 1, 2, \dots k$$

To solve for the optimal control history, we need first assume the number of switching times, say k, then try to find the solutions { t_j, j = 1, 2, ... k } and { p^j_0, i = 0, 1, 2, ... r } for (2.5)-(2.7). If (2.5)-(2.7) really admit solutions and they satisfy (2.8) as well, by uniqueness of the solution of the optimal control problem, we have the unique solution.

We have found that, in general, k is always equal to r. Only when { ω_i , i = 1, 2, ... r } satisfy some special conditions, k is less than r. For the case where k is equal to r, Theorem 1.3 can be simplified by omitting (2.7).

Reduced Order Model

We now consider how many flexible modes should be retained in the reduced order model. The question is answered by analyzing the degradation in the performance of the designed system on the control evaluation model. This performance degradation is associated with 'unmodelled dynamics' of the uncontrolled residual modes in the control evaluation model (from r+1-th term to n-th term). The effects, therefore, result in post maneuver free vibration of the system, due to control spillover. We need to make sure these vibrations have amplitudes within a specified performance error bound during the optimization.

There are two ways to quantify the performance degradation : (i). the residual or spillover energy $\mathcal{E}_r(t)$, and (ii). the pointing error of the rigid central body after completion of the maneuver $\theta e(t)$ (where $t \geq t_f^*/2$). From the recent investigation on these [1], the latter is the better one because the maximum pointing error continues to decrease as we suppress additional modes at the final time, while the spillover energy does not necessarily decrease. Also [1] gives three closed form expressions for the upper bound $|\theta_e(t)|$, based on the control evaluation model. Among them, the most useful according to our experience is

$$|\theta_e(t)| \leq 2\{(2+2k)U_0/J^{*2}\} \sum_{i=r+1}^{n} (\beta_0^i)^2 \; ; \quad t \geq t_f^*/2, \qquad (2.9)$$

We use this upper bound to determine the size of the control design model in order to obtain a prespecified post-maneuver pointing accuracy of the rigid central body.

Characteristics of the Optimal Design Problem

Theorem 2.
Suppose the number of flexible modes retained in the model is fixed. The optimal maneuver time solved from the (2.5)-(2.8) is a continuous function of the structural design variables, ξ.
Corollary 1. The objective function, $\mu(\xi)$, is a continuous function of ξ.
Corollary 2. There exists a solution to the optimal design problem (2.2).

We have observed that the objective function is always a differentiable function of the structural design variables, ξ. Consider the generic case where k is equal to r. The optimal maneuver time can be obtained from (2.5)-(2.6). Actually (2.5)-(2.6) represent a system of implicit equations of the form :

$$\mathbf{F}\,(t_f^*\,,\mathbf{t_j}\,,\omega,\beta,J^*) = 0. \qquad (2.10)$$

The gradient of the optimal maneuver time with respect to structural parameters can be obtained using the Implicit Function Theorem as follows : Let $\mathbf{x}=(t_f^*\,,\mathbf{t_j})$ and $\mathbf{y}=(\omega,\beta,J^*)$

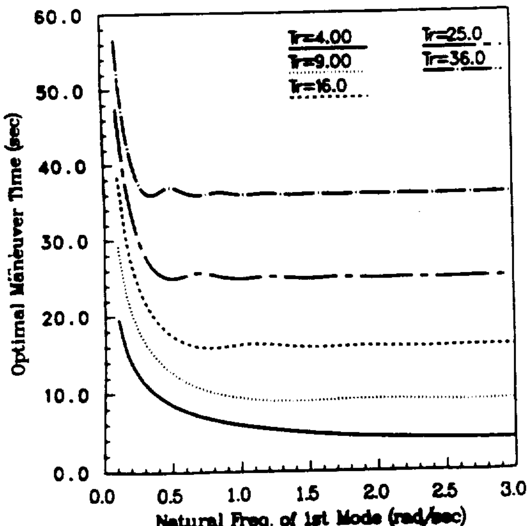
Theorem 3. (Implicit Function Theorem)
Suppose $(\mathbf{x_0},\mathbf{y_0})$ is such that $\mathbf{F}(\mathbf{x_0},\mathbf{y_0}) = 0$ and $\mathbf{F}(\mathbf{x_0},\mathbf{y_0}) \in C^k$, and the Jacobian matrix $[\partial F/\partial \mathbf{x}]$ is nonsingular (regular) at $(\mathbf{x_0},\mathbf{y_0})$. Then there exist a neighborhood of $\mathbf{y_0}$, say $N(\mathbf{y_0})$, and a mapping $\mathbf{G} : N(\mathbf{y_0}) \to R^n$ such that $\mathbf{x_0} = \mathbf{G}(\mathbf{y_0})$ and $\mathbf{G}(\mathbf{y_0}) \in C^k$, and $\mathbf{F}(\mathbf{G}(\mathbf{y}),\mathbf{y}) = 0$ on $N(\mathbf{y})$. Moreover, we have

$$[\partial G/\partial y]^t|_{y0} = -[\partial F/\partial x]^{-t}|_{x0}\,[\partial F/\partial y]^t|_{x0,\,y0} \qquad (2.11)$$

By the Chain Rule, we can obtain the gradient of the objective function with respect to the design parameters. A candidate optimal design must satisfy the Kurash-Kuhn-Tucker necessary conditions [18]. We use mathematical programming to find it.

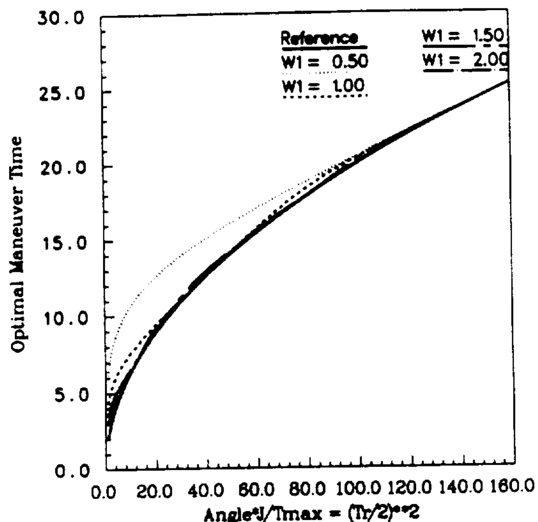
From (2.10), t_f^* is an implicit function of (ω,β,J^*). Furthermore, for the generic case where k is equal to r, t_f^* actually is a function of ω and J^* only. We show the behavior of $t_f^*(\omega,J^*)$ for the simplest case where there is only one flexible mode in Figures 2 and 3.

C-3



Tr is the maneuver time for a rigid spacecraft with fixed rotation moment J*, maneuver angle, and total torque limit Uo

Figure 2.



Tr is the maneuver time of rigid spacecraft with the rotation moment J and the total torque limit Tmax.
Reference is the curve : Tr = 2*SQRT(Angle*J/Tmax)

Figure 3.

We have found the following results : Assume a spacecraft has only one flexible mode.

(i). For a spacecarft with very small ω_1 (usually a very flexible spacecarft), t_f^* is quite large. On the other hand, for a spacecraft with large ω_1 (as shown in the Figure 2, greater than 2.0), t_f^* is almost the same as that of the equivalent rigid spacecraft.

(ii). For a spacecraft with $\theta J^*/U_0 \geq 120.0$ (the torquer limit is very small or the maneuver angle is very large), t_f^* is almost the same as that of the equivalent rigid spacecraft.

Of course, a typical spacecraft has more than one flexible mode, and we can not say much about it. However, Figure 2 and 3 provide important information. If the spacecraft is very flexible or the torquer limit is very large (usually this implies very large maneuver speed), the result of the optimal design can provide substantial improvement.

Problem Solving Algorithm

The control design model is chosen according to the analysis of control spillover. In order to take advantage of Theorem 2, we assume that the control design model is fixed during the optimization and formulate the optimization procedure as

$\mathcal{P}_1$:

Begin with a reasonable baseline design of the spacecraft.

Step	0 :	Set up the reduced model by (2.9). (Set the value of r)
Step	1 :	Initialize the design variables.
Step	2 :	Get the cross section of the appendage for the current value of the design variables.
Step	3 :	Finite Element Analysis.
Step	4 :	Calculate the natural frequencies of the modes in the reduced model by the Dynamic Reduction Method.
Step	5 :	Solve the time-optimal control problem to obtain the optimal maneuver time.
Step	6 :	Find the next values of the design variables by the Nonlinear Programming.
Step	7 :	If the result is convergent, Step 8. Otherwise, go to Step 2.
Step	8 :	If the spillover constraint (ii of (2.2)) is satisfied, then Stop. Otherwise, Step 0.

Although the algorithim $\mathcal{P}_1$ is able to solve the optimal structural design problem (2.2), unfortunately, in our experience, there exist a lot of numerical difficulties associated with it :

i). To solve the time-optimal control problem, we need to know the number of switching times.

ii). Actually the set of nonlinear equations (2.5-2.7) admit many solutions, of which only one satisfies the inequality conditions (2.8). Thus, even though we have a good nonlinear equation solver, it would not be able to guarantee to find the solution we want.

Given all the difficulties above, it seems a formidable task to solve the optimal design problem by $\mathcal{P}_1$ without any simplfication, especially if one expects to find the global optimal design. Therefore, we introduce approximate design methods as described in the next section.

III. Approximate Design Method

The fundamental idea of this solution procedure is to formulate an approximate design problem without violating any constraint of the original problem. The solution of the approximate design problem is a 'near-optimal design' in the sense that there is little difference of objective function between the two solutions. We need to quantify the difference without solving the original problem and make it as small as possible. However, there is a trade-off between accuracy and efforts for solving problems. Thus an important capability of the approximation algorithm is that we can adaptively upgrade the approximation procedure to obtain a reasonable result according to the specific application requirement. Since design models can not exactlly represent the real system, it is unreasonable to concern oneself so much about a relatively small improvement of accuracy of the solution based on a design model. In this section we introduce two approximate design methods : the Adaptive Frequency Tuning method, and the Minorant method. The former one is suitable for the single maneuver case; the latter one requires more computation work but is suitable for the multimaneuver case.

Frequency Tuning Approach

There are two basic assumptions :

Assumption 1. : The natural frequencies of the modes retained in the reduced order model can be freely assigned by adjusting the values of the design variables.

Assumption 2. : During the design iteration, the mass distribution of the appendage is taken to be independent of the stiffness distribution, i.e., the total rotational moment of the spacecraft, J* , does not change when the stiffness distribution is modified.

Considering (2.5, 2.6), if for a spacecraft the natural frequencies of all modes in the reduced order model happen to satisfy :

$$\omega_i \cdot t_f^* = j_i \cdot 4\pi \;, \quad i = 1,2, \ldots r \tag{3.1}$$

where t_f is the maneuver time and j_i is some integer multiplier,

then, the solution in terms of switching times and the optimal maneuver time satisfy :

$$k = 0\,, \text{ and } \quad t_f^* = 2\sqrt{\theta J^* / U_0} \tag{3.2}$$

It also satisfies the inequality condition (2.8). Thus we solve the time-optimal control problem for $\{\omega_i, i = 1,2, \ldots r$ satisfying (3.1) $\}$. Moreover, (3.2) imply that there is no switch of the control history between 0 and $t_f^*/2$, and only one switch at the mid-maneuver. This means that all flexible modes in the reduced model are dead beat at the end of maneuver by the same control which maneuvers a rigid body of the same value of total rotational moment J*. We have the new optimization problem :

$$\mathcal{P}_2 \qquad \min_{\Xi} \; t_f^* = 2\sqrt{\theta J^* / U_0}$$

subject to :

the constraints I and II of (2.2), and (3.1) (3.3)

Proposition : Under the Assumptions 1 and 2 above, the solution of $\mathcal{P}_2$ solves our original problem, (2.2), and it is a global optimum.

The rigid-body control strategy is the simplest to implement, and we don't need to solve any nonlinear equations (2.5-2.7). Furthermore the optimal design of appendage which satisfies (3.1) may be very flexible (in the sense that natural frequencies of the first few flexible modes are very small), and very light (in the sense that J* is small). This idea for design appears to be original.

Adaptive Upgrade Algorithm

Unfortunately, Assumption 2 above is not always satisfied in general applications. For example, in designing an appendage of rectangular cross section with high density material, the stiffness is highly coupled with the design of mass distribution. Actually, $\mathcal{P}_2$ implicitly assumes that the global optimal design of the appendages is such that the time-optimal control is the same as the rigid-body control strategy. We restrict ourselves to solve the original problem in a subspace of the feasible design variable space. Therefore, the result of $\mathcal{P}_2$ in general does not apply and needs to be modified or upgraded.

We first quantify the index of improvement in approximation as the difference of objective function between the exact optimal design (the solution of original problem) and the solution of approximate design problems. Let tf be the maneuver time of the exact optimal design, which is equal to the minimum of t_f^* over the entire feasible design space.

Also we note that μ is equal to $\int p(\theta_i)\cdot tf(\theta_i)\,d\theta_i$. Let tf^a, μ^a be the approximated solution of tf and μ respectively.

Then we introduce : Index of approximation : $\mathcal{E}_0 = |\, tf^a - tf \,|$ or $|\mu^a - \mu|$ (3.4)

An approximated solution is better if the index of approximation is smaller. However, this doesn't mean the two designs are close to each other. For example, they may be substatially different in shape. In order to avoid difficulties in computing tf, we modify (3.4) : $\mathcal{E}_1 = |\, tf^a - \mathcal{L}^b(tf)|$ or $|\mu^a - \mathcal{L}^b(\mu)|$,

where $\mathcal{L}^b(\cdot)$ is a lower bound of $\cdot$, and it is very easy to compute. (3.5)

Also, we have $$\mathcal{L}^b(\mu) = \int p(\theta_i)\,\mathcal{L}^b(tf(\theta_i))\,d\theta_i \qquad (3.6)$$

There are two ways to define such a lower bound :

(i). the maneuver time for a rigid spacecraft with the least fesible total rotational moment J^*:

$$\mathcal{L}^b_1(tf) = 2\sqrt{\theta J^* / U_0} \qquad (3.7)$$

It is usually unreasonable to define the lower bound in this way because (3.7) is very conservative. The appendage with the least total rotational moment is usually too slender, too flexible, and likely requires a long maneuver time.

(ii). the optimal maneuver time of the optimal design which is based on a reduced model with only one flexible mode. Let the superscript 1 of tf indicate that the value is based on a reduced model with only one flexible mode. Thus

$$\mathcal{L}^b_2(tf) = tf^1 = \text{minimum of } t_f^{*1} \text{ over the entire feasible design space.} \qquad (3.8)$$

Since we need more maneuver time for the reduced model with more flexible modes, we know tf^1 is a lower bound of the maneuver time for the design problem of any reduced order model. We need some computation effort to calculate tf^1; however, the calculation is not very difficult. It is more reasonable to define the lower bound of the maneuver time to be tf^1.

We propose the modified approximate problem $\mathcal{P}_3$ according to the following facts :

Fact 1 : For a specified reduced order model with r flexible modes, we can divide the feasible design space into :

$\mathcal{D}_0$: { ξ : the time-optimal control history of this design admits only one switch at mid-maneuver, without any switch in (0, $t_f^*/2$)},

$\mathcal{D}_1$: { ξ : the time-optimal control history of this design admits at most one switch in (0, $t_f^*/2$ },

$\mathcal{D}_2$: { ξ : the time-optimal control of this design admits at most two switches in (0, $t_f^*/2$)},

....

and $\mathcal{D}_0 \subseteq \mathcal{D}_1 \subseteq \mathcal{D}_2 \subseteq \mathcal{D}_3 \subseteq$ .... (3.9)

Fact 2 : t_f^* over $D_r \geq t_f^*$ over $\mathcal{D}_{r+1}$ (3.10)

Fact 3 : the solution of (2.2) is the t_f^* over $\mathcal{D}_r$ for some $r \geq 0$.

Actually, the solution of $\mathcal{P}_2$ is nothing but tf over $\mathcal{D}_0$. Similarly, $\mathcal{P}_3$ is the problem of solving for tf over $\mathcal{D}_r$, $r \geq 1$, adaptively upgrading with respect to the index of improvement, and with a stopping criterion based on sufficiently small change of improvement. We can eventually obtain the exact global optimal design if the upgrade goes on. However, we have restricted ourselves to solveing for tf over $\mathcal{D}_r$, $r \leq 2$.

The Minorant Design Method

$\mathcal{P}_2$ and $\mathcal{P}_3$ are not suitable for the general multiple maneuver case because it is difficult to find ω_i , i = 1,2, ... r which satisfy (3.1) for many different maneuvers, $\{\theta_i\}$. In this section we discuss an algorithm, the minorant method, which is more difficult to implement, but, suitable for the multiple maneuver case. While solving the time optimal control problem, we find that for any design of spacecraft, $t_f^{*\,r+1} \geq t_f^{*\,r}$; however, the difference becames smaller and smaller as r increases. From our numerical studies, it is observed that the maneuverability is most influenced by the total rotational moment J*, and then from the few lowest flexible modes. An appendage with smaller total rotational moment or with more rigidity, in the sense that the natural frequencies of the lowest few flexible modes are large tends to be very maneuverable.

$\mathcal{P}_4$ is based on the following assumption and fact.

Assumption : For any fesible design of the spacecraft $\xi \varepsilon \Xi$, we have $| t_f^*(\xi)^{i+2} - t_f^*(\xi)^{i+1} | \leq | t_f^*(\xi)^{i+1} - t_f^*(\xi)^{i} |$, $i \geq 0$, where the superscript i indicate that the quantity is obtained based on a reduced model with i flexible modes.

Fact 4 : $| tf^{i+2} - tf^{i+1} | \leq | tf^{i+1} - tf^{i} |$, $i \geq 0$, and $| tf^{i} - tf^{r} | \rightarrow 0$ as r and i are sufficiently large.

Furthermore, $|\mu^{i+2} - \mu^{i+1} | \leq | \mu^{i+1} - \mu^{i} |$, $i \geq 0$, and $| \mu^{i} - \mu^{r} | \rightarrow 0$ as r and i are sufficiently large.

$\mathcal{P}_4$:

Step 1 : Let i = 0, and Solve μ^i by $\mathcal{P}_1$.

Step 2 : Obtain the index of improvement $\mathcal{E}$. If there is no relative change of improvement, stop. Otherwise, i = i+1. Go to Step 1.

The exact optimal design can be obtained for i = r. However, we do not go beyond $i \geq 2$. The capability of $\mathcal{P}_4$ will be investigated later with numerical examples.

IV Numerical Examples

In our examples, we consider designing appendages by adjusting the cross section. We use practical examples with realistic scale and material. Furthermore, we try to investigate the design of large flexible space structures, such as huge antenna or space stations.

In what follows, we perform the modal analysis with the finite element method, and model the flexible spacecraft with one rigid body mode and twenty flexible modes. There are r flexible modes, obtained by the dynamic reduction method, retained in the reduced order model for control design. The reduced order model is specified according to the post-maneuver spillover constraint. In the examples, we specify the maximum angular deviation of the central rigid body post maneuver as 0.05 deg. The appendages are I-beams (as shown in Figure 4). Our goal is to obtain the optimal flange depth distribution of the appendages, and assume the width of the web, and thickness of the web and flange to be constant. The flange depth is symmetric about a central line passing through the cross section. We use two spline polynomials as the assumed shape functions to describe the half flange depth :

$$h_1(x) = c_1 + (c_2/L)x + (c_3/L)^2 x^2 + (c_4/L)^3 x^3, \qquad 0 \leq x \leq L/2$$

$$h_2(x) = h_1(L/2) + h_1'(L/2)(x - L/2) + (c_5/L)^2 (x - L/2)^2 + (c_6/L)^3 (x - L/2)^3, \quad L/2 \leq x \leq L$$

where c_i, i = 1, 2, ... 6 are design variables. (4.1)

We note that all design variables c_i , i = 1, 2, ... 6 are almost of the same order, and h(x) and dh(x)/dx are continuous at x = L/2.

We consider a spacecraft with two identical flexible appendages. For simplicity, we assume the appendages are made of a single uniform material.

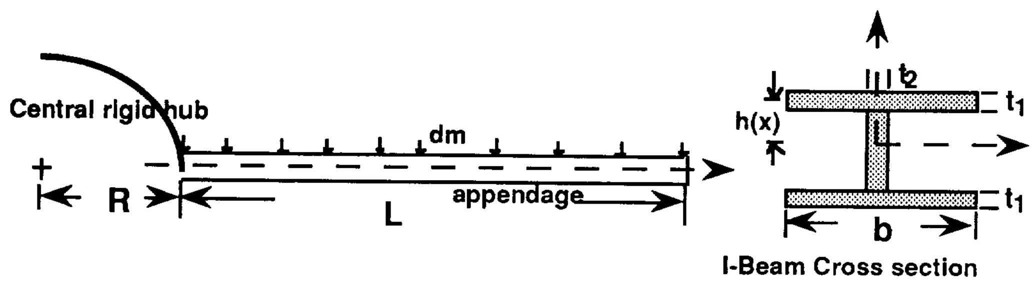


Figure 4 Design of the cross section of appendages

We begin solving the problem by finding a reasonable number of modes in the reduced order model. We use a reasonable baseline design with flange equal 4.00 cm. As shown in Table 1, we note that it is appropriate to retain three flexible modes for a postmaneuver maximum angular deviation to be guaranteed less than 0.05 deg.

Number of modes retained in the model	0	1	2	3	4	5	6	7
Max. angle deviation post maneuver (deg)	1.495	0.082	0.0113	2.6e-3	8.3e-4	3.2e-4	1.5e-4	7.1e-5

Table 1.

Spacecraft Data	
Appendage material density, ρ	1880.00 kg/m^3
Appendage Material Elasticity, E	2.76E11 N/m^2
Radius of the rigid central body, R	12.00 m
Mass of the rigid central body	4500.00 kg
Length of one appendage, L	50.00 m
Maximum torque available, U_0	3.0 E4 N-m
Width of the web, b	5.00 cm
Thickness of the web, t1	1.75 cm
Thickness of the flange, t2	0.75 cm
Distributed pay load mass, dm	9.00 kg/m
Concentrated pay load mass (at x = L), M	None
The resource constraint of two appendages	450.0 kg
The minimal flange depth	2.00 cm
The maximal flange depth	12.00 cm

Case 1 : Single maneuver case

Command slew angle, θ 90.00 deg

Thus the exact solution is t_f, which is equal to t_f^{*3} over the entire feasible design space.

Result :

(a). $\mathcal{L}^{b}_{1}(t_f) = 2\sqrt{\theta J * U_0}$ = 21.9814 sec, but t_f^{*3} of this design is 24.6213 sec.

(b). $\mathcal{L}^{b}_{2}(t_f) = t_f^{1}$ = 22.3126 sec, and t_f^{*3} of this design is 22.41457 sec. The switching times between 0 and t_f^{*3} of the time-optimal control history are 1.5547E-8, 0.21945, 0.48124 sec (one switching time is almost zero).

(c). From $\mathcal{P}_2$: t_f over domain $\mathcal{D}_0$ is 22.3218 sec. Let it be t_f^{a}.

$| t_f^{a} - \mathcal{L}^{b}_{2}(t_f) |$ = 9.2E-3. We can accept this design as the solution (as shown in Fig. 5).

Properties of this optimal design of Case 1 :

Structural mass of two appendages	379.687 kg
Total pay load mass along the appendages	900.00 kg

Total mass of the spacecraft	5779.687 kg
Total rotational moment	2375330.68 kg-m^2
Natural frequency ω_i, i = 1, 2, ..., 4	0.5642, 1.6942, 4.4738, 8.9745 (rad/sec)
The max. angle deviation from the uncontrolled modes	0.00908 deg
Number of switches between 0 and $t_f^*/2$ of the time-optimal control history :	None

Case 2 : General Multiple Maneuvers

The set of maneuvers are { θ_i } = {9, 15, 30, 45, 60, 90 (deg) }, and assume that they occur at the same frequency. Thus the objective function (maneuverability index) is

$$\mu(\xi) = \frac{1}{6}\sum_{i=1}^{6} t_f^*(\theta_i) \tag{4.3}$$

The solution μ equals μ^{*3} over the entire feasible design space.

Result :

(a). $\mathcal{L}^b{}_1(\mu) = 2\sqrt{\theta J * U_0}$ = 13.1753 sec.

(b). $\mathcal{L}^b{}_2(\mu) = \mu^1 =$ 15.0436 sec, and μ^{*3} for this design is 15.30617 sec.

As $\mathcal{P}_4$: if we let μ^a = 15.30617, we have $|\mu^a - \mathcal{L}^b{}_2(\mu)|$ = 0.26257, as 1.7454 %.

(c). μ^2 is 14.8580 sec, and μ^{*3} for this design is 14.96326 sec.

As $\mathcal{P}_4$: if we let μ^a = 14.96326, we have $|\mu^a - \mathcal{L}^b{}_2(\mu)|$ = 0.06526, as 0.4392 %. We accept it as the solution (as shown in Fig. 5).

(d). We investigate the exact solution by $\mathcal{P}_1$ and obtain μ^3 is 14.9455 sec.

Properties of this optimal design of Case 2 :

Structural mass of two appendages	425.075 kg
Total pay load mass along the appendages	900.00 kg
Total mass of the spacecraft	5825.075 kg
Total rotational moment	2379168.55 kg-m^2
Natural frequency ω_i, i = 1, 2, ..., 4	0.8460, 2.0276, 5.5051, 10.6193 (rad/sec)
The max. angle deviation from the uncontrolled modes	0.02436 deg
Number of switches between 0 and $t_f^*/2$ of the time-optimal control history :	Three

V. Conclusion and Future Work

The problem of combined design of structures and controls for optimal maneuverability of an elastic spacecraft has been considered. The main results of the present work are

i). The problem formulation is consistent with bang-bang forms of time optimal controls.
ii). The performance degradation constraint is considered in the design problem.
iii). The optimal design problem is well defined. There always exists a solution.
iv). The optimization is done by mathematical programming.
v). The gradient of the objective function is computed using the Implicit Function Theorem.
vi). Efficient and practical approximate methods have been developed.

Our experience with various numerical examples leads to the following assertions :

i). The best structural designs often are those for which the designs of mass distribution and stiffness distribution have very little coupling.

ii). The benefit of multiple controls is not apparent, since we can use scalar control to achieve good results.

Since spacecraft structure is modelled to be linear, with small displacement and inextensible deformation, the performace for a realistic system which violates these assumptions is worth investigating. The constraints of structural dynamic response, such as maximal displacement and stress, should be considered in the examples as well. Those topics are indicated for future study.

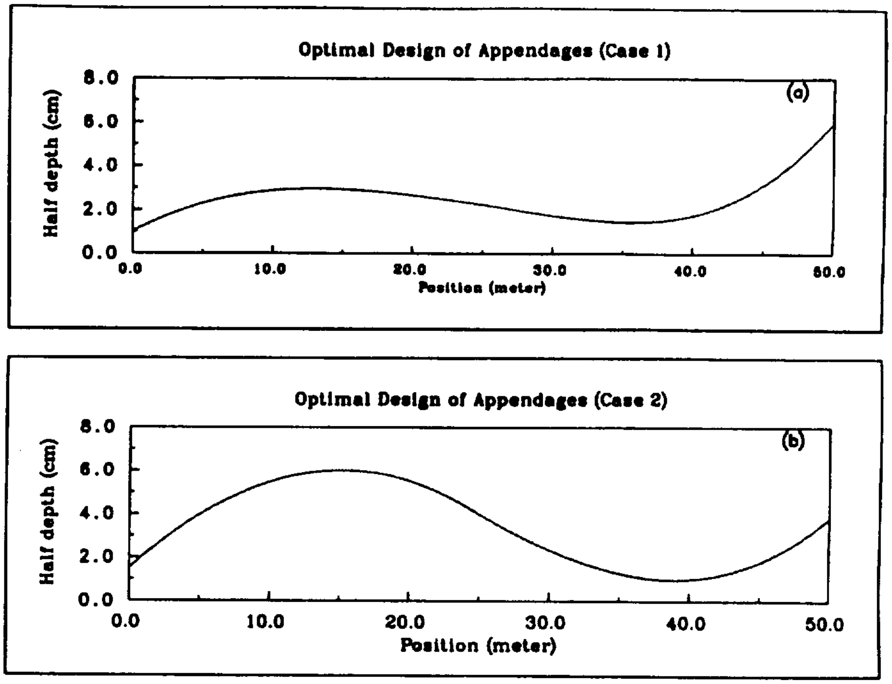


Figure. 5

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