FRACTIONAL CLOUDINESS IN SHALLOW CUMULUS LAYERS

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1. Introduction. Fractional cloudiness influences the planetary boundary layer by controlling the cloud-top radiative cooling rate, and regulating the buoyant production and consumption of turbulence kinetic energy. Betts, Hanson, and Albrecht have modeled partly cloudy PBLs by assuming a single family of convective circulations. The same idealized model has been used in observational studies, based on conditional sampling and/or joint distribution functions, by Lenschow, Albrecht, and others. This approach is extended in the present paper. None of these authors has proposed a method to determine σ, the fractional area covered by rising motion; finding such a method has been a key objective of the present study.

2. Model formulation. As a starting point, we adopt Lilly's mixed-layer model. It is assumed that in the interior of the PBL, the turbulent fluxes are entirely due to the convective circulations, with rising branches covering fractional area σ, and sinking branches covering fractional area $1 - \sigma$. The vertical flux of an arbitrary scalar ψ due to the convective circulations is given by

$$F_{\psi} = - \frac{\omega_*}{g}(\Psi_u - \Psi_d) ,$$

(2.1)

where g is the acceleration of gravity; ω_* is the "convective mass flux"; and subscripts u and d denote upward and downward branches, respectively. Near the lower boundary is a "ventilation layer" (essentially the same as the surface layer) within which the turbulent fluxes have to be carried by small eddies, since the organized vertical motions associated with the convective circulations must vanish there. The ventilation layer is assumed to be thin in the sense that the turbulent fluxes at its top are approximately equal to those at the surface. The surface fluxes are assumed to satisfy the usual bulk aerodynamic formula,

$$(F_{\psi})_S = V (\Psi_g - \Psi_S) ,$$

(2.2)

where V is the "ventilation mass flux." Here subscripts g and S denote the earth's surface and a level in the ventilation layer, respectively. At level S, the parcels rising away from the lower boundary must be "charged" with the properties of the boundary. We cannot assume, however, that the properties of the updrafts at level S are the same as the those of the boundary, because there can be very strong gradients across the ventilation layer. The small eddies of the ventilation layer rapidly dilute air that has been in contact with the boundary, by mixing it with air that has recently descended from the interior of the PBL. In order to take this mixing into account, we introduce a nondimensional parameter, M_V, such that

$$(\Psi_u)_S - \Psi_S = M_V (\Psi_g - \Psi_S) ;$$

(2.3)

in case $M_V = 1$, we get $(\psi_u)_S \approx \psi_g$. Smaller values of M_V indicate stronger mixing by the small eddies of the ventilation layer. We expect $0 < M_V \ll 1$. By combining (2.1-3), we find that

$$-M_V \omega_{\cdot S} = gV(1 - \sigma).$$

$$(2.4)$$

This is a kind of "continuity equation" for the eddies, expressing a relationship between the convective mass flux and the ventilation mass flux.

Near the PBL top is an "entrainment layer" within which the organized vertical motions associated with the convective circulations become negligible, and smaller eddies carry the turbulent fluxes. The entrainment layer is assumed to be thin in the sense that the turbulent fluxes at its base are approximately equal to those at the PBL top. We can show that

$$-M_E \omega_{\cdot B} = gE\sigma_B.$$

$$(2.5)$$

This is another "continuity equation", analogous to (2.4). Here subscript B denotes a level in the entrainment layer, and M_E is another mixing parameter, analogous to M_V. We expect $0 < M_E \ll 1$.

We now assume that σ is independent of height between the top of the ventilation layer and the base of the entrainment layer. This allows us to drop the subscripts B and S from σ. Comparing (2.4) and (2.5), we find that

$$\sigma = \cfrac{1}{1 + \cfrac{E\omega_{\cdot S} M_V}{V\omega_{\cdot B} M_E}}.$$

$$(2.6)$$

The form of (2.6) ensures that σ is between zero and one, so long as the ratio in the denominator is positive. To develop a useful expression for σ, a logical next step would be to introduce parameterizations for M_E and M_V. Because M_E and M_V represent the effects of small eddies with brief lifetimes, they should be highly amenable to parameterization. Unfortunately, however, no such parameterization currently exists.

As an alternative to parameterizing M_E and M_V, we assume that the expression in the denominator of (2.6) is equal to one. This implies that an increase in the convective mass flux at B or S is accompanied by more vigorous mixing there. Using this assumption, we find that

$$\sigma = \cfrac{1}{1 + \cfrac{E}{V}}.$$

$$(2.7)$$

According to (2.7), σ decreases as the entrainment mass flux increases relative to the ventilation mass flux. *Rapid entrainment implies small σ.*

3. Flux profiles in a partly cloudy well mixed layer. Lilly showed that, in a well mixed layer, the turbulent fluxes of conservative variables are linear with pressure, and also vary linearly with the entrainment rate. He further showed that in a mixed layer of horizontally uniform cloudiness the fluxes of liquid water and buoyancy have simple dependencies on height and the entrainment rate, even though liquid water and buoyancy are not conservative. In this Section we generalize Lilly's results to include the case of partly cloudy layers, by drawing on the results of Randall (*J. Atmos. Sci.*, 1987, pp. 850-858; hereafter referred to as R87), and using (2.7). An example is used for clarity.

We assume that $(F_r)_S$ satisfies a bulk aerodynamic formula, and $(F_r)_B$ satisfies Lilly's "jump" relation. The effects of drizzle are neglected for simplicity. As an example, we consider the following parameters: $\delta p_M = 70$ mb, $r_M = 7.5$ g kg^{-1}, $T_g = 21$ °C, $gV = 2.2$ mb hr^{-1}, $R_0 = 70$ W m^{-2}, $\Delta\sigma_d = 2$ K, $\Delta\sigma_m = -2$ K, $p_S = 1020$ mb. Here $\Delta\sigma_d$ and $\Delta\sigma_m$ are the usual dry and moist inversion stability parameters. Fig. 1 shows how the latent heat flux varies with σ. At the earth's surface, the latent heat flux is always equal to its prescribed surface value. Near the PBL top, F_r varies strongly with E. Since E and σ are related by (2.7), $(F_r)_B$ increases rapidly as σ decreases for $\sigma \ll 1$.

We assume that $(F_h)_S$ satisfies a bulk aerodynamic formula, and $(F_h)_B$ satisfies Lilly's "jump" formula. (Distributed radiative cooling is neglected for simplicity.) The area-averaged radiative cooling obviously depends on the fractional cloudiness. We assume for simplicity that when only the updrafts are saturated

$$\overline{\Delta R} = \sigma Q_u \Delta R_0.$$

(3.1)

Here ΔR_0 denotes the radiative cooling that occurs above a fully overcast optically thick cloud layer. The factor Q_u is introduced to allow continuous transitions as clouds form and dissipate; for thick clouds, $Q_u = 1$. The details are omitted here for brevity.

For the case in which only the updraft is saturated, the radiative cooling depends explicity on σ through (3.1), and the turbulent fluxes also depend on σ implicitly because the entrainment rate satisfies (2.7). The radiative cooling rate and the entrainment rate depend on the cloudiness, but to find the cloudiness we need to know the turbulent flux. *In short, we have to solve simultaneously for the cloudiness, the radiative cooling rate, and the moist static energy flux.*

Fig. 1 shows how the updraft cloud depth varies with σ, for the parameters given above. For s greater than about 0.7, no cloud occurs. For smaller values of σ, the updrafts are cloudy, but the downdrafts remain cloud-free. Fig. 2 shows how $\overline{\Delta R}$ varies with σ . The maximum value of $\overline{\Delta R}$ occurs for $\sigma \cong 0.3$; for larger values of σ the cloud is absent or thin, and for smaller values it covers little of the area.

Up to this point, we have not had to consider latent heat effects, since both the total mixing ratio and the moist static energy are conserved under both dry adiabatic and moist adiabatic processes. Virtual temperature is not conserved under moist processes, however. As shown by R87, *for the case in which only the updrafts are saturated* the convective mass flux model implies that the buoyancy flux satisfies

$$F_{sv} = (1 - \sigma)(F_{sv})_{CLD} + \sigma(F_{sv})_{CLR} - \frac{\omega}{g}[1 - (1 + \delta)\varepsilon]\, L\tilde{\Gamma},$$

(3.2)

where $\tilde{\Gamma}$ is a measure of the relative humidity of the mean state. In (3.2), the forms of F_{sv} for the clear-sky and overcast cases are denoted by $(F_{sv})_{CLR}$ and $(F_{sv})_{CLD}$, respectively; L is the latent heat of condensation; and δ and ε are the usual positive nondimensional thermodynamic parameters. Notice that in (3.2) the "cloudy" flux is paradoxically weighted by the "clear-sky" fractional area, and vice versa. A derivation and interpretation of (3.2) is given by R87.

Fig. 3 shows how the fluxes of moist static energy, total water, and liquid water vary with with σ, at the PBL top level. All three fluxes increase rapidly as σ decreases for $\sigma \ll 1$. This is due to the rapid increase in the entrainment rate as σ decreases, which follows from (2.7). As σ decreases, the liquid water mixing ratio of the updrafts increases, while that of the downdrafts remains constant (at zero). This favors an increase in the liquid water flux near the PBL top. Further discussion is given by R87.

Fig. 4 shows the variation of F_{sv} with σ, at the surface, the updraft cloud base, and the PBL top. Of course, the surface value is independent of σ. For $\sigma \le 0.5$, F_{sv} decreases upward from the surface to cloud base, and increases upward continuously above cloud base. As E increases and σ decreases, the vertical profile of F_{sv} responds to several competing factors. First, increasing E tends to reduce F_{sv} below cloud base, because of the inversion at the PBL top. There is a similar but weaker tendency for Fsv to decrease above cloud base, unless cloud-top entrainment instability occurs. A second factor is that cloudiness leads to radiative cooling for $\sigma \le 0.5$. For $0.3 < \sigma < 0.5$, $\overline{\Delta R}$ increases as σ decreases because the cloud gets thicker, even though its fractional area decreases. This increase in $\overline{\Delta R}$ tends to increase F_{sv} at all levels above the surface. For $\sigma \le 0.3$, $\overline{\Delta R}$ decreases as σ decreases, and this tends to reduce F_{sv} at all levels.

4. Plans for comparisons with FIRE data.

There are many ways in which the FIRE data can be used to test the assumptions on which the present model is based. Lenschow, Greenhut and others have demonstrated that conditional sampling methods can be used with aircraft data to determine the convective mass flux profile, and the updraft and downdraft properties, including the fractional area covered by rising motion. Using such methods, it should be possible to determine M_E and M_V. This can best be done by using (2.4) for M_V and the analogous definition for M_E, with measured values of the updraft and downdraft properties. The values of M_E and M_V so determined should be independent of "species;" they should be the same for water vapor and ozone, for example.

The ventilation and entrainment mass fluxes can also be determined observationally, using standard methods. It then becomes possible to check (2.5) and (2.6), which are the key equations used in the derivation of our method to determine σ. In addition, the assumption that σ is independent of height can be tested. This assumption leads to (2.6), which is the most general form of our prescription for σ. In addition, we can check (2.7), which has been used to determine σ in this paper.

5. Concluding Remarks.

The fractional cloudiness parameterization described here is suitable, with minor modifications, for use in a general circulation model. It represents a break with earlier cloudiness parameterizations, because in the present parameterization, the cloud amount is partly determined by the turbulence. Of course, the more familiar couplings among clouds, radiation, and turbulence are retained. The cloud, turbulence, and radiation parameterizations give rise, therefore, to a coupled system of equations that must be solved simultaneously. Dealing with this added complexity is a challenge for the future.

Normalized Updraft Cloud Depth

σ, %

Figure 1

Radiative Cooling Rate, W m^{-2}

σ, %

Figure 2.

(Fh)B, L(Fr)B, and L(Fl)B, W m^{-2}

σ, %

Figure 3.

Buoyancy fluxes, W m^{-2}

σ, %

Figure 4.