PROPERTIES OF LARGE SCALE PLASMA FLOW DURING THE EARLY STAGE OF THE PLASMASPHERIC REFILLING

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ABSTRACT

Refilling of the outer plasmaspheric flux tube during the initial stage after geomagnetic storms involves both macroscopic and microscopic plasma processes. The former type of plasma processes includes the interhemispheric plasma flow, while the latter one includes microinstabilities and wave–particle interactions. These processes are mutually interlinked. The purpose of this paper is to better characterize the macroscopic properties of the interhemispheric plasma flow by solving a more complete set of hydrodynamic equations than that solved previously. Specifically, we solve the ion continuity, momentum and energy equations for the plasma flow along the closed magnetic field lines. Such a characterization is essential for understanding the microscopic flow. We show that during the initial stage of the supersonic outflow starting with $T_e = T_i$ in the equatorial region, the ions cool substantially creting the condition $T_e >> T_i$ in the equatorial region, where T_e and T_i are the electron and ion temperatures. Such a temperature condition is a prerequisite for microinstabilities involving ion streams in a plasma. The ion cooling along with the super thermal flow velocities of the ion streams create the possibility of microinstabilities affecting the large scale flow. However, such microscopic effects are not studied here. Using the hydrodynamic model for the large–scale plasma flow, we have examined the dynamics of shocks which form in the geomagnetic flux tubes during the early stage of the refilling. These shocks are more like those forming in neutral gases than the electrostatic shocks driven by microinstabilities involving ion–ion interactions. Therefore, the shocks seen in the hydrodynamic model are termed as hydrodynamic shocks. We show that such shocks ar generally unsteady and therefore the usual shock jump conditions given by the Rankine–Hugoniot relations are not strictly applicable to them.

The density, flow velocity and temperature structures associated with the shocks are examined for both asymmetrical and symmetrical interhemispheric flows. In the

asymmetrical flow the outflow from one of the two conjugate ionospheres is dominant. On the other hand, in the symmetrical case outflows from the two ionospheric sources are identical. Both cases are treated by a two—stream model. In the late type of flow, the early—time refilling shows a relaxation type of oscillation, which is driven by the large—scale interactions between the two identical streams. After this early stage, the resulting temperature structure shows some interesting features. In the equatorial region the streams are isothermal, but in the off—equatorial regions the streams have quite different temperatures, and also densities and flow velocities. The dense and slow stream is found to be warmer than the low—density fast stream. In the late stage of the refilling, the temperature is found to steadily increase from the conjugate ionospheres towards the equator; the equatorial temperature is found to be as high as about $8000^{\circ}k$ compared to the ionospheric temperature of $3600^{\circ}k$.

1. INTRODUCTION

After nearly two decades of research, the refilling of plasmaspheric flux tubes after geomagnetic storms still remains a challenging problem from both theoretical and observational standpoints. From the theory viewpoint, the difficulties arise from the fact that the refilling involves plasma processes having scale lengths ranging from plasma Debye lengths and ion and electron Larmor radii to geophysical distances. A self–consistent treatment of such plasma processes is a formidable task. Therefore, most of the progress in tackling this problem has been made on two separate fronts; the macroscopic plasma flows [Banks et al., 1971, Greboski, 1972; Richards et al., 1983; Khazanov et al., 1984; Singh et al., 1986; Singh, 1988; Rasmussen and Schunk, 1988] and the microscopic plasma processes [Schulz and Koons, 1972; Singh and Schunk 1983; Singh et al., 1986 a,b; Singh and Hwang, 1987; Singh, 1988; Singh and Torr, 1988]. The real challenge lies in incorporating the effects of microscopic processes on the macroscopic flow properties. However, before such a challenge is addressed, it is imperative that we first develop an in depth understanding of the microscopic and macroscopic processes separately.

In previous studies on macroscopic flow properties during the plasmaspheric refilling, time–dependent hydrodynamic equations have been solved by several authors; Richards et al. [1983] studied subsonic plasma flow. Khazanov et al. [1984] and Singh et al. [1986] adopted a single–fluid model capable of handling both supersonic and subsonic stages of the interhemispheric flow. Singh [1988] and Rasmussen and Schunk [1988] have adopted a two–fluid model, in which only the continuity and momentum equations have been solved. The purpose of this paper is to characterize the properties of the macroscopic plasma flow using a more complete set of hydrodynamic equations than that solved earlier. Specifically, we solve the continuity, momentum and also the energy equations for the ions. Such a treatment of the interhemispheric plasma flow allows us to examine some of the important issues concerning the plasmaspheric refilling as discussed below.

Banks et al., [1971] postulated that when the fast supersonic interhemispheric flows collide at the equator, a shock pair forms. Between the shocks, the plasma is relatively warm and dense. The refilling occurs behind the shocks as the shocks propagate down the flux tube, one in each hemisphere. Hydrodynamic plasma models of Khazanov et al. [1984] and Singh et al. [1986] show the shock formation. In these hydrodynamic models based on a single—stream treatment, the shock formation is artificial because when the interhemispheric flows overlap in the equatorial region, the average flow velocity suddenly becomes zero. This leads to a density enhancement. The shock is launched by such a compressive perturbation just like the shocks in neutral gases. However, in a single—stream hydrodynamic model, there is no mechanism for the momentum coupling between the interhemispheric streams. This led Singh [1988] to use a two—stream model for the refilling. In this model the streams originating from the conjugate ionospheres were treated as separate fluids. This model showed that when the ion streams overlap in the equatorial region, they continue to counterstream. Recently, Rasmussen and Schunk [1988] used the same model and allowed the counterstreaming to continue for a long enough time so that the flows reached the opposite ionospheres. Where shocks propagating towards the equator originate such a shock propagation causes refilling from "bottom to top" implying that the flow originating from a given ionosphere begins to refill the flux tube starting at its opposite end. Such a refilling was seen in the single—stream model of Singh et al. [1986] for an asymmetrical flow in which one of the two ionospheric plasma sources was disabled making the outflow from it negligible small. When the shocks propagating towards the equator in a symmetrical flow cross each other at the equator, the refilling appears to be from "top to bottom" as seen from single stream models [Khazanov, et. al., 1984; Singh, et. al., 1986].

The counterstreaming of ion beams seen in the macroscopic hydrodynamic models can be trusted only if it can be further corroborated by more sophisticated plasma models. Singh [1988] has examined the microinstability of counterstreaming ion beams showing that

they couple together through the ion—ion instability provided that

$$1.3 V_{ti} \leq V_b \leq M \, C_s \tag{1}$$

$$T_e \lesssim 3 \, T_i \tag{2}$$

where V_{ti} is the ion thermal velocity, V_b is the ion flow velocity (see Figure 1), C_s is the ion—acoustic speed, T_e and T_i are the electron and ion temperatures, and the Mach number lies in the range 2 to 4 [Singh, 1988; Forslund and Shonk, 1971].

The macroscopic models show that the condition (1) on the flow velocity in the counterstreaming plasma flows is likely to be satisfied [Khazanov, et. al., 1984; Singh et al., 1986 a; Singh, 1988; Rasmussen and Schunk, 1988]. However, previous large—scale hydrodynamic models are generally based on isothermal assumptions according to which the electron and ion temperatures remain constant through the entire refilling process. Starting with $T_e \sim T_i$ in the ionosphere, this condition of equal electron and ion temperatures prevails in the models. This seems unrealistic, especially for the ions during the early stage of the refilling when supersonic flow conditions are likely to prevail. Thus, the condition (2) on electron and ion temperatures for the coupling of the ion streams through the ion—ion instability during the early stage of the refilling remains yet untested. Khazanov, et. al. [1984] have apparently solved the temperature equation including the temperature anisotropy. However, strangly enough they have not reported any information on the temperature structure, except for the temperature anisotropy.

The two—stream model of Singh [1988], extended to included the energy equations for the ions, allows us to determine the self—consistent flow velocity and the ion temperature distributions in the flux tube. The knowledge of such distributions is crucial to further developments on the effects of the instabilities on the refilling, even though the large scale hydrodynamic model can not handle the microstabilities.

As mentioned earlier, one of the key questions regarding the ion—ion instability during equatorial counterstreaming is whether or not the temperature condition $T_e > 3 T_i$ can be satisfied. Under such a condition the Landau dumping by the ions is sufficiently

diminished to allow the instability to occur. We show in this paper that during the early stage of the refilling, when supersonic flow conditions prevail, ions cool considerably. Since the electron thermal velocity is much larger than the plasma flow velocity, the electrons are expected to remain isothermal. Thus, starting with $T_e \sim T_i$ in the ionosphere, it is possible to satisfy the temperature condition for the ion–ion instability. However, when such instabilities occur, the dynamics are complicated and the isothermal assumption even for the electrons fails. The inclusion of such dynamics in the refilling models is of paramount importance, but it is a difficult problem [Singh, 1988] and we have not attempted to do this in this paper.

Recent papers on refilling based on hydrodynamic models show the formation of shocks [Singh et a., 1986; Singh, 1988; Rasmussen and Schunk, 1988]. In plasma physics literature, the shocks driven by supersonic plasma flows parallel to the ambient magnetic field are known as electrostatic shocks [e.g. see Tidman and Krall, 1971]. This is primarily due to the fact that the ion beams are slowed down by the electric potential jump across the shock front. So far the nature of the shocks seen in the hydrodynamic models has not been quantitatively examined to determine whether the ion beams ar slowed down by the electric fields or by the pressure gradients in the shock front. We have carried out such an inquiry here. Furthermore, we have examined the shock jump conditions given by the Rankine–Hugoniot (R–H) relations for the conservation of mass, momentum and energy. We find that due to the prevailing unsteady behavior of the shocks, such conditions are not satisfied in the early stages of refilling.

The two–stream model, including energy equation and interstream collisions, shows that the refilling of the flux tube is not monotonic in time. Up to about five hours in the refilling time, all the physical quantities including density, flow velocity and temperature, show a relaxation type of oscillation. Physically such oscillations are the interplay between the collisional force on one hand and the electric field and pressure forces on the other.

The rest of the paper is organized as follows. The theoretical model is discussed in Section 2. Numerical results from the model are presented in Section 3. In Section 3.1, the plasma flow from a single ionospheric plasma source along a depleted flux tube is discussed, including the formation of a hydrodynamic shock and its dynamics. The Rankine–Hugoniot relations for the shock are presented and discussed here. The symmetrical refilling with the plasma flow from both the conjugate ionospheres is presented in Section 3.2. It is shown here that during the early stage of the refilling, the flow undergoes a relaxation oscillation. The paper is concluded in Section 4.

2. THEORETICAL MODEL

We solve the following hydrodynamic equations along a closed magnetic flux tube.

$$\frac{\partial n_i}{\partial t} + \frac{1}{A} \frac{\partial}{\partial s} (A n_i U_i) = 0 \tag{3}$$

$$\frac{\partial U_i}{\partial t} + U_i \frac{\partial U_i}{\partial s} + \frac{1}{m_i n_i} \frac{\partial P_i}{\partial s} - \frac{q_i}{m_i} E - g(r) \cdot \hat{e}_s = \left[\frac{\delta U_i}{\partial t} \right]_c \tag{4}$$

$$\left[\frac{\partial}{\partial t} + U_i \frac{\partial}{\partial s} \right] \left[\frac{3}{2} n_i k T_i \right] + \frac{5}{2} n_i k T_i \frac{1}{A} \frac{\partial}{\partial s} (A U_i) = \left[\frac{\delta \xi_i}{\partial t} \right]_c \tag{5}$$

$$P_i = n_i k T_i \tag{6}$$

$$\left[\frac{\delta U_i}{\partial t} \right]_c = \sum_j n_i m_i \nu_{ij} (U_j - U_i) \phi_{ij} \tag{7}$$

$$\left[\frac{\delta \xi_i}{\partial t} \right]_c = \sum_j \frac{n_i m_i \nu_{ij}}{m_i + m_j} \left[3k(T_j - T_i)\psi_{ij} + m_j(U_i - U_j)^2 \phi_{ij} \right] \tag{8}$$

$$\phi_{ij} = \frac{3\sqrt{\pi}}{4} \frac{erf(\epsilon_{ij})}{\epsilon_{ij}^3} - \frac{3}{2} \frac{exp(-\epsilon_{ij}^2)}{\epsilon_{ij}^2} \tag{8a}$$

$$\psi_{ij} = exp(-\epsilon_{ij}^2) \tag{8b}$$

$$\epsilon_{ij} = \frac{|U_i - U_j|}{(2kT_{ij}/\mu_{ij})^{1\backslash 2}} \tag{8c}$$

$$T_{ij} = \frac{m_i T_j + m_j T_i}{m_i + m_j} \tag{8d}$$

$$\mu_{ij} = \frac{m_i m_j}{m_i + m_j} \tag{8e}$$

where s is the distance along a closed dipolar magnetic field line (Figure 1), t is time, n_i, U_i, P_i and T_i are the density, flow velocity, pressure, and temperature associated with the ion stream i; m_i is the ion mass; k is the Boltzmann constant; q_i is the ionic charge; g(r) is the gravity; $\hat{e}_s$ is a unit vector along the magnetic field; A is the cross–sectional area of the flux tube and $A \propto B^{-1}$. The expressions for $\left[\frac{\partial U_i}{\partial t} \right]_c$, $\left[\frac{\partial E_i}{\partial t} \right]_c$, and the velocity dependent correction terms ϕ_{ij} and ψ_{ij} are taken from Schunk [1977].

The electric field $\underline{E}$ is calculated by assuming that electrons are a massless isothermal fluid. This gives the electron density in terms of the electric potential ϕ.

$$n_e = n_o \, exp \, (e\phi/k \, T_e) \tag{9}$$

Further assuming that the quasineutrality is maintained, the electric field is given by

$$E = -\frac{d\phi}{ds} = -\frac{kT_e}{e} \frac{1}{n} \frac{dn}{ds}, \quad n = \sum_i n_i \tag{10}$$

where T_e is the electron temperature, n_o is the density where $\phi = o$, n is the total ion density and n_i is the density of the ith ion stream or ion species.

The above set of equations are solved for a specified number of ion species or streams as the need may arise using the flux–corrected transport technique of Boris and Book [1976]. This technique has been previously used for the refilling problem [Singh, et. al., 1986a; Singh, 1984; Singh and Rasmussen, 1988].

3.   NUMERICAL RESULTS

As mentioned earlier in the Introduction, one of our aims is to critically examine the nature of the shocks seen in the hydrodynamic models of the refilling. In symmetrical refilling when the streams originating from the conjugate ionospheres are equally strong, the plasma dynamics gets quite entangled. This overshadows some important physical processes affecting the formation and evolution of shocks. Therefore, we begin our discussion in this section with a much simpler situation in which the flow originating from one ionosphere is dominant compared to the flow from the other conjugate ionosphere. We designate such an interhemispheric flow as asymmetrical refilling. After discussing the plasma processes in asymmetrical refilling, we will consider the case of symmetrical filling, in which both the ionospheric plasma sources are equally strong in supplying plasma for the refilling.

3.1   ASYMMETRICAL REFILLING

Previously the asymmetrical flow was considered by Singh et al. [1986a], who solved the set of equations in section 2, without the energy equation (5), and without the collision terms. Ions were assumed to remain isothermal. They showed that the dominant stream originating, say, from the northern ionosphere, flows out to the southern ionosphere, where it increases the plasma density substantially. The pile up of plasma in the southern hemisphere launches a shock travelling towards the northern ionosphere. The refilling occurs behind the shock from the southern to the northern hemisphere as the shock propagates.

More recently, Rasmussen and Schunk [1988] have reproduced essentially the same refilling scenario using a two–stream model. However, in these previous studies the ion energy equation was not solved; the isothermal conditions for both electrons and ions were assumed. We now present results based on a model in which the ion energy equation has been solved, but the electrons are still assumed to obey the Boltzmann relation. The geometry of our calculation is shown in Fig. 1. We consider a flux tube with L=5; the boundary at s=0 corresponds to the northern ionosphere, while that at $s=s_{max}$ to the southern ionosphere. The geomagnetic latitudes of these boundaries are $\pm \lambda_0$ and the corresponding altitude is $v_0 = 3000$ km. The distance is measured along the flux tube from the northern ionospheric boundary at s = 0.

Initially at time t = 0, we assume a depleted flux tube with latitudinal density distributions given by

$$n(\lambda) = n_N \left(\sin\lambda/\sin\lambda_0\right)^{12} \quad \lambda > 0 \tag{11a}$$

$$n(\lambda) = n_S \left(\sin\lambda/\sin\lambda_0\right)^{12} \quad \lambda < 0 \tag{11b}$$

where n_N and n_S are the plasma densities at the northern (s = 0, $\lambda = \lambda_0$) and southern (s = s_{max}, $\lambda = -\lambda_0$) boundaries of the flux tube. In the calculations presented here we have arbitrarily chosen $n_N = 500$ cm^{-3} and $n_S = 50$ cm^{-3}. Other choices for n_N are n_S, as long as $N_n >> N_s$, show essentially the same physical processes. The initial (t=0) velocity and temperature profiles are assumed to be $V(\lambda) = 0$ and $T_i(\lambda) = T_0 = 0.31$ eV. At the boundaries the ion temperature is maintained at $T = T_0$ for t > 0. The asymmetry in the interhemispheric flow is caused by $n_N >> n_S$; thus the plasma stream originating from the northern boundary dominates the plasma flow in the flux tube. In the two–stream models used here the boundary conditions on the stream densities, velocities and temperatures are as follows. For a given stream, at the boundary of the ionosphere of origin these quantities are fixed while they are kept floating at the conjugate ionospheric boundary. For example, for stream originating in the northern ionosphere n(s=0) = n_N,

$V(s=0) = 0$ and $T(s=0) = T_o$, while they are determined self–consistently by the flow conditions at $s = s_{max}$.

EARLY TIME FLOW FEATURES

The early time development of the plasma flow in response to the initial conditions given above is shown in Figure 2. Panels (a), (b) and (c) show the temporal evolutions of the profiles of ion densities $n_i(\lambda)$, flow velocities $U_i(\lambda)$ and temperatures $T_i(\lambda)$ for the two streams originating from the northern ($\lambda = \lambda_o$) and southern ($\lambda = -\lambda_o$) ionospheres. The solid curves are for the dominant stream while the dashed curves are for the weak one. The profiles are at time increments of about 18 minutes. The profiles at $t = 0.3$ hr show outflows from both the ionospheres. The flows are essentially expansions of the ionospheric plasmas. The expansion fronts are characterized by steep gradients in the density, and velocity profiles. An interesting feature of the temperature profiles at $t = 0.3$ hr is as follows. Behind the expansion points, where plasma flows are supersonic as seen from the velocity profile, the ion streams are seen to cool significantly, the minimum temperature just behind the expansion front is only about 100° K, compared to the ionospheric temperature of 3560° K at $\lambda = \pm \lambda_o$. Since in the region of supersonic outflow behind the expansion fronts the profiles of the flow properties are not seen to change with time, the energy equation can be simplified to

$$U_i \frac{dT_i}{ds} = -2/3\, T_i \frac{1}{A} \frac{\partial}{\partial s}(U_i\, A) \tag{12}$$

This equation clearly shows that as the product $(U_i\, A)$ increases, $dT_i/ds < 0$, which implies ion cooling. This is the situation for the flow behind the expansion front before the equatorial crossing. A comparison of velocity (Fig. 2b) and temperature profiles (Fig. 2c) shows that cooling of the ions in the region of supersonic outflow from the ionosphere is a persistent feature. Such a cooling of ions is important for the excitation of microinstabilities which require the temperature condition $T_e > 3T_i$, as mentioned earlier in the introduction.

The profiles at $t = 0.6$ hr show that the expansion of the dominant stream continues down the flux tube after crossing the equator and its density at the southern boundary ($\lambda = -\lambda_o$, $s = s_{max}$) becomes as large as the source density n_N. At $t = 1.2$ hr, the density of this stream at the southern boundary has become nearly 700 cm^{-3}, compared to the ionospheric source density $n_N = 500$ cm^{-3}. In association with the density increase and the sharp drop in the velocity near the southern boundary (Fig. 2b), the ion temperature of the stream has also increased (Fig. 2c).

The pile up of plasma in the southern hemisphere before $t = 1.2$ hr by the dominant stream originating in the northern ionosphere can be understood as follows. Assuming steady flow conditions and ignoring the effects of the minor stream, the continuity and momentum equation can be combined to yield

$$\frac{dn}{n} \frac{(U^2 - U_s^{\;2})}{U^2} = \frac{k}{m_i U_i^2} dT_i - \frac{dA}{A} \tag{13}$$

where U_s is a sound speed and it is given by

$$U_s^{\;2} = k(T_e + T_i)/m_i \tag{14}$$

We see from the foregoing equations that for $U > U_s$, $dn > 0$ if $dT_i > 0$ and $dA < 0$. When the dominant stream crosses the equator $dA < 0$ and dT_i is generally positive. Thus, as the dominant flow penetrates into the southern hemisphere the stream density continually increases. This is one of the important features of the flow for $t \lesssim 1.2$ hr as shown in panel a of Figure 2.

SHOCK FORMATION

The evolution of the density profile of the dominant stream for $t \geq 1.2$ is shown in Figure 3. For the purpose of comparison we have also plotted in this figure the initial density profile and that at $t = 0.6$ hr. Note that the horizontal axis in this plot in not λ, but it is the distance from the northern boundary at $s = 0$. The important feature of the evolution of the density profile is its steepening, which leads to the formation of shock

propagating towards the equator. The build up of plasma near the southern boundary $(s = s_{max})$ is a compressive perturbation. The steepening of such a perturbation into discontinuities is a well known phenomenon both in gas dynamics [e.q. see Roshko and Liepmann, 1957] and plasma physics [Tidman and Krall, 1971]. The basic reason for the steepening is that the perturbation with a relatively higher density propagates faster than the perturbation with the lower density. For shocks in gases, the steepening stops when collisional effects become important; thus the shock width is of the order of a mean free collision length. In collisionless plasma, the steepening stops when it is counterbalanced by the dispersive effects [Tidman and Krall, 1971]. For the ion–acoustic types of perturbation the dispersive effects become important at scale lengths of the order of tens of Debye length, in such a case the steepening can lead to very thin shocks with thicknesses determined by the plasma Debye length. In our present numerical scheme, the grid spacings are of the order of several tens of kilometers. Thus, steepening to Debye length scales is not possible, and the shock width is limited to a few grid spacings $\sim 10^2$ km. However, in reality the shocks are likely to steepen further until dispersion and anomalous effects stop further steepening [Singh et al., 1986 a; Tidman and Krall, 1971].

SHOCK DYNAMICS

We now examine the dynamics of the propagating shock shown in Figure 3. This shock appears to form at the southern boundary between t = 0.6 and t = 1.2 hrs. In contrast to the expansion front that exist in the flux tube prior to t = 0.6 hrs and that propagates from north to south, the shock discussed here travels from the southern boundary $(s = s_{max})$ through the equator to the northern boundary (s = 0). The northern boundary is the origin of the dominant plasma stream in which this shock forms. After crossing the equator, the shock continues to travel toward the northern boundary, but does so at an erratic pace. The shock will sometimes remain in place for several tens of minutes, then accelerate and move forward only to slow again. It eventually reaches the northern

boundary where it slows and dissipates. These later times (t > 2.45 hrs) are discussed as the late time behavior of the shock.

Normally, the shock properties are examined in terms of conservation of the flow of mass, momentum and energy across the shock. Under steady conditions when the density, velocity and temperature do not explicitly depend on time, the conservation relations are the usual Rankine–Hugoniot (R–H) jump conditions, which are obtained by integrating the steady–state continuity, momentum and energy equations across the shock. For the hydrodynamic model of the plasma used in this paper the R–H jump conditions are given by [Liberman and Velikovich, 1986].

$$n_{i1} V_{i1} = n_{i0} V_{i0} \tag{15}$$

$$n_{i1}(m_i V_{i1}^2 + kT_{i1} + kT_e) = n_{i0}(m_i V_{i0}^2 + kT_{i0} + kT_e) \tag{16}$$

$$\frac{5}{2} kT_{i1} + \frac{1}{2} m_i V_{i1}^2 = \frac{5}{2} kT_{i0} + \frac{1}{2} m_i V_{i0}^2 \tag{17}$$

where subscript "1" and "0" refer to the shocked and unshocked plasmas respectively. It is worth mentioning that the cross sectional area (A) of the flux tube does not appear in the continuity Equation (15) because the shock is quite thin and, therefore, the relative change in A across the shock is negligibly small. The state of the shocked plasma at t = 2.45 hr is shown in Figures 4a to 4d which are, respectively, the plots of density, flow velocity, temperature and pressure of the dominant stream. The transition between the unshocked (0) and shocked (1) plasmas is shown by the dashed vertical line. In the unshocked plasma the flow velocity U_{i0} is high and density n_{i0} is low near the shock. On the other hand, in the shocked plasma the density n_{i1} is relatively high and the flow velocity $U_{i1} \simeq 0$. Furthermore, the ion temperature T_{i0} near the shock in the unshocked plasma is quite low while in the shocked plasma the temperature T_{i1} is generally high. The velocities V_{i1} and V_{i0} appearing in (15) to (17) are flow velocities in the rest frame of the shock. Therefore,

$$V_{i0} \equiv V_{i0} + V_{sh} \tag{18}$$

$$V_{i1} \simeq V_{sh} \tag{19}$$

If the shock is steady (15), (18) and (19) can be combined to give the shock velocity

$$V_{sh} \simeq \frac{n_{i0}}{n_{i1} - n_0} U_{i0} \simeq \frac{n_{i0}}{n_{i1}} U_{i0} \qquad (20)$$

The values of n_{i1}, n_{i0} and U_{i0} are tabulated in Table 1. The shock velocity obtained from (20) is also tabulated. Table 1 shows that during the early stages (t < 2.4 hr) the shock velocity should be quite small ($\sim$ .1 V_{th}) and only after the shock crosses the equator at about t $\simeq$ 2.5 hr, V_{sh} abruptly increases to V_{th}. We now show that for the flow in a dipolar flux tube this is not true and, in fact, the shock velocity shows an exactly opposite temporal behavior.

Figure 5 shows the shock position s from s = 0 as a function of time. The shock velocity obtained from V_{sh} = ds/dt is also plotted in the figure. After the equatorial crossing (t > 3 hr), the shock velocity shown in this figure, is averaged over one hour. It is seen that during the early stage (t < 2.5 hr) the shock velocity is relatively large ($V_{sh} >$ 0.85 V_{th}). When the shock approaches the equator, it accelerates achieving a velocity up to $V_{sh} \simeq 1.1 \ V_{th}$. After crossing the equator, the shock slows down considerably, traveling with an average velocity of $V_{sh} \simeq 0.1 \ V_{th}$.

The behavior of the shock velocity seen in Figure 5 is dramatically opposite to that calculated from the mass conservation given by (1) for a steady shock and tabulated in Table 1. This contrast between the actual dynamics of the shock and that predicted from the mass conservation highlights that in the early stages of refilling the shock is highly unsteady, implying that $\partial n / \partial t$ in the continuity Equation (3) is not ignorable at any stage of the shock evolution. For the early stage (t < 3 hr) of the shock evolution, Figure 3 clearly shows the unsteady behavior of the shock as the density behind it is seen to steadily decrease.

For the time interval between the shock formation and its equatorial crossing, it is found that the flux $n_{i0}U_{i0}$ in the unshocked region next to the shock front, in the region of the supersonic flow (see Fig. 4), is negligibly small compared to the flux $n_{i1}V_{sh}$, where V_{sh}

is the actual shock velocity shown in Figure 5. Therefore, the supersonic flow ahead of the shock which is moving from south to north does not play a significant role in the shock dynamics, and the dynamics of the propagating shock is controlled by the expansion of the plasma accumulated in the southern hemisphere during the very early stage ($t < 1.2$ hr, Fig. 2a). It turns out that the equilibrium of the plasma distribution on a converging position of the flux tube, supported by a supersonic flow towards the direction of convergence is quite unstable. When the ram pressure of the tenuous supersonic flow is exceeded by the pressure in the denser plasma, the latter plasma expands while the supersonic flow recedes into the diverging portion of the flux tube. This reduces the density of the supersonic flow (see eq. (13)). This reduces the effectiveness of the supersonic flow and the expanding plasma advances at a faster speed. This is specially true as the expansion approaches the equator (see Fig. 5).

After the equatorial crossing of the shock, the flux $n_{i0}U_{i0}$ into the shocked plasma is found to become larger than $n_{i1}V_{sh}$, indicating that a major part of the plasma flux is spent in increasing the plasma density in the shocked region, and only a small part is spent in filling of the increased volume of the flux tube behind the shock due to its motion. These features, pertaining to the imbalance of the plasma flux across the shock, highlight its non—steady behavior.

In view of the above discussion on the unsteady behavior of the shock in the flux tube, the energy balance given by (17) is not applicable to the shock. The derivation of the energy balance relation in the form of (17) explicitly uses the continuity relation (15).

The unsteady feature discussed above is also seen from the scrutiny of the momentum balance Equation (16). In Table 2, we have given the sums on the right (RHS) and left (LHS) hand sides of this equation at some selected times. It is seen that before the equatorial crossing ($t < 2.5$ hr), the total pressure, consisting of both the thermal and the ram pressures, in the shocked plasma (LHS) considerably exceeds that in the unshocked plasma (RHS). This again points to the fact that the shock is not steady as concluded

earlier. However, after the equatorial crossing it is seen from Table 2 that the pressure balance improves markedly, despite the unsteady behavior of the mass flow as discussed above.

This improvement is easily understood as follows: In the momentum balance, unsteady features may arise due to the term $\partial(n_i V_i)/\partial t = V_i \partial n_i/\partial t + n_i \partial V_i/\partial t$ as seen in the rest frame of the shock. Before the equatorial crossing by the shock, both V_{sh} and n_{i1} are relatively large (see Fig. 5) and therefore $\partial(nV)/\partial t$ is not negligible. This gives rise to the unsteady feature before the equatorial crossing. On the other hand, after the equatorial crossing the shock velocity is drastically reduced (see Fig. 5) making $\partial(nV)/\partial t$ negligibly small. Thus, the momentum balance givenn by (16) improves significantly, after the equatorial crossing.

LATE TIME FEATURES OF ASYMMETRICAL REFILLING

So far we have been primarily concerned with the evolution of the shock in the flow of the dominant plasma stream. We now discuss some other noteworthy features in the flow of the dominant stream. We highlight here the thermal structure during the refilling. The minor stream is found to be controlled by the dynamics of the dominant stream. Therefore we make only brief remarks on the dynamics of the minor stream.

Panels (a), (b) and (c) of Fig. 6 show the temporal evolutions of the profiles of densities, velocities and temperatures for both the solid streams after $t \gtrsim 2.5$ hr. The density profiles for the dominant stream in panel (a) show that shocked plasma develops some interesting structures. The profile at at = 2.45 hr shows a density minimum (marked with an arrow)on the right—hand side of the shock. This density minimum is colocated with a density minimum for the minor stream. The velocity profiles at t = 2.45 show that for the dominant stream $U \simeq 0$, but the minor stream has a relatively large velocity near the density minimum. The temperature profile of the dominant stream at t = 2.45 hr in panel (c) shows that the temperature has a peak colocated with the density minimum. The dynamical processes which lead to the formation of the density minimum and temperature

maximum in the flow of the dominant stream are as follows.

In the early stage of the shock formation near the southern boundary $(s = s_{max}, \lambda = -\lambda_0)$ before $t = 1.2$ hr, the density of the high speed supersonic flow near the shock is relatively large. The kinetic energy associated with this flow is conserved by conversion to thermal energy of the plasma. Thus, the jump from a large to small flow velocity is accompanied by a rapid change in the stream temperature; for example at $t = 1.2$ hr (Fig. 2c) it increases from a low value ($\sim 200^\circ$ K) to a relatively large value of about 9000° k. The above feature can be seen from the density, velocity and temperature profiles at $t = 1.2$ hr in Fig. 2. However, when the shock moves towards the equator, the density of the high speed flow near the shock steadily decreases till the shock crosses the equator. This can be seen from Table 1. When the density of the supersonic flow becomes sufficiently low over the time interval 1.5 hr $< t < 2.5$ hr, the shocked plasma is no more affected by the fast supersonic flow on on the left—hand side of the shock. It neither significantly contributes to the continuity relation nor to the momentum balance as noted earlier. This results in a nearly free expansion of the previously shocked plasma during the time interval mentioned above, $(1.5$ hr $< t < 2.5$ hr$)$. A consequence of this is that the sudden jump from $U \simeq 4$ to $U = 0$ at $t = 2.45$ hr (see Figs. 4b and 4c) is not accompanied by a corresponding sudden change in the temperature unlike for $t \leq 1.2$ hr. Figure 7a shows the transition from heating associated with the shock to the situation with no heating. The transition occurs at about $t = 1.5$ hr. Under such a condition the high speed flow does not affect the dynamics of the plasma behind the shock, and the previously shocked plasma evolves self—consistently sot hat the momentum equation is satisfied with the flow velocity $U \simeq 0$. This requires that in the regions where the temperature is relatively high, due to the previous heating by the shock, the density be relatively low. Neglecting collisions and $\frac{\partial}{\partial t}$ in the ion momentum equation, we have

$$\frac{e}{m_i}\frac{d\phi}{ds} + \frac{1}{n_i}\frac{1}{m_i}\frac{d}{ds}(nkT_i) \simeq 0 \qquad (21)$$

Since $d\phi/ds = (kT_e/e)\frac{1}{n_i}\frac{dn_i}{ds}$, the above equation becomes

$$nkT_e + P_i = \text{const} \qquad (22)$$

In the above equations, n is the plasma density ($n = n_i = n_e$). Equation (22) shows that P_i alone is not a constant, but the quantity ($P_i + nkT_e$) is. Thus, the tendency of the ion pressure (P_i) to increase due to the temperature increase is balanced by that of a decrease in the plasma density n. This is clearly seen by comparing the profiles of density (Fig. 4a), temperature (Fig. 4c) and pressure (Fig. 4d) in the shocked plasma. Equations (21) and (22) merely state that the pressure forces are balanced by the electric forces.

The velocity and temperature profiles at t = 3.1 hr in Figure 6 shows that, unlike for t = 2.5 hr, the jump in the velocity from U $\simeq$ 4 to U $\simeq$ 0 is accompanied by the heating of the ions creating a new peak in the temperature, exactly colocated with the velocity jump. The emergence of this additional peak in ion temperature and its further evolution in time is seen clearly from Figure 7b. We note that at t = 3.1 hr, the shock has crossed the equator and is approximately in steady state so that the R–H equations can be used. In addition, at the shock front, $n_{i0} < n_{i1}$, $T_{i0} < T_{i1}$ and $V_{sh} << U_{i0}$. With these conditions, the momentum balance across the shock is given by (see Eq. 16).

$$n_{i0}m_iU_{i0}^2 + n_{i0}kT_e = n_{i1}kT_e + n_{i1}kT_{i1} \qquad (23)$$

Solving this equation for the ion temperature T_{i1} in the shocked plasma, we obtain

$$kTi_1 \simeq (n_{i0}/n_{i1} - 1)kT_e + m_iU_{i0}^2\, n_{i0}/n_{i1} \qquad (24)$$

Using the numerical values of n_{i0}, n_{i1} and U_{i0} given in Table 1, we find that $T_{i1} \simeq 3\,T_H \simeq 10{,}700^\circ$ K, which is in good agreement with the numerical value of the peak temperature at t = 3.07 hr as shown in Fig. 7b. If we were to use (24) at t = 2.45 hr, we find that $T_{i1} \sim 0$, showing again that at this time (t = 2.45 hr) the momentum balance

condition is not satisfied and the kinetic energy of the supersonic flow does not affect the ion temperature behind the shock. As mentioned earlier, this is primarily due to the fact that the density of the supersonic flow $n_{i0} << n_{i1}$, the density behind the shock at t = 2.45 hr.

Figure 7b shows that the effect of the shock heating expands in both the hemispheres after the shock crosses the equator. The expansion into the northern hemisphere is associated with the northward motion of the shock in the dominant ion stream. After the equatorial crossing, the northbound shock moves with an average velocity of about 0.6 km/s. The expansion of the warm plasma into the southern hemisphere is only slightly slower; its velocity is about 0.5 km/s. Figure 8 shows the density, temperature and pressure profiles associated with the southbound warm plasma expansion front. As indicated by the arrows, the sudden drop in the temperature near the front is accompanied by a sudden increase in the density. We note that the temperature profile, in the region of the shocked plasma where $U \simeq 0$, develops a deep minimum; it is the consequence of the history of the evolution of the ion heating by the shock. During the very early stage (< 1.2 hr, Fig. 2c) the shock heats the plasma in the southern hemisphere. Such a heating stops for sometime (1.5 < t < 2.5 hr) when the density of the supersonic flow near the shock becomes sufficiently low. The shock heating reoccurs after the equatorial crossing of the shock. Thus, in a limited region immediately southward of the equator the ions remain quite cold. When the shock heating restarts and the heated plasma begins to expand southward, this region of very cold ions progressively shrinks. The temperature minimum just on the right–hand side of the arrow in Figure 8b is the consequence of this temporal evolution of the ion heating by the shock. The other temperature minimum at $\lambda \simeq 40^\circ$ in Fig. 8b is associated with the supersonic flow.

It is found that the density, temperature and pressure structures on the right side of the arrows in Figure 8 closely obey (22), which was derived from the momentum equation

in the shocked plasma. The pressure forces in the region of decreasing pressure (Fig. 8c) are balanced by the electric forces given by the density gradients (Fig. 8a).

3.2 SYMMETRICAL REFILLING

The discussion in the previous section on the dynamics of a single dominant stream revealed that the temporal evolution of the flow yields structures in the density, temperature and pressure distributions which are quite interesting and complicated in the sense that they cannot be predicted without the type of calculations carried out in this paper. When the interhemispheric flow is symmetrical with equally strong flows from both ionospheric sources, the structures in the distributions of the physical quantities in the flow and their temporal and spatial evolutions are likely to be even more complicated due to the interactions between the equally strong plasma streams. Rasmussen and Shunk [1988] have studied such interhemispheric flow, but they did not discuss the dynamical features of the flow as their main aim was to demonstrate that the relatively late time behavior of the refilling as seen from single—stream [Singh, et. al., 1986] and two—stream models are similar. Furthermore, our model includes the energy equation. This enables us to study the temporal and spatial evolution of the temperature structures during the refilling.

The calculations on the symmetrical refilling are carried out in the same way as described for the asymmetrical flow, discussed earlier, except for setting $n_N = n_S$ in Equations (11a) and (11b). We have chosen n_S and $n_N = 500$ cm^{-3} in the example discussed here. Figures 9a and 9b show the temporal and spatial evolutions of the density and velocity profiles of the two steams over a time period from t = 0.3 hr to 2.7 hr. Thick line curves in the density panels are the total plasma density $n = n_N + n_S$. Figure 9c shows the average flow velocity given by $V_{av} = (n_N V_N + n_S V_S)/(n_N + n_S)$, where the subscripts "N" and "S" refer to the streams originating from the northern and southern hemispheres, respectively.

The velocity profiles at t = 0.3 hr show that supersonic outflows from both the ionospheres have already set up a counterstreaming in the equatorial region. The flows are

headed by expansion fronts. The density and velocity profiles at t = 0.6 hr show that counterstreaming continues as the expansion fronts continue to move into the opposite hemispheres. For the sake of convenience in referring to the plasma streams, henceforth, the streams originating in the northern and southern ionospheric sources are referred to as stream–N and stream–S, respectively. A comparison of the density and velocity profiles in Figures 2 and 9 show that the features of stream–N are quite similar to those of the dominant stream in the case of asymmetrical refilling until about t ≃ 0.9 hr. However, as stream–N and stream–S penetrate deep into the opposite hemispheres, they begin to affect each other's flow properties. At t = 0.6 hr, the total equatorial density has increased to about 10 cm^{-3} from the initial density of 0.01 cm^{-3}. This refilling has occurred due to the transport of the plasma from the conjugate ionospheres, as clearly seen from the average flow velocity, $V_{aV} = (n_N V_N + n_S V_S)/(n_N + n_S)$, at t = 0.3 hr shown in Fig. 9c. The average flow velocity t = 0.6 hr shows that the equatorial plasma has begun to be transported downward in each hemisphere leading to the decrease in the equatorial density (n_{eq}) as seen from the density plot at t = 0.9 hr; the density has decreased to about 4 cm^{-3}. This trend of plasma depletion from the equatorial region continues as seen from the density and velocity profiles at t = 1.2 hr, when the equatorial density has decreased to $n_{eq} \simeq 2.3$ cm^{-3}.

The depletion of the equatorial plasma, after the very early (t < 0.6 hr) refilling, occurs for to the following reasons. When streams penetrate deep into the opposite hemispheres they modify the total density profile considerably. As we saw earlier in the asymmetrical case (Figures 2 and 3), the plasma that accumulated at high latitudes evolved into a shock and then expanded almost freely equatorward. In the present case of the symmetrical refilling, the interstream collisions become effective in slowing down the streams as they penetrate deep into the opposite hemispheres. This helps in the plasma buildup. The stream–N launchs the shock in the southern hemisphere and stream–S in the northern one. Such equatorward moving shocks and expanding plasma fronts can be seen

by comparing the density profiles of the two streams at t = 0.9 hr and 1.2 hr in Fig. 9a. The modification in the total density profile, forming density "shoulders" where $dn_i/ds \simeq 0$, changes the distribution of the polarization electric fields (see eq. 10). Since such fields play a significant role in the outflow of the plasma [Banks and Holzer, 1969; Singh and Schunk, 1985], the streams are significantly modified in the hemispheres of their origins. The interstream collisions are another factor which modifies the supersonic outflow when the density is enhanced. These factors reduce the flow of plasma towards the equator as evidenced by the reduced flow velocities of relatively high latitudes at t = 1.2 hr in Fig. 9a. But at the same time, the fast streams crossing the equator continue to transport plasma away from the equator. Thus, the equatorial plasma density reduces after some refilling in the very early state (t < 0.6 hr). This density reduction is an interesting example of the affects of the mutual interaction between the two streams. Such a decrease in the equatorial density was not seen with only one dominant stream.

The equatorial density depletion stops when the plasma streams expanding towards the equator cross each other; recall that the stream–N is expanding from the southern hemisphere toward the equator, while stream–S is expanding from the northern hemisphere towards the equator. When the expansion fronts reach the equator, the equatorial density is enhanced to about 60 cm^{-3} at t = 1.5 hr (Fig. 9b). As this equatorward expansion of the streams proceed their densities decrease in the opposite hemispheres and the supersonic outflows of the streams in the hemispheres of their origin reestablishes (compare profiles at t = 1.2 hrs and 1.5 hrs in Figures 9a and 9b. Eventually, the plasma transported by each stream into the opposite hemisphere accumulates near the equator forming a plasma cloud into which the supersonic outflow terminates. The density profile of such a cloud in the opposite hemisphere undergoes a relaxation type of oscillation. These features can be seen from the density profiles at t ≥ 1.8 hr in Fig. 9b. The plasma clouds are seen to have sharp edges just off the equator as indicated by the arrows at t = 1.8 hr. The density and velocity profiles at t = 2.1 hr show that the plasma clouds have rapidly expanded into the

opposite hemispheres. This again modifies the supersonic outflows from the conjugate ionospheres as clearly seen from the density profiles at t = 2.4 hr. At this time the equatorial density is depleted again, N_{eq} has fallen from 60 cm^{-3} at t = 1.5 hr to 37 cm^{-3} at t = 2.4 hr. However, from the velocity profile at t = 2.7 hrs it appears that the plasma transported to the low latitudes flows back again to the equatorial region and N_{eq} rebuilds to 72 cm^{-3} at t = 2.7 hr.

The above discussion shows that the refilling processes during the early stage undergo a relaxation type of oscillation, which is more clearly seen from Figure 10, which gives the temporal evolution of the densities of the stream–N at the equator (solid–line curve) and at $\lambda = +55°$ near the northern boundary (broken–line curve). The oscillations in the density at $\lambda = +55°$ is much stronger than that in the equatorial density. It is interesting to note that the oscillations in the two densities are 180° out of phase, implying that when the equatorial density maximizes, the density at $\lambda = +55°$ minimizes. This is in agreement with our earlier discussion showing that the oscillations in the densities are caused by the back and forth transport of plasma from the equator to relatively higher geomagnetic latitudes.

Figure 10 shows that the relaxation oscillations in the refilling last until about t = 5 hr, after which the refilling becomes nearly monotonic as the densities continuously increase with time, although the rate of increase in the equatorial density is seen to decrease continuously. It is worth mentioning here that the picture of the early time refilling seen from our model differs from that suggested by Rasmussen and Schunk [1988]. According to these authors the refilling occurs monotonically as the shocks formed near the conjugate ionospheres and first propagated towards the equator then downward towards the opposite ionospheres.

LATE TIME REFILLING FEATURES

After the initial stage dominated by the relaxation oscillations, the refilling becomes quite monotonic. In this late stage, the refilling is characterized by downward motions of

shocks, _____ each hemispheres. Figures 11a and 11b show some important features of the monotonic refilling. The density profiles in Figure 11a show that each ion stream has a region of monotonically decreasing ion density. The flow in this region is supersonic. This flow terminates in a shocked plasma, in which the plasma density is enhanced and the flow velocity $U \simeq 0$. These features are quite similar to that of the asymmetrical refilling with a dominant stream as seen earlier in panels (a) and (b) of Figures 6. The profile of the total density (thick line curves in Fig. 11a) shows shocks, one in each hemisphere propagating downward. The shock propagation velocity is found to be about 0.5 km/s. The shocks disappear at about $t = 19$ hr, after which the total plasma density does not show any significant variation with time and the equatorial density $n_{eq} \simeq 316$ cm^{-3} while the densities at the boundaries are about 750 cm^{-3}, which is larger than the densities of the individual streams as imposed by the boundary condition $n_N = n_S = 500$ cm^{-3}. The main feature of the quasisteady density profiles, as shown at $t = 24.6$ hr is that the equatorial density is nearly half of the densities at the boundary.

The density distributions shown in Figure 11a are associated with temperature distributions shown in Figure 11b. The regions of the decreasing density where the flow is supersonic each stream is relatively cold. The jump in the temperature near the shock for each stream is associated with the conversion of the kinetic energy into heat as we discussed earlier in Section 3.1. Between the shocks, where the stream densities are large, the temperature of the two streams are nearly equal for $t > 5.5$ hr. The nearly isothermal condition for the two streams between the two shocks is found to be the consequence of the inter—stream collisions. In order to show this we have compared in Figure 12 the density and temperature distributions obtained from calculations with and without inter—stream collisions. The distributions shown are for $t = 8.6$ hr. There are some significant differences in both the density and temperature distributions. In the case without the collisions, the two streams have quite different temperature distributions between the shocks, the temperature of stream—N peaks near the shock in the stream—S and vice—versa.

The temperature and density distributions of the two collisionless streams between the shocks are found to give a constant pressure, which satisfies the momentum equation. On the other hand, as noted earlier, in the case of the calculation with collisions the two streams have nearly the same temperature in the region bounded by the shocks. This isothermal condition is brought about by the collision term proportional to the temperature difference between the two streams.

Another significant effect of collisions is found on the temperature of one stream in the region of the supersonic flow of the other. For example, the maximum temperature of stream—N in the region of the supersonic flow of stream—S is found to be about $12,000^\circ$ K. Since for the stream—N the flow velocity $U \simeq 0$ in this region and the temporal variation in its temperature is found to be small, the temperature distribution can be estimated by the balance between the two collisional terms in the energy equation (see Eq. (8)):

$$(T_{is} - T_{in}) \, \psi_{sn} = -\frac{1}{3k} m_H (U_{is} - U_{in})^2 \phi_{sn} \tag{25}$$

where ϕ_{sn} and ψ_{sn} are defined by Equations (8a) and (8b), respectively. The foregoing transcendental equation is solved numerically for values of U_{is} in the range $0 \leq U_{is} \leq 4V_{tH}$, and the results are shown in Figure 13 as the solid line. The dashed line is the temperature profile found from exact calculation. Equation (25) yields the same trend in the temperature variation as seen from the model, but it yields a higher value of the temperature. This is attributed to the assumptions in the flow velocity for stream—S. Since ψ_{sn} and ϕ_{sn} involve exponetial functions involving the difference in the flow velocity, the errors are magnified in the temperatures.

10.   ACKNOWLEDGEMENTS

5. Conclusion and Discussion

We have examined, for the first time, the properties of large—scale plasma flows in a dipolar flux tube using a two—stream model [Singh, 1988; Rasmussen and Schunk, 1988] based on a more complete set of plasma transport equations than that used previously. Specifically we have solved the continuity, momentum and energy equations for each ion stream in the flow. The large—scale coupling and interactions between the streams and their affects on the plasmaspheric refilling processes have been examined.

Typically the interhemispheric plasma flow in an empty flux tube after geomagnetic storms is believed to have two streams, which originate in the conjugate ionospheres. We have studied both asymmetrical and symmetrical flows. The significance of the studies on asymmetrical flow is that by making one of the streams dominant, the basic properties of the plasma flow from a single ionospheric source plasma along a closed flux tube can be illucidated, without the complications caused by the mutual interactions between two equally strong streams in a symmetric flow. Some basic properties of a dominant stream in a flux tube is found to be as follows:

(i)   As the supersonic flow develops from the source ionosphere an expansion front forms, behind which the ions cool considerably. By the time expansion front reaches the equator, ions cool by a factor of about 10 in a flux tube with L = 5. Such ion cooling has a direct bearing on generation of plasma waves by the ion streams.

(ii)   When the expansion front crosses the equator and penetrates into the opposite hemisphere the density behind the front continually increases and so does the ion temperature. The buildup of a large plasma density in the opposite hemisphere is an interesting result. This buildup continues until the plasma pressure near the boundary in the opposite hemisphere exceeds the ram pressure of the supersonic flow.

(iii) When the plasma pressure exceeds the ram pressure the high density plasma profile steepens into a shock front traveling from the southern to northern hemisphere (Fig. 3). However, the shock motion is quite unsteady before the equatorial crossing (Fig. 5); shock motion does not satisfy the usual Rankine–Hugoniot shock jump conditions. It is found that before the equatorial crossing, the shock motion is like a free expansion of the dense plasma behind the shock. This is specially true over a time period (1.5 hr < t < 2.5) when the supersonic flow density is sufficiently low so that the ram pressure hardly affects the shock dynamics. During this interval of nearly free expansion, the shock is seen to accelerate (Fig. 5).

(iv) After the equatorial crossing the shock slows down abruptly to quite small velocities. During this phase it is found that although the steady–state continuity relation (15) is not satisfied, but the corresponding momentum balance (16) is approximately satisfied. The steady–state condition for the density is not met because $\partial n / \partial t$ term behind the shock is never zero. On the other hand, the steady state condition for the momentum equation is approximately.

(v) The shock motion from the southern to northern boundary produces interesting structure in the temperature (Fig. 7) and density (Figs. 6 and 7) distributions, which evolve with time.

Some noteworthy features of the symmetric flow are as follows.

(i) At early times when the flows originating from the conjugate hemispheres do not overlap, the flow properties are similar to those of the dominant stream in the asymmetric flow, including the ion cooling behind the expansion fronts of each stream before equatorial crossing of the fronts.

(ii) When the streams begin to overlap after equatorial crossing the stream mutually interact. We note that such interactions do not involve any ion–ion instability, which can lead to the formation of electrostatic shocks [Singh, 1988]. Our

large—scale hydrodynamic code is not capable of including such microscopic plasma processes. The coupling seen in this model is through the polarization electric fields and interstream collisions. These interactions lead to a relaxation type of oscillations in the refilling of the flux tube. The oscillations manifest an interchange of plasma from the equator to high latitudes and vice—versa. The oscillations last for about 5 hours after which the refilling becomes quite monotonic. The relaxation type of oscillations have not been noted in previous studies [Rasmussen and Schunk, 1988].

(iii)   During the phase of the monotonic refilling, the main feature of the plasma flow is the propagation of shocks away from the equator as known from previous studies [Singh, et. al., 1986; Rasmussen and Schunk, 1988]. However, the temperature structure in the flow associated with the shock motion is found to be quite interesting.   Between the shocks two streams become isothermal with temperatures appreciably enhanced over the ionospheric source temperature. In the regions of the supersonic flows, the temperatures of the slow streams are further enhanced. This additional enhancement in the temperature occurs due to the interstream collisions; the high—speed stream heats the slow stream. Thus in the region of the supersonic outflows from the conjugate ionosphere, two ion populations with different densities, flow velocities and temperatures simultaneously exist. This may partly explain the coexistence of different ion populations, having different thermal properties, seen in the depleted flux tubes [Sojka, et. al., 1983].

(iv)   It is shown that in the symmetrical flow the interstream collisions play a significant role in determining both the density and temperature structures (Fig. 12).

(v)   We find that the plasma in the flux tube reaches a quasi–steady state in about 24 hours. In this state, flow velocity is approximately zero, density profile is concave with density minimum at the equator. On the other hand, the temperature profile shows a temperature enhancement at the equator.

In our present model of the interhemispheric plasma flow, we have solved the energy equation without the term giving the heat flow. This term is $(2/3n_i)$ $\nabla \cdot q$, where q is the ion heat flow vector. Assuming that $q_i = -\lambda \nabla T_i$, where λ is the thermal conductivity, the time scale of change in temperature due to thermal conduction can be estimated from the equation.

$$\frac{\partial T_i}{\partial t} \simeq -\frac{2}{3n_i} \lambda_i \nabla^2 T_i \qquad (26)$$

For the thermal conductivity we use the expression [e.g., see Clemmov and Dougherty, 1969].

$$\lambda_i \simeq 2.1 \frac{n_i k_B^2 T_i}{m_i} \tau_i \qquad (27)$$

where τ_i is the ion–ion collision time. Carrying out a dimensional analysis of eq. (26), we find that the approximate diffusion time for the change in the temperature over a scale length L is given by

$$\Delta t_i \simeq \frac{3}{2} \frac{n_i}{\lambda_i} L^2 \simeq 0.7 \left[\frac{m_i}{k_B T_i} \right] \frac{L^2}{k_B \tau_i} \qquad (28)$$

where

$$\tau_i \simeq 3.3 \left[\frac{m_i}{m_e} \right]^{1/2} \times 10^5 T_{iev}^{3/2} / \ell n \Lambda \qquad (29)$$

where m_i is the H$^+$ mass and m_e is the electron mass, T_{iev} is the ion temperature in eV and $\Lambda = 1.24 \times 10^7 T^{3/2} n^{-1/2}$, with T in $^\circ$k and plasma density n_i in m^{-3}. Assuming $T = 10^4$ $^\circ$k and $n \sim 10^8$ m^{-3}, $\ell n \Lambda \simeq 21$ and $\tau_i \simeq 7 \times 10^5$s. With these values we find that the Δt is approximately given by

$$\Delta t_i \simeq 7 \times 10^{13} L_{km}^2 / T_{iev}^{5/2} \qquad (30)$$

where L_{km} is the scale length expressed in kilometers.

From our model we see that maximum temperature $T_{iev} \sim 1$ eV and $L_{km} > 10$ km giving $\Delta t_i \gtrsim 7 \times 10^{15}$ s. This time is so large that the heat conduction due to the motion of ion alone is insignificant.

It can be argued that the electron and ions thermalize by means of coulombs collisions and the heat flow can be affected by the electron motion. Replacing m_i by m_e and λ_i by $\lambda_e = 2.88$ n $k_B^2 T_e \tau_e / m_e$, where τ_e is the electron–electron collision time and, the above equation can be manipulated to give

$$\Delta t_e \simeq (m_e/m_i)^{1/2} \Delta t_i \qquad (31)$$

where Δt_e is the heat diffusion time through the electron motion. Even this time is found to be too large to affect the temperature structures calculated in this paper. Therefore, it appears that the heat flow does not play a significant role at the time scale of the refilling of the flux tube.

It is worth mentioning that recently Rasmussen and Schunk [1988] suggested that an isothermal model for the ion is valid for the interhemispheric flow during refilling. They quote Holzer [1972] to substantiate their assumption. It appears that such an assumption is valid for the outflow of plasma along an open flux tube as shown by Holzer [1972]. Our calculations presented here show that this is not a valid assumption for the interhemispheric flow; the solution of the energy equation shows interesting temperature structures in the flux tube which are suppressed by the isothermal model. Another consequence of our model based on the two–stream approach and the ion energy equation is that ion populations with different thermal characteristics can coexist.

In conclusion, it must be mentioned that in this paper we have studied only the large–scale properties of the plasma flow. How such flow properties affect the small–scale processes, which in turn modify the large–scale flow, remains a challenging problem yet to be tackled.

Table 1

Using Equation (20), shock velocity V_{sh} is calculated. This calculation is based on the assumption that the shock is steady.

time(hr)	$\dfrac{n_{io}}{n_N}$	$\dfrac{n_{i1}}{n_N}$	U_{io}/V_{tH}	V_{sh}/V_{tH}
1.80	3.4×10^{-3}	0.24	3.85	0.14
2.40	5.5×10^{-3}	0.17	3.96	0.13
3.07	7.5×10^{-3}	0.038	3.86	0.95
4.20	9.4×10^{-3}	0.05	3.80	0.88
4.80	1.36×10^{-2}	0.065	3.70	0.98
6.75	2.2×10^{-2}	0.11	3.56	0.90
9.20	4×10^{-2}	0.145	3.37	1.28

Table 2

Sum of the terms on the left and right hand sides of the momentum balance
Equation (16) at some selected times are tabulated. The sums are in the units of $(N_0 k T_0)$.

time(hr)	LHS (shocked plasma)	RHS (unshocked plasma)a
1.2	0.40	0.27
2.4	0.35	0.123
3.0	0.16	0.14
4.8	0.25	0.21
5.5	0.31	0.28
6.1	0.289	0.23
8.6	0.4	0.48

References

1.   Banks, P. M. and T. E. Holzer, High Latitude Plasma Transport: The Polar Wind,
      J. Geophys. Res., Vol. 74, p. 6317, 1969.

2.   Banks, P. M., A. F. Nogy, E.W.I. Axford, Dynamical Behavior of Thermal Protons
      in the Mid—Latitude Ionosphere and Magnetosphere, Planet. and Space Sci., Vol.
      19, p. 1053, 1971.

3.   Boris, J. P. and D. L. Book, Solution of Continuity Equations by the Method of
      Flux—Corrected Transport, Methods Comput. Physics, Vol. 16, 85, 1976.

4.   Clemmow, P. C., and J. P. Dougherty, Electrodynamics of Particles and Plasmas,
      Addison Wiley Publishing Company, Reading, Massachusetts, Chap. 19, 1969.

5.   Forslund, D. W. and C. R. Shonk, Formation of Electrostatic Collisionless Shocks,
      Phys. Rev. Lett., Vol. 25, 1699, 1971.

6.   Grebowsky, J. M., Model Development of Supersonic Trough Wind with Shocks,
      Planet. Space Sci., Vol. 20, p. 1923, 1972.

7. Khazonov, G. V., M. A. Koen, Yu. V. Konikov, and I. M. Sidorov, Simulation of Ionosphere–Plasmasphere Coupling Taking into Account Ion Inertia and Temperature Anisotropy, Planet. Space Sci., Vol. 32, p. 585, 1984.

8. Liberman, M. A. and H. L. Velikovich, Physics of Shock Waves in Gases and Plasmas, Springer–Verlag, Berlin, Chapter 2, 1986.

8. Liepmann, H. W. and A. Roshko, Elements of Gas Dynamics, Chap. 3, John Wiley and Sons, Inc., New York, 1957.

10. Rosmussen, C. E. and R. W. Schunk, Multistream Hydrodynamic Modeling of Interhemispheric Plasma Flow, J. Geophys. Res., Vol. 93, p. 14557, 1988.

11. Richards, P. G., R. W. Schunk, and J. J. Sojka, Large–Scale Counterstreaming of H^+ and He^+ Along Plasmaspheric Flux Tubes, J. Geophys. Res., Vol. 88, p. 7879, 1983.

12. Schulz, Michael and H. C. Koons, Thermalization of Colliding Ion Streams Beyond the Plasmapause, J. Geophys. Res., Vol. 77, p. 248, 1972.

13. Schunk, R. W., Mathematical Structure of Transport Equations for Multispecies Ion Flows, Rev. Geophys., Vol. 15, p. 429, 1977.

14. Singh, Nagendra and R. W. Schunk, Numerical Simulation of Counterstreaming Plasmas and their Relevance to Interhemispheric Flows, J. Geophys. Res., Vol. 88, 1983.

15. Singh, Nagendra and R. W. Schunk, Temporal Behavior of Density Perturbations in the Polar Wind, J. Geophys. Res., Vol. 90, p. 6487, 1985.

16. Singh, Nagendra, R. W. Schunk, H. Thiemann, Temporal Features of the Refilling of a Plasmaspheric Flux Tube, J. Geophys. Res., Vol. 91, p. 13433, 1986a.

17. Singh, N., H. Thiemann, and R. W. Schunk, Studies on Counterstreaming Plasma Expansion, Physica Scripta, Vol. 33, p. 355, 1986b.

18. Singh, Nagendra and K. S. Hwang, Perpendicular Ion Heating Effects on the Refilling of the Outer Plasmasphere, J. Geophys. Res., Vol. 92, p. 13513, 1987.

19. Singh, Nagendra, Refilling of a Plasmaspheric Flux Tube—Microscopic Plasma Processes, In Modeling Magnetospheric Plasma, (Ed's. T. E. Moore and J. H. Waite, Jr.), Geophysical Monograph 44, American Geophysical Union, p. 87, 1988.

20. Singh, N. and D. G. Torr, Review of Microscopic Plasma Processes Occurring During Refilling of the Plasmasphere, Adv. Space Res., Vol. 8, p. (8)5, 1988.

21. Tidman, D. A. and N. A. Krall, Shock Waves in Collisionless Plasmas, John Wiley and Sons, Inc., Chapters 1 and 6, 1971.

Figure Caption

Fig. 1   Geometry of the flux tube. Ionospheric boundaries are at $s = 0$ (northern ionosphere) and $s = s_{max}$ (southern ionosphere), $\pm\lambda_0$ are the geomagnetic latitudes of the two boundaries and v_0 is their geocentric altitude.

Fig. 2   Temporal evolutions of the profiles of (a) density, (b) velocity and (c) temperature for the dominant (——) and the weak(——) streams.

Fig. 3   Temporal evolutions of the density profile showing the formation of a shock. The profiles at $t = 0$ are initial profile in the empty flux tube. Note the steepening of the profiles from $t = 0.6$ hr to $t = 2.4$ hr.

Fig. 4   State of the shock at $t = 2.4$ hr: (a) density profile, (b) velocity profile, (c) temperature profile and (d) pressure profile.

Fig. 5   Location and velocity of the shock as a function of time. The distance is measured from $s = 0$.

Fig. 6   Same as Fig. 2 but for later times.

Fig. 7   Evolution of the ion temperature profile in response to the shock motion. (a) before the equatorial crossing and (b) after the equatorial crossing.

Fig. 8   State of the plasma at $t = 7.8$ hr: (a) density profile, (b) temperature profile and (c) pressure profile.

Fig. 9   Evolution of the profiles of (a) density, (b) velocity, and (c) average velocity. The thick line curves in (a) give the total density $n_i = n_s + n_n$.

Fig. 10   Same as (a) and (b) in Fig. 9, but for later times.

Fig. 11   States of plasma during late stage of the refilling, (a) density profiles and (b) temperature profiles. In panel (a) the thick solid line curve gives the total density, which is the sum of the densities of the two streams.

Fig. 12   Comparison between calculations with and without collisions (high–hand panels). The top panels show the density profiles, while the bottom ones show temperature profiles.

Fig. 13   Broken line curve shows the enhancement of the temperature of stream—N in the southern hemisphere where stream—S is supersonic. Solid line curve shows the temperature obtained by balancing the two collision terms in the energy equation (see eq. 25).