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Non Linear Evolution of a Second Mode Wave in Supersonic Boundary Layers

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Recent advances in supersonic and hypersonic aerospace technology have led to a renewed interest in the stability and transition to turbulence of high speed flows. The last 30 years have intermittently witnessed some vigorous attempts to understand some of the fundamental routes to transition for incompressible flows. While a fairly comprehensive picture of the initial stages leading to the breakdown of an incompressible laminar boundary layer has emerged (mostly under controlled conditions) the non-linear effects responsible for transition at high speeds are still very much a mystery. However, the current nonlinear incompressible theories, numerical simulations and experiments will, hopefully, serve as a guide in gaining a better understanding of the mechanisms present in the supersonic and hypersonic regimes.

Compressible linear stability theory diverges from incompressible linear stability theory in several ways. Incompressible inviscid instabilities, linked with the existence of an inflection point in the mean streamwise velocity profile are replaced by a correlation between inviscid compressible instabilities and a generalized inflection point, which brings into play the mean density profile. Furthermore, as the Mach number increases, the growth rate of the 3-D modes begin to overtake those of the 2-D modes. Beyond Mach 2.2, multiple unstable modes (at fixed Reynolds number and frequency) can coexist. The higher modes are inviscid in nature and have different behaviors with regard to wall cooling. Not only are they more unstable than their first mode counterparts (viscous in nature) above a certain Mach number, but they are destabilized by wall cooling, which is detrimental for high altitude hypersonic aircraft.

It is of vital importance that the nonlinear nature of these second mode instabilities be understood, and that their role in the context of transition be elucidated. The objective of the work is to understand the possible equilibrium state of a second mode wave, before initiating a study of 3-D wave interactions.

Two years ago, a spectral code was developed to perform direct simulations of subsonic and supersonic flows over flat plates. In this paper, we present several direct simulations of one 2-D second mode perturbation wave, superimposed upon a prescribed mean flow. Periodicity is assumed in the streamwise direction (Fourier) and the variables are expanded in Chebyshev series in the direction normal to the plate. The code is fully explicit and is time advanced with a 3rd order Runga-Kutta scheme. The second mode wave ($R_{\delta^*} = 8000$), interacts with itself to generate higher streamwise harmonics. Physical parameters are chosen to maximize the linear growth rate at the prescribed Reynolds number. Initial results indicate that the nonlinear processes begin in the critical layer region and are the result of the cubic interactions in the momentum equations, rather than due to the higher streamwise harmonics. Analysis of the various terms in the momentum equations combined with numerical experiments in which various modes are artificially suppressed, lead to the conclusion that asymptotic methods will produce the saturated state in one or two orders of magnitude less computer time than that required by the direct numerical simulations.

CURRENT STATUS OF FLAT PLATE STABILITY AND TRANSITION

- **Theory**
  - Inviscid Stability Theory (Lees and Lin, 1946)
  - Linear Parallel Theory (Mack 1965)
  - Linear Non-Parallel Theory (Nayfeh and El-Hady, 1980)
  - Non-Linear Theories
    * Wave-Interactions: NONE
    * Secondary Instability: (El-Hady 1988, Ng 1988)
- **Experiment**
  - Stability of Supersonic Boundary Layers (Laufer and Vrebalovich 1960)
- **Numerical**
  - Linear Stability of Ideal Gases (Mack 1975, Malik 1982, Macaraeg & Streett 1988, Erlebacher & Herbert 1988, Ng 1988)
  - Linear Stability of real gases (equilibrium air) (Malik 1988, Macaraeg & Streett 1988)
  - Non-Linear Stability of (Erlebacher and Hussaini 1987)

PHYSICAL PARAMETERS

Non-Dimensionalization

length:	δ^*
$\vec{u}, \rho, T$:	free-stream values
P:	ρU_∞^2

- boundary layer flow
- ideal gas (air)
- $Pr = 0.70$, $Re = 8000$
- M=4.5
- $\alpha = 2.25$, $\psi = 0°$
- – linear frequency: 2.05 $\Longrightarrow$ period = 3.07
  – linear growth: .0215
  – 20 periods in time gives amplification of 3.76
  – e^9 amplification in 136 periods

DISCRETIZATION

- Fully explicit
- 3^{rd} order low-storage Runga-Kutta in time
- Fourier collocation in two periodic directions (stream and span)
- Chebyshev collocation in vertical direction
- Zero normal stress boundary-conditions in the free-stream
- Continuity equation is imposed at the wall and in the far-field
- Zero temperature perturbations at the wall

MODAL ANALYSIS

$$u \propto e^{-\iota\omega t}$$
$$\frac{\dot{u}}{u} \approx -\iota\omega$$
$$\omega_i = Real(\frac{\dot{u}}{u})$$
$$\omega_r = Imag(\frac{\dot{u}}{u})$$

Let

$$u(x,y) = \sum_{n=-\infty}^{\infty} u_n(y,t)e^{\iota n\alpha x}$$

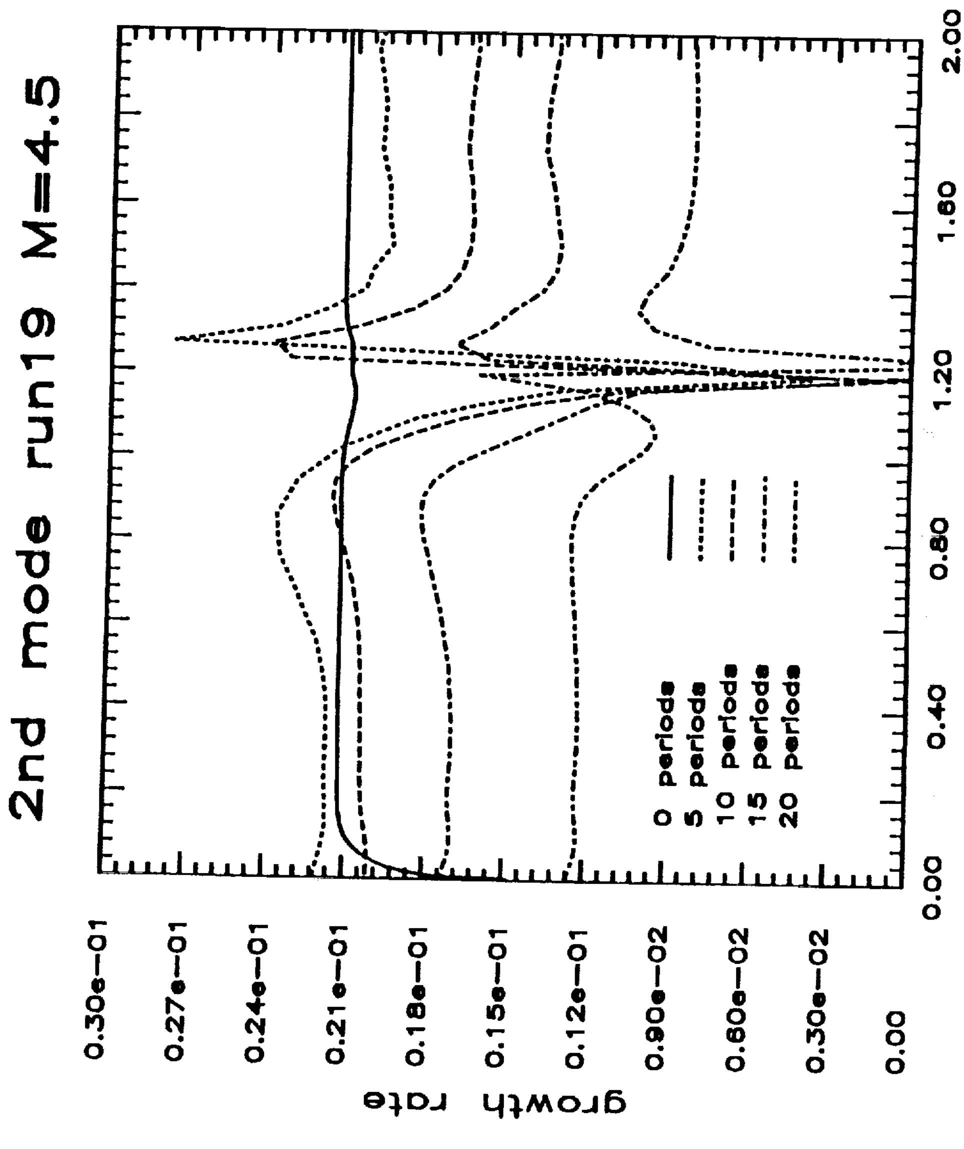
After substitution into the x-momentum equation,

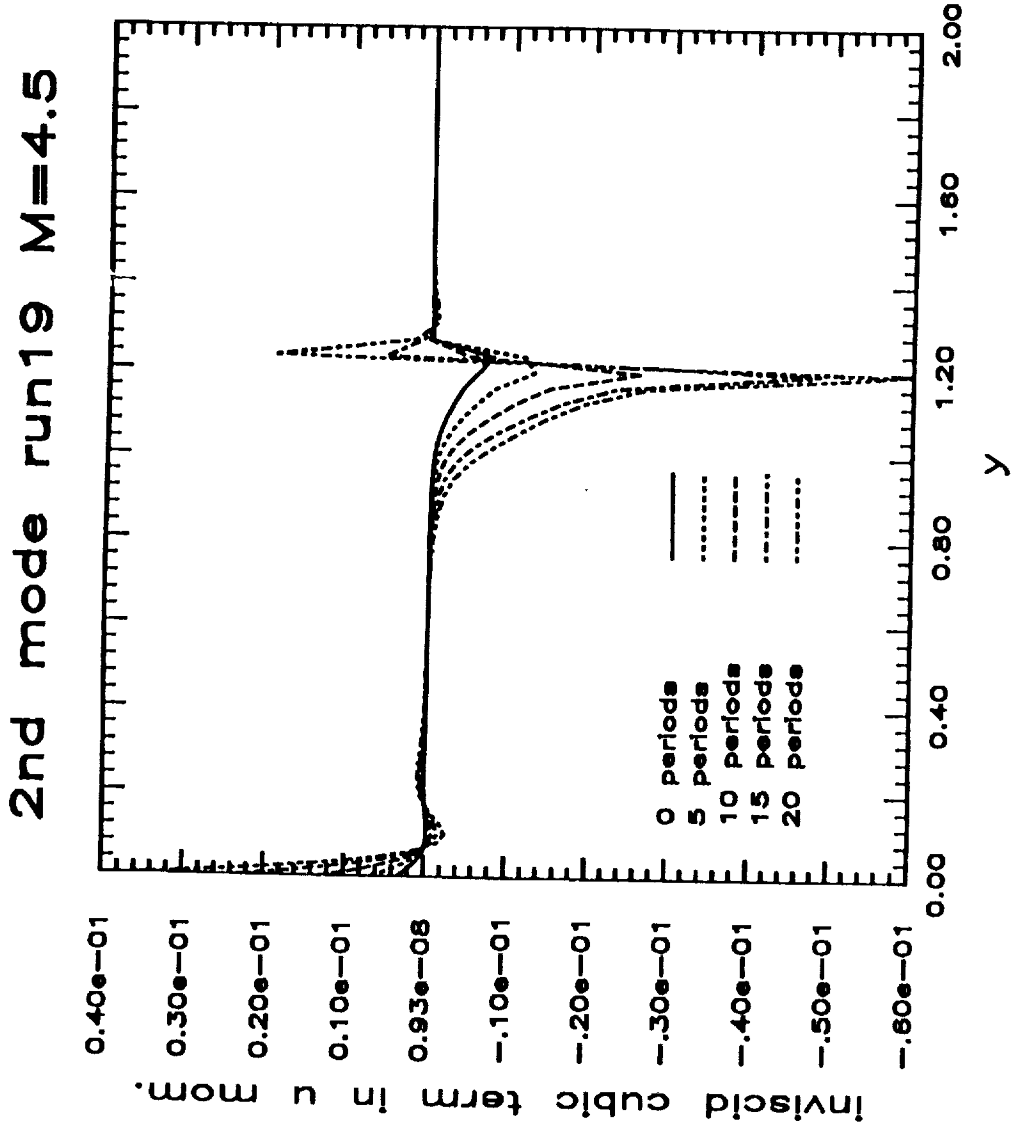
$$\dot{u} = \text{linear} + \text{cubic} + \text{viscous terms}$$

where

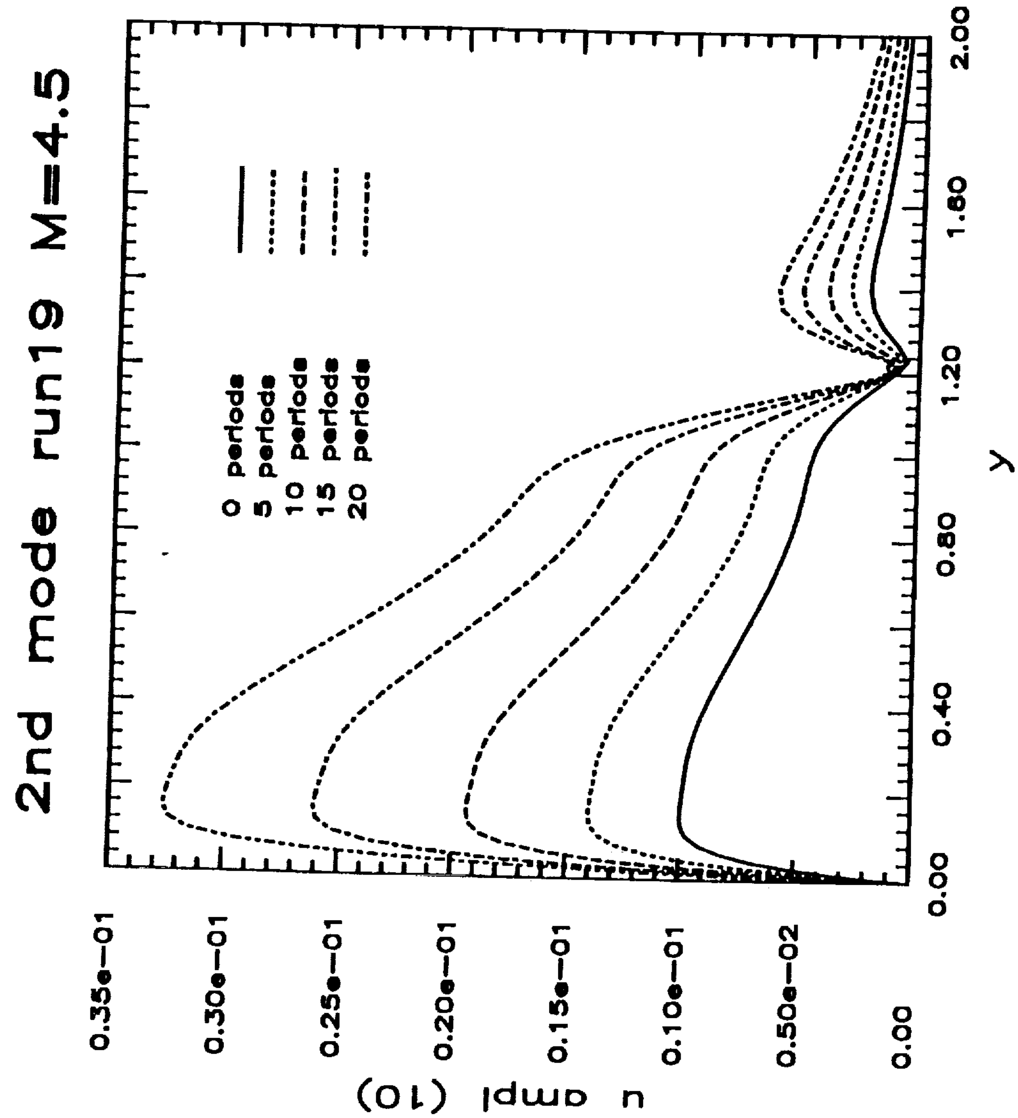
$$\text{Linear} = \iota\alpha u_0\, u_1\, \rho_0 + \cdot$$

$$\text{Cubic} = \iota\alpha u_1\, u_{-1}\, \rho_1 + \cdot$$

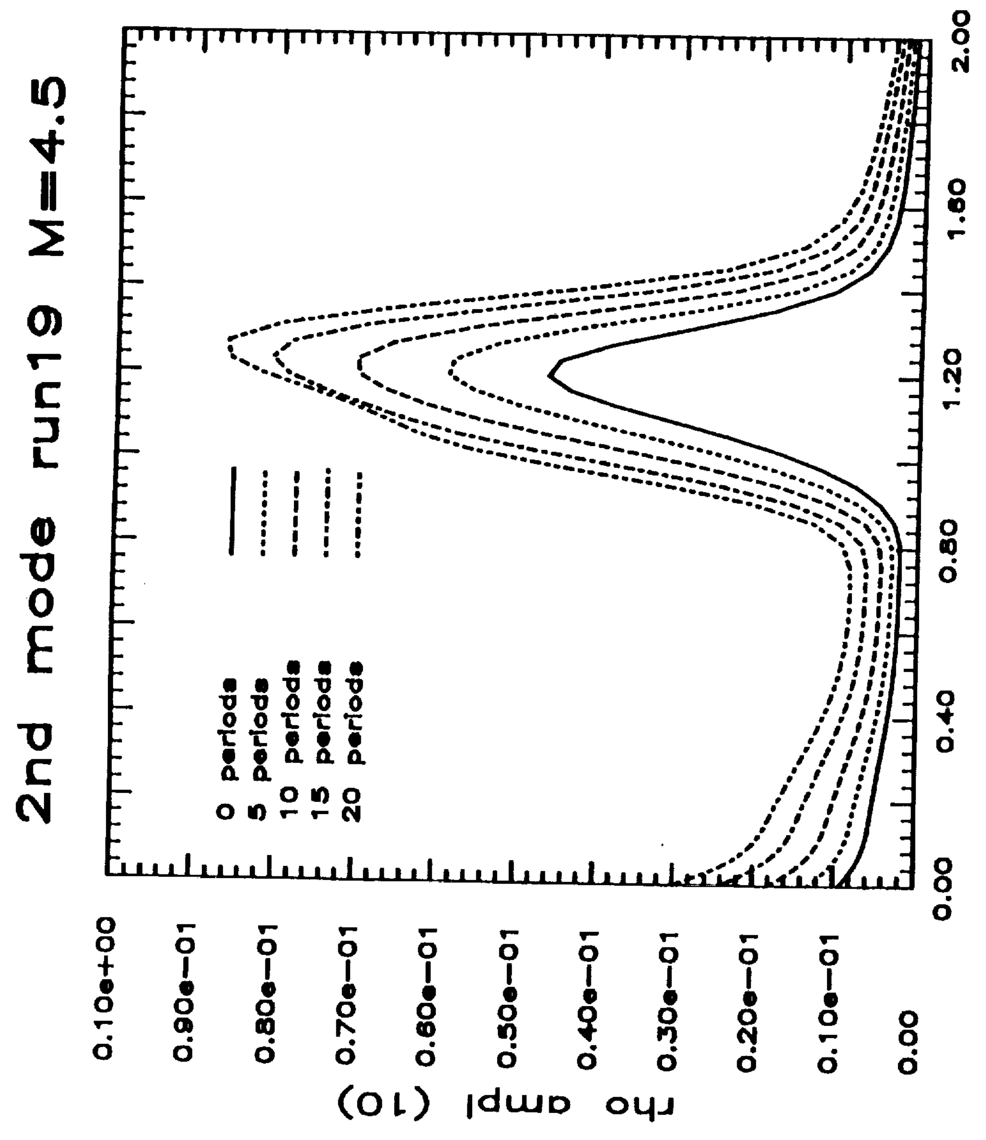

2nd mode run19 M=4.5
growth rate
0.30e-01
0.27e-01
0.24e-01
0.21e-01
0.18e-01
0.15e-01
0.12e-01
0.90e-02
0.60e-02
0.30e-02
0.00
y
0.00
0.40
0.80
1.20
1.60
2.00
0 periods
5 periods
10 periods
15 periods
20 periods

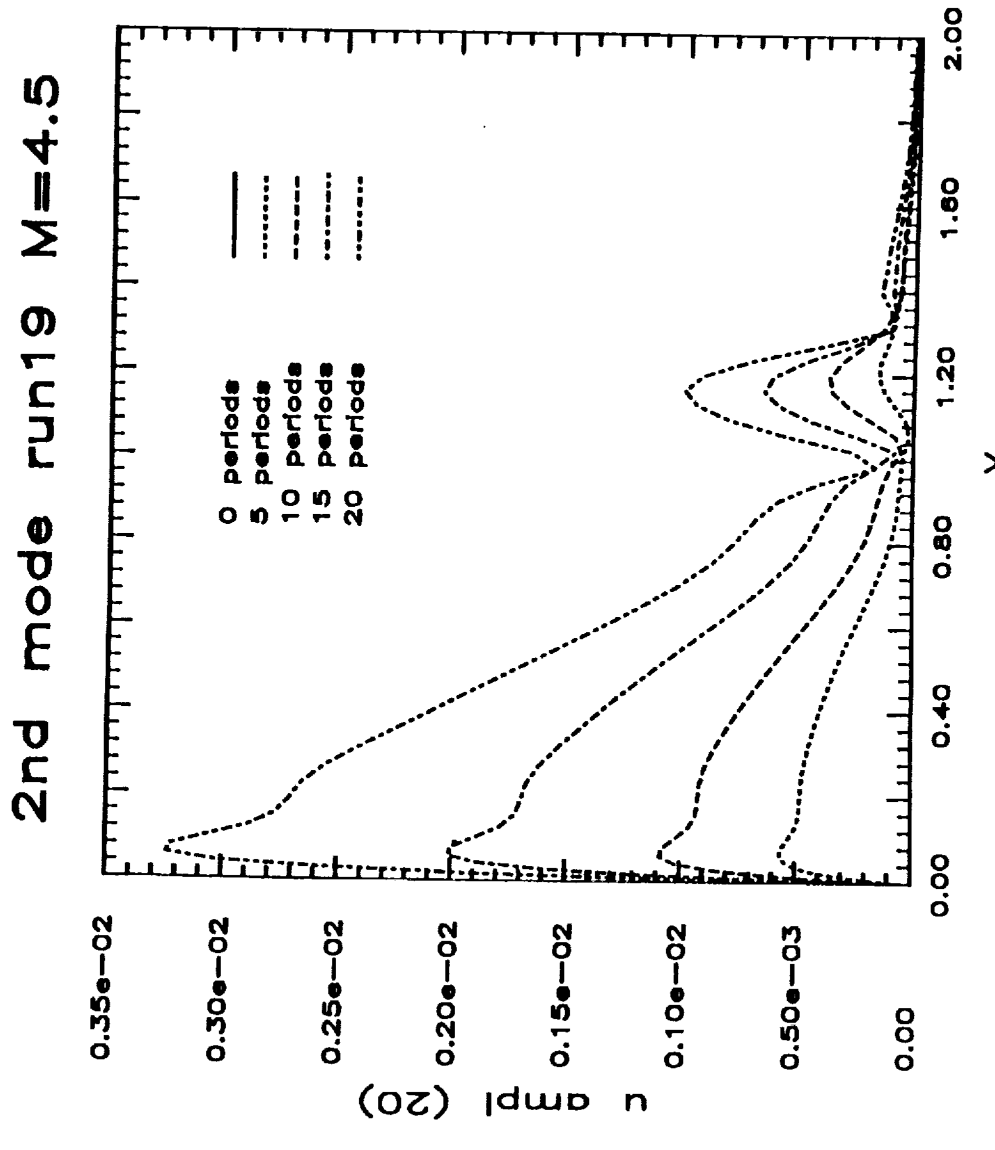


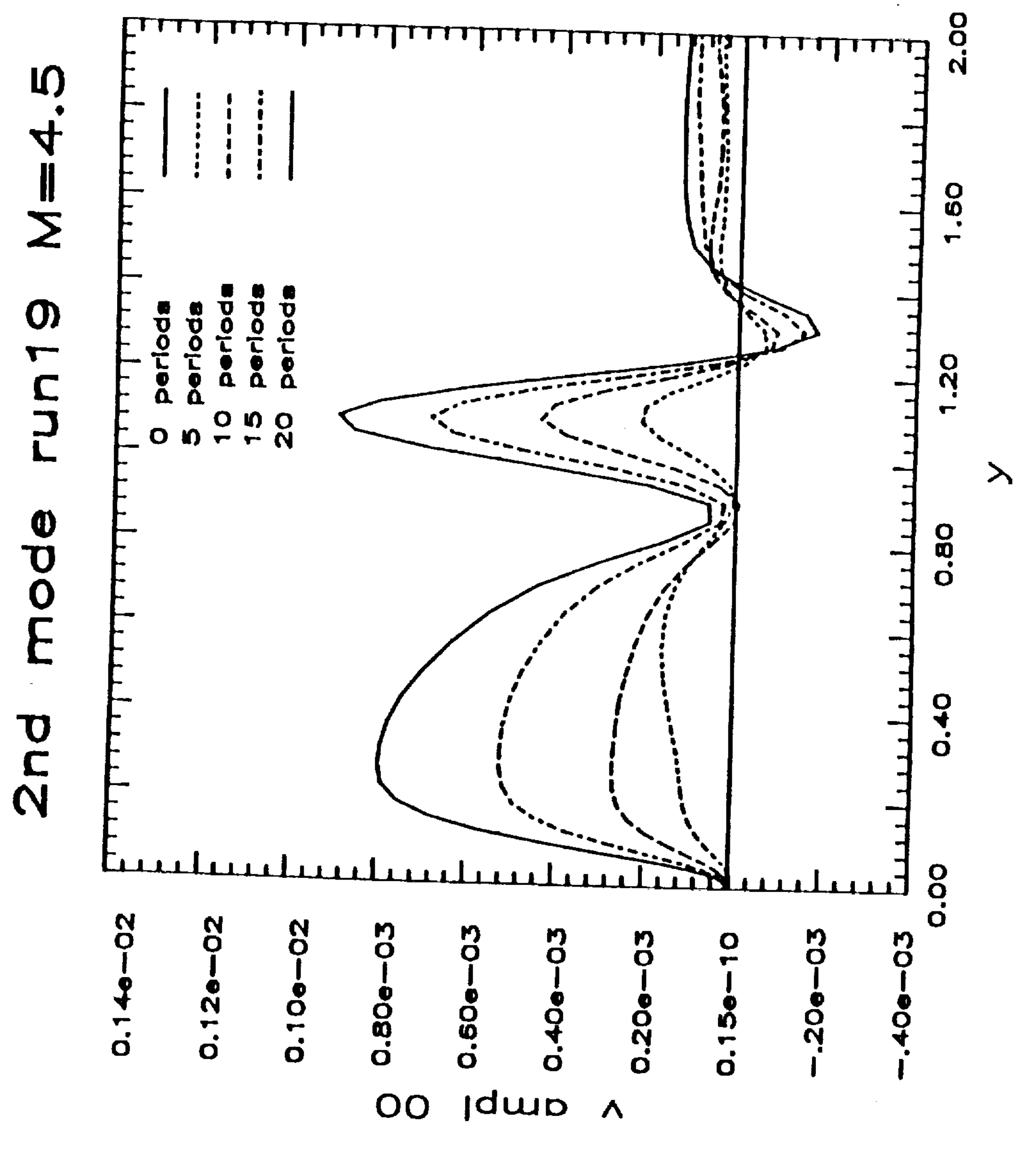
2nd mode run19 M=4.5
inviscid cubic term in u mom.
0.40e-01
0.30e-01
0.20e-01
0.10e-01
0.93e-08
-.10e-01
-.20e-01
-.30e-01
-.40e-01
-.50e-01
-.60e-01
0.00
0.40
0.80
1.20
1.60
2.00
y
0 periods
5 periods
10 periods
15 periods
20 periods

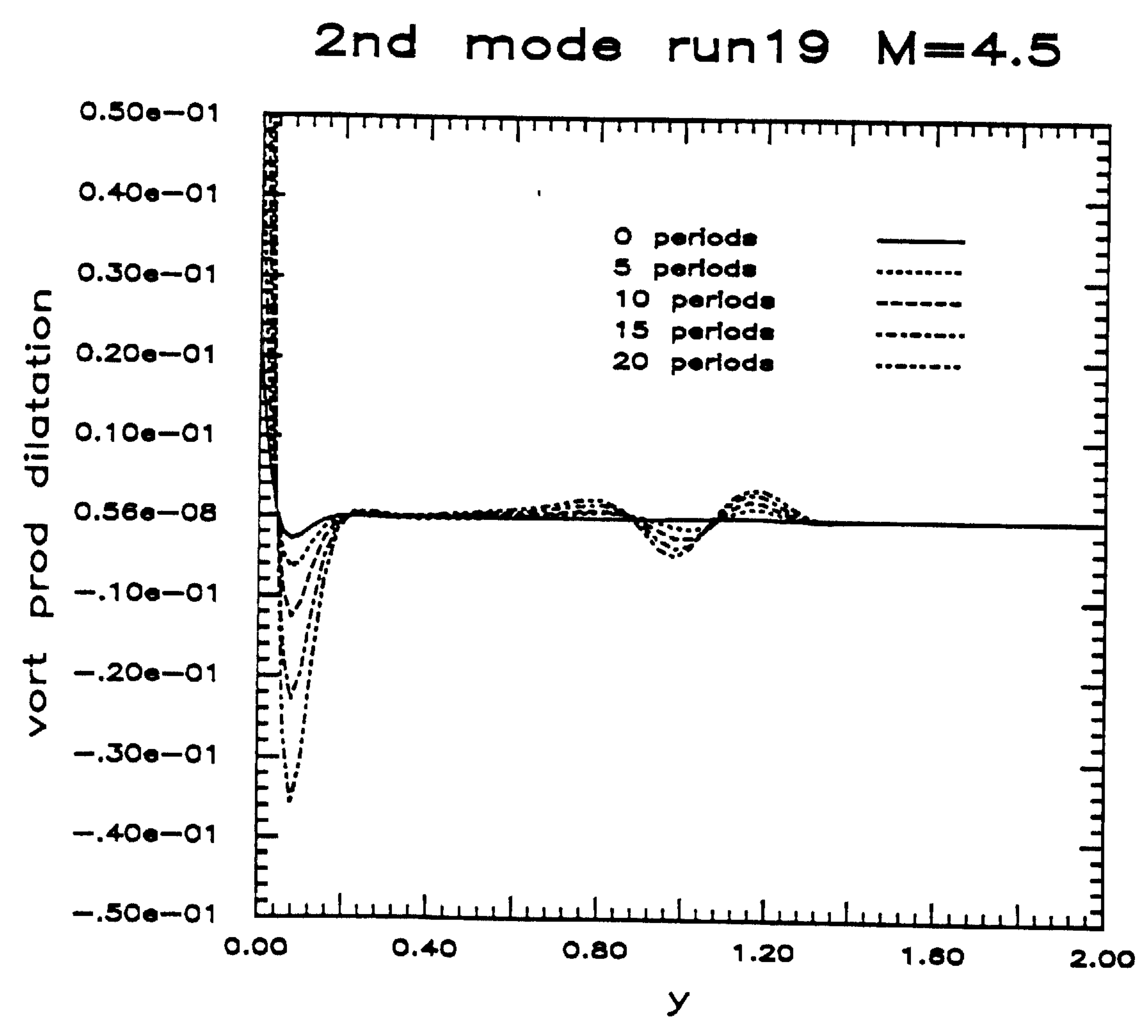


2nd mode run19 M=4.5
0 periods
5 periods
10 periods
15 periods
20 periods
u ampl (10)
0.35e-01
0.30e-01
0.25e-01
0.20e-01
0.15e-01
0.10e-01
0.50e-02
0.00
0.00
0.40
0.80
1.20
1.60
2.00
y

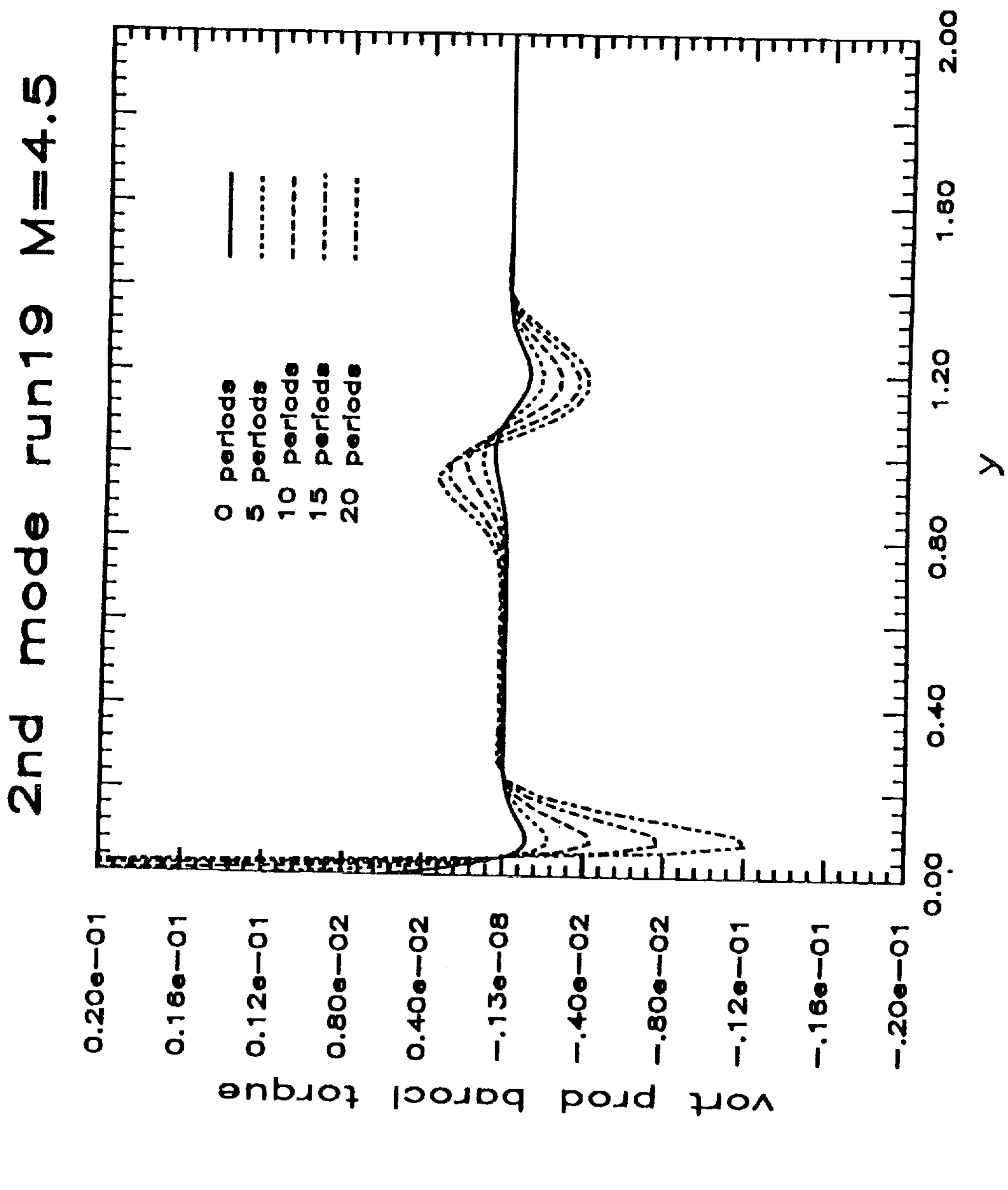

2nd mode run19 M=4.5
0 periods
5 periods
10 periods
15 periods
20 periods
rho ampl (10)
0.00
0.10e-01
0.20e-01
0.30e-01
0.40e-01
0.50e-01
0.60e-01
0.70e-01
0.80e-01
0.90e-01
0.10e+00
y
0.00
0.40
0.80
1.20
1.60
2.00


2nd mode run19 M=4.5
0 periods
5 periods
10 periods
15 periods
20 periods
y
u ampl (20)
0.00
0.40
0.80
1.20
1.60
2.00
0.00
0.50e-03
0.10e-02
0.15e-02
0.20e-02
0.25e-02
0.30e-02
0.35e-02


2nd mode run19 M=4.5
0 periods
5 periods
10 periods
15 periods
20 periods
0.14e-02
0.12e-02
0.10e-02
0.80e-03
0.60e-03
0.40e-03
0.20e-03
0.15e-10
-.20e-03
-.40e-03
v ampl 00
0.00
0.40
0.80
1.20
1.60
2.00
y



2nd mode run19 M=4.5
0.50e−01
0.40e−01
0.30e−01
0.20e−01
0.10e−01
0.56e−08
−.10e−01
−.20e−01
−.30e−01
−.40e−01
−.50e−01
vort prod dilatation
0.00
0.40
0.80
1.20
1.60
2.00
y
0 periods
5 periods
10 periods
15 periods
20 periods


2nd mode run19 M=4.5
0 periods
5 periods
10 periods
15 periods
20 periods
y
vort prod barocl torque
0.20e−01
0.16e−01
0.12e−01
0.80e−02
0.40e−02
−.13e−08
−.40e−02
−.80e−02
−.12e−01
−.16e−01
−.20e−01
0.00
0.40
0.80
1.20
1.60
2.00

COST OF DIRECT SIMULATION

- The number of operations for a single iteration is approximately twice that of an incompressible direct simulation
- Acoustic effects must be treated implicitly for low Mach number simulations. At high Mach numbers, time step restrictions are a function of the Reynolds number
- Steep gradients of u' and T' in the critical layer increase the initial resolution
- Compressibility effects weaken the secondary instability, further increasing the required computer time to capture the initial stages of breakdown
- Cost can decrease with use of static or dynamics adaptive grids, or multidomain decompositions in the vertical direction

$\Longrightarrow$ VERY EXPENSIVE !!!

25 periods on $8 \times 2 \times 65$ grid: 10 CPU hrs on Cray II at 100 Mflops average yields factor 4 amplification

CONCLUSIONS

- Compressible boundary-layer code is suitable for the study of second modes, albeit is still very expensive due to the slow growth and high spatial and temporal frequencies of the instabilities
- Non-linearities (not due to higher streamwise harmonics) induces strong growth rate departures from the linear values in the critical layer region. The actual cause is still unknown.
- Density perturbations probably play (as expected) a fundamental role in the development of the non-linear saturated state. Further information awaits more detailed considerations of intermodal energy transfers.