

NUMERICAL SIMULATION OF NONLINEAR DEVELOPMENT OF INSTABILITY WAVES

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SUMMARY

The present work is concerned with the nonlinear interactions of high amplitude instability waves in turbulent jets. In plane shear layers Riley and Metcalf (1980) and Monkewitz (1987) have shown that these interactions are dependent, among other parameters, on the phase-difference between the two instability waves. Therefore, in the present work we consider the nonlinear development of both the amplitudes and the phase of the instability waves. The development of these waves are also coupled with the development of the mean flow and the background turbulence. In formulating this model it is assumed that each of the flow components can be characterized by conservation equations supplemented by closure models. Results for the interactions between the two instability waves under high-amplitude forcing at fundamental and subharmonic frequencies are presented here. Qualitative agreements are found between the present predictions and available experimental data.

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CONSERVATION EQUATIONS

Each flow component is split in the form:

$$U_i(x, r, t) = \bar{U}_i(x, r) + \tilde{u}_i(x, r, t) + u_i'(x, r, t)$$

\bar{U} is the time-averaged mean flow velocity which is taken to be given by the two-stage hyperbolic tangent profile. \tilde{u} is the periodic component which is split into two frequency-components in the form:

$$\tilde{u}_i = A_1 \hat{u}_{1i}(r, \theta) e^{i\phi_1(x) - i\omega_1 t} + A_2 \hat{u}_{2i}(r, \theta) e^{i\phi_2(x) - i\omega_2 t} + c.c.$$

\hat{u} is the radial shape taken as the eigen-function solution of the locally-parallel linear stability equation for each frequency-component. A and ϕ are the amplitude and phase to be obtained from the nonlinear interaction equations, and θ is the momentum thickness u' is the turbulence component which is related to the turbulence energy T through an assumed Gaussian profile.

THE NONLINEAR INTERACTION EQUATIONS

Time-averaging and phase-averaging techniques are applied to the full unsteady Navier-Stokes equations to derive the governing equations for each

flow component. These equations are manipulated to obtain nonlinear equations for the amplitudes A_1, A_2 , phases ϕ_1, ϕ_2 , momentum thickness θ and the turbulence energy T :

Mean flow

$$\frac{1}{2} \frac{dI_{ma}}{d\theta} \frac{d\theta}{dX} = -I_{mt}T - I_{\omega m}A_1^2 - I_{2\omega}A_2^2$$

Turbulence

$$\frac{d}{dX} [I_{ta}T] = I_{mt}T + I_{\omega t}A_1^2T + I_{2\omega t}A_2^2T - \frac{I}{\epsilon} T^{3/2}$$

$\omega\omega$ component

$$\frac{d}{dX} [I_{\omega a}A_1^2] = I_{\omega m}A_1^2 - I_{\omega t}A_1^2T + I_{2\omega\omega}A_2A_1^2 \cos(2\phi_\omega - \phi_{2\omega} - \phi_0 + \sigma)$$

$$I_{\omega a} \frac{d\phi_\omega}{dX} = \pi S_\omega + I_{\omega m}^* + A_2 I_{2\omega} \sin(2\phi_\omega - \phi_{2\omega} - \phi_0 + \sigma)$$

2ω -component

$$\frac{d}{dX} [I_{2\omega a}A_2^2] = I_{2\omega m}A_2^2 - I_{2\omega t}A_2^2T - I_{2\omega\omega}A_2A_1^2 \cos(2\phi_\omega - \phi_{2\omega} - \phi_0 + \sigma)$$

$$I_{2\omega a} \frac{d\phi_{2\omega}}{dX} = \pi S_{2\omega} + I_{2\omega m}^* - \frac{A_1^2}{A_2} I_{2\omega\omega} \sin(2\phi_\omega - \phi_{2\omega} - \phi_0 + \sigma)$$

The integrals I appearing in the above equations are functions of θ , frequencies, and the closure assumptions. S is the Strouhal number defined as $\omega d/(2\pi U)$. The solution of the above system of equations is subject to the initial conditions at $x = 0$: $\theta_0, T_0, A_{10}, A_{20}$, and ϕ_0 .

RESULTS AND DISCUSSIONS

The calculated fundamental and subharmonic components at Strouhal numbers 0.3 and 0.6 are shown in figure 1 for several initial phase angles. The initial momentum thickness is 0.026 R , initial turbulence energy levels is 0.0001 the initial energies of the fundamental and subharmonic are taken such that the initial instability velocity components at the jet centerline are 1.2 and 0.6 percent, respectively. Figure 1 shows that the fundamental is not sensitive to the phase-difference as much as the subharmonic does. Bradley and Ng's (1989) measured integral spectral amplitude shows similar features. The fundamental is less dependent than the subharmonic on the phase angle. Maximum subharmonic amplification occurs at $\phi_0 = 180^\circ$ and minimum subharmonic's amplification occurs at $\phi_0 = 0^\circ$, same as the present results in figure 1(b).

The calculated centerline phase-averaged velocities are shown in figure 2 in comparison with the data of Arbey and Ffowcs-Williams (1984). The Strouhal

numbers are 0.3 and 0.6, the initial turbulence energy level is 0.00001, initial momentum thickness is 0.012. The initial centerline velocity of the $S = 0.3$ component is taken 1.5 percent and that of the $S = 0.6$ is 0.38 of the $S = 0.3$ component as in the experiment. At $S = 0.3$, figure 2(a) shows that calculate peak occurs further down stream as compared to the measured ones. However, the calculated peak has the same level as the measured one. The peak increases when ϕ_0 is changed from 0 to 180°, as the present computations also predict. The calculated phase averaged velocities at $S = 0.6$ shown in figure 2(b) has the same features as the measured one; same level of amplification and same dependency on ϕ_0 . The measured component increases again after it decays which is probably due to its interaction with other frequency components. This mechanism is not accounted for here.

The dependency of the subharmonic amplification on the initial phase angle is shown in figure 3 for Strouhal numbers 0.2 and 0.4. The initial levels are $\bar{u}_{f0} = 7$ percent, $\bar{u}_{s0} = 0.5$ percent. The peak of the subharmonic at $\phi_0 = 270^\circ$ is three times higher than its peak at $\phi_0 = 90^\circ$. The corresponding momentum thickness is shown in figure 4, compared to the unexcited momentum thickness. The figure shows that the momentum thickness is only weakly dependent on the phase angle. This indicates that the direct role of the subharmonic in turbulent jets in controlling the mixing is less pronounced as compared to its role in controlling the mixing in Laminar jets. However, the subharmonic can still have a strong role in the mixing process through enhancing the background turbulence which in turn increases the mixing.

If both the fundamental and subharmonic's initial levels are high, the dependency on the phase angle is less pronounced as figure 5 indicates. The Strouhal numbers are 0.3 and 0.6 and the initial levels are $\bar{u}_f = \bar{u}_s = 3$ percent. At high initial levels, large energy levels are drained from the mean flow and therefore the fundamental-subharmonic energy exchanges are relatively smaller and consequently less pronounced.

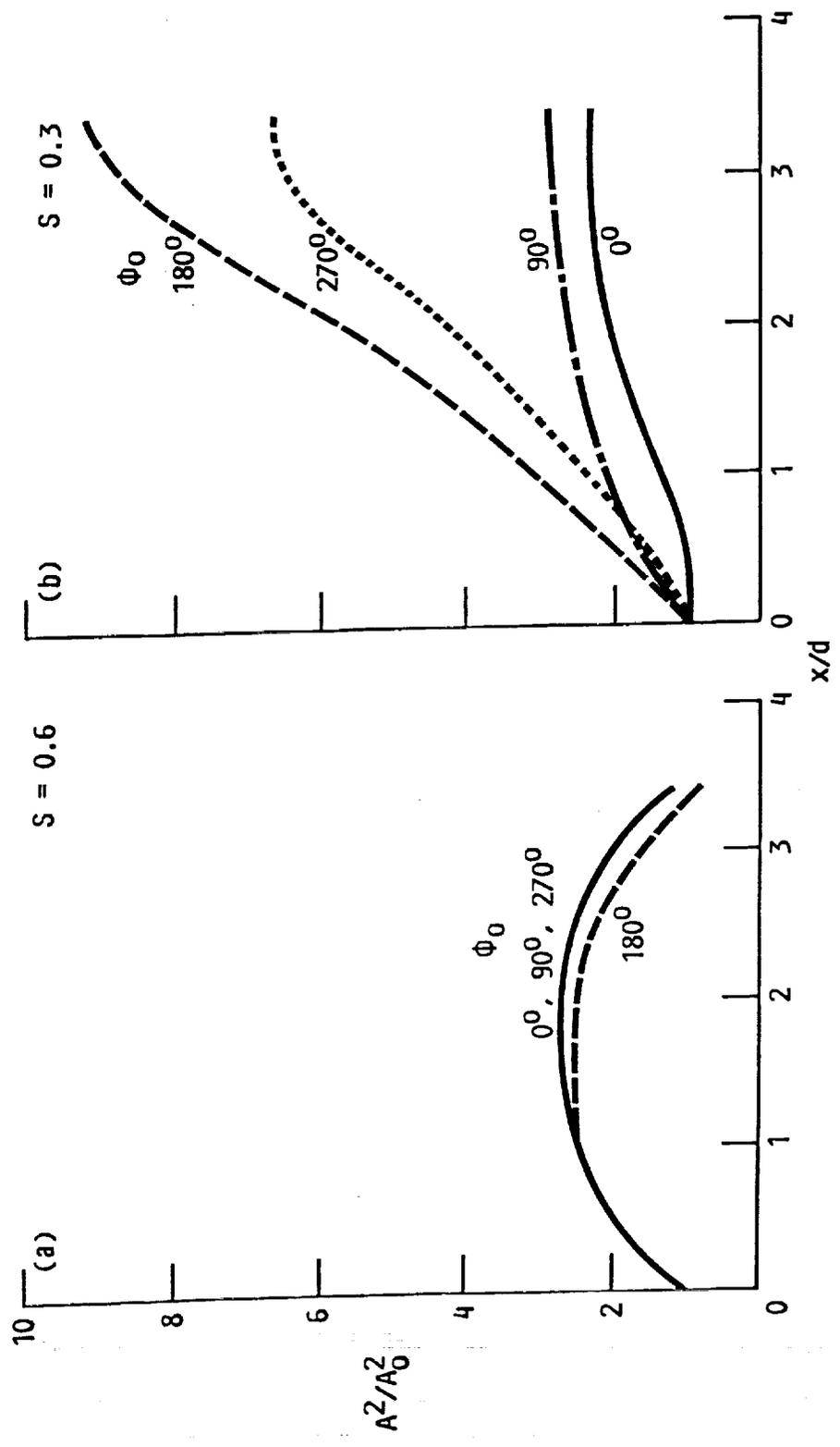
The effect of the forcing level at a fundamental frequency of $S = 0.4$ on the subharmonic's amplification is shown in figure 6. The figure shows that the peak of the subharmonic increases with increasing the forcing level. However, a saturation condition occurs around a forcing level of 10 percent. Higher forcing levels result in no further increase of the subharmonic's peak over 20 percent.

REFERENCES

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Figure 1

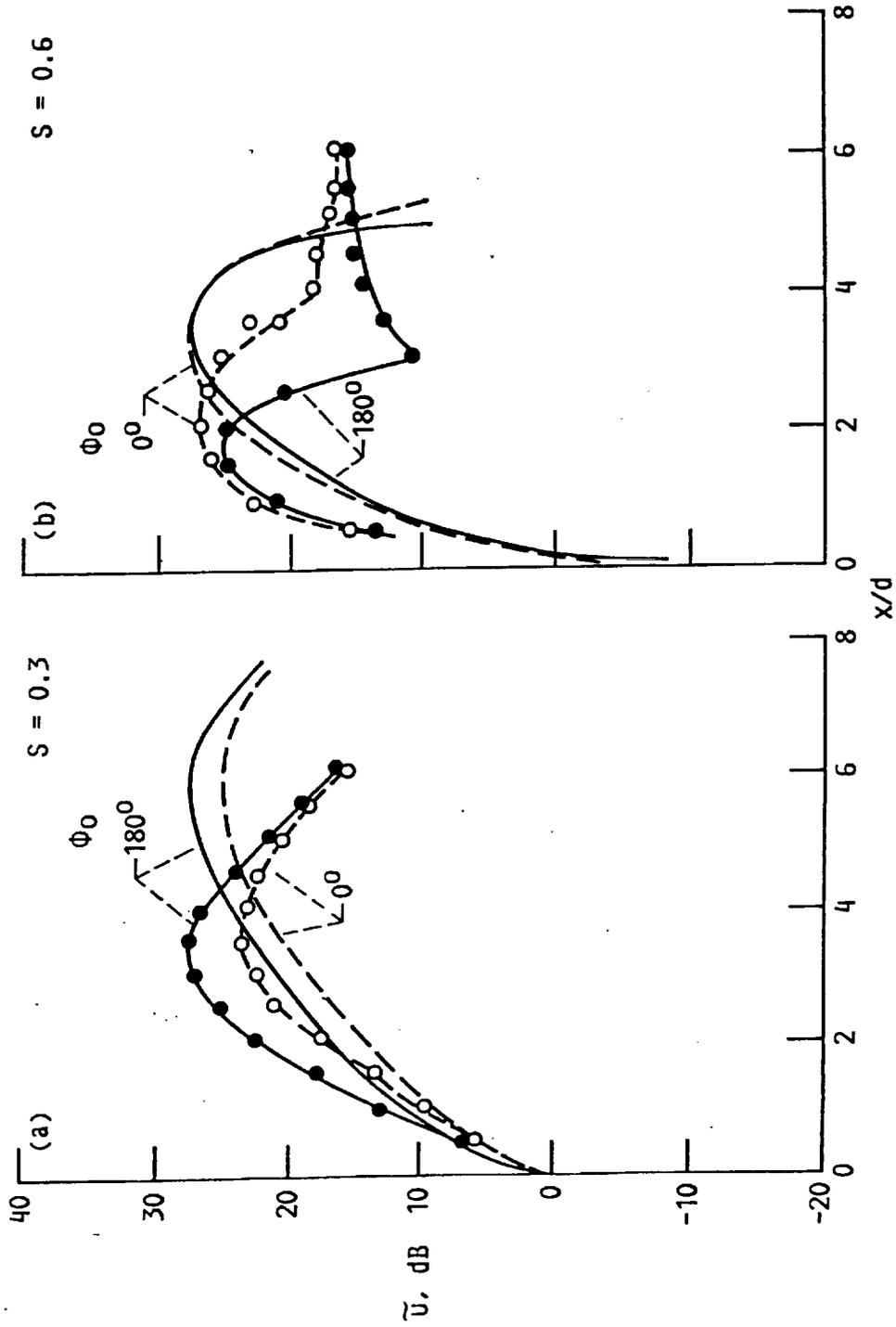
DEPENDENCY OF THE STABILITY COMPONENTS ON THE INITIAL PHASE ANGLES



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Figure 2

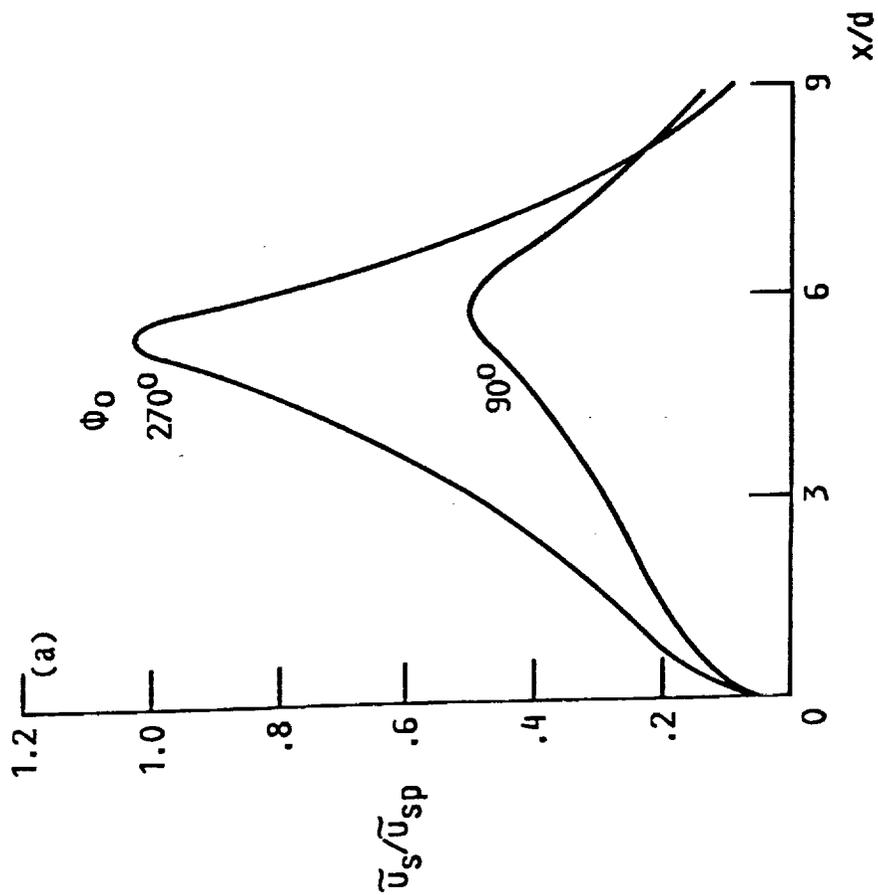
COMPARISON BETWEEN CALCULATED PHASE-AVERAGED CENTERLINE VELOCITIES AND ARBEY & FLOWCS-WILLIAMS' (1984) DATA



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Figure 3

DEPENDENCY OF SUBHARMONIC'S AMPLIFICATION ON THE INITIAL PHASE-ANGLE AT STROUHAL NUMBERS OF 0.4 AND 0.2



DEVELOPMENT OF THE MOMENTUM THICKNESS UNDER TWO-FREQUENCY EXCITATION AND FOR THE UNEXCITED CASF

Figure 4

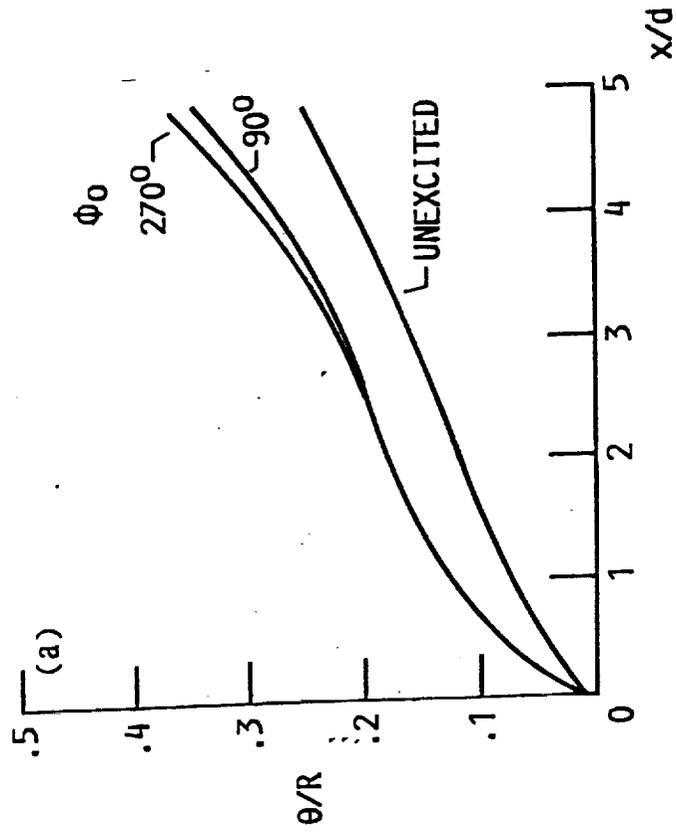


Figure 5

DEPENDENCY OF THE SUBHARMONIC'S PEAK ON THE PHASE-ANGLE AT HIGH LEVELS BOTH THE FUNDAMENTAL AND THE SUBHARMONIC

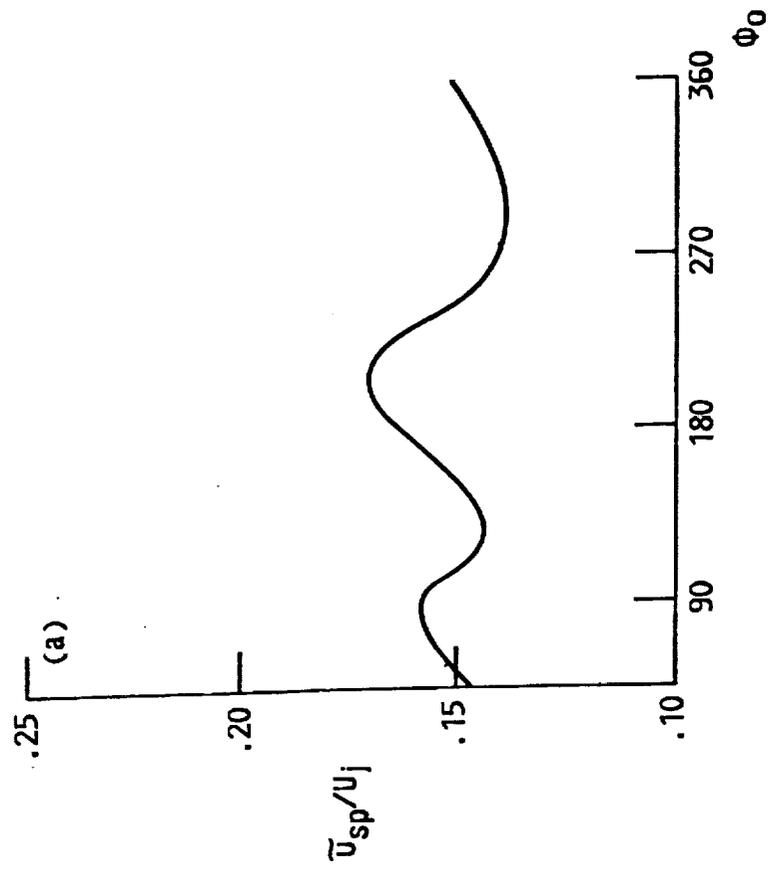


Figure 6

DEPENDENCY OF THE SUBHARMONIC'S PEAK ON THE FORCING LEVEL, AT STROUHAL NUMBERS OF 0.2 AND 0.4

