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The First Geocenter Estimation Results Using GPS Measurements

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The center of mass of the Earth is the natural and unambiguous origin of a geocentric satellite dynamical system. A geocentric reference frame assumes that the origin of its coordinate axes is at the geocenter, in which all relevant observations and results can be referred and in which geodynamic theories or models for the dynamic behavior of Earth can be formulated. In practice, however, a kinematically obtained terrestrial reference frame may assume an origin other than the geocenter. A fast and accurate method of determining origin offset from the geocenter is highly desirable. Global Positioning System (GPS) measurements, because of their abundance and broad distribution, provide a powerful tool to obtain this origin offset in a short period of time. Two effective strategies have been devised. Data from the first Central and South America (Casa Uno) global GPS experiment have been studied to demonstrate the ability of recovering the geocenter location with present-day GPS satellites and receivers.

I. Introduction

Reference frames are established in order to represent positions and motions of objects with respect either to the Earth (terrestrial frames) or to celestial bodies in space (celestial inertial frames). One of the geophysical requirements of a reference frame is that other geophysical measurements can be related to it; for example, the reference frame used for expressing the Earth's gravity field as a spherical harmonic expansion adopts the center of mass of the Earth as origin. This frame must be related to the adopted terrestrial frame as well as to the inertial frame in which the satellite orbits are calculated. The geocenter is at a focus of the orbital ellipse of a geocentric satellite dynamical system. It is therefore directly accessible through dynamical methods. But in practice the origin assumed by a kinematically obtained terrestrial reference frame can be at some location other than at the Earth's center of mass. Any time-dependent offset in a geocentric terrestrial frame origin from the geocenter can be misinterpreted as plate motions. In order to avoid such confusion, it is important to determine as accurately as possible the translational offset of the adopted terrestrial reference frame origin from the geocenter.

Two effective strategies were devised to determine the reference frame origin offset from the geocenter using

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Global Positioning System (GPS) observations. The results of covariance analyses performed to investigate the accuracy with which the geocenter position can be determined with these two strategies have been published in [1]. The analyses indicate that the geocenter position can be determined to an accuracy of a few centimeters with just one day of precise GPS pseudorange and carrier phase data collected by a global GPS tracking network.

Several regional GPS experiments have been carried out since 1985, but the tracking ground networks have not been extensive enough to provide the global coverage needed for accurate geocenter estimation. The Casa Uno experiment, however, used a semiglobal network stretching over the South Pacific and across the continental United States and Europe, thus providing an opportunity to demonstrate the capability of GPS data for geocenter determination. The analyses of the results using this set of data show that the accuracy is strongly limited by a nonglobal GPS constellation, the received data quality, and the geometrical distribution of the semiglobal ground tracking sites.

II. Two Effective Strategies to Determine the Geocenter

The two strategies devised in [1] to determine the origin offset from the geocenter are briefly reviewed here. A fiducial network consists of two or more GPS tracking stations whose positions have been determined in an Earth-fixed coordinate frame to a very high accuracy, usually by Very Long Baseline Interferometry (VLBI) or Satellite Laser Ranging (SLR). Several GPS receivers at other, less accurately known stations also observe the GPS satellites along with the fiducial network. The data can be processed simultaneously to adjust GPS satellite states and the positions of the nonfiducial sites. The fiducial stations established by VLBI provide a self-consistent, Earth-fixed coordinate frame; thus the improved GPS satellite orbits and the nonfiducial stations can be expressed with respect to this coordinate frame to a greater accuracy. The filter process is designed so that the baselines between a reference site and all other nonfiducial sites are adjusted along with GPS orbits and the absolute coordinates of the reference site. The correction to the reference site coordinates infers the adjustment of the geocenter position coordinates. This concept has been used in Strategy 1, where one or two fiducial baselines are fixed or constrained by their a priori uncertainties [1]. The orientation of the adopted coordinate frame is defined by the fixed baselines, and the absolute scaling can be fixed either by the length of these baselines or by the Earth's gravitational constant (GM). Both are known to an accuracy of about one part in 10^8. The absolute scale derived from the fixed baseline length allows the coordinate frame thus established to be consistent with the VLBI frame of the fiducial baselines.

In Strategy 2, only the longitude of a reference site is held fixed; all other site coordinates are adjusted simultaneously along with GPS satellite states. The absolute scale is provided by the Earth's GM. The geocentric radius at a station can be obtained from the adjusted periods of GPS orbits and pseudorange measurements. The time signature of the measurements defines the latitude. The coordinate system thus defined will be an Earth-centered, Earth-fixed (ECEF) coordinate frame.

The covariance study carried out in [1] assumed a full constellation of eighteen GPS satellites distributed in six orbital planes. A data arc spanning over 34 hours from a network of six globally distributed tracking stations was also assumed. For Strategy 1, the a priori uncertainty for the relative positions of the fiducial sites was assumed to be 3 cm. P-code pseudorange and carrier phase data noise were assumed to be 5 cm and 0.5 cm respectively when integrated over 30 minutes and corrected for ionospheric effects by dual-frequency combination. Carrier phase biases were adjusted with a large a priori uncertainty. The abundance and broad distribution of the GPS measurements allow the GPS and station clocks to be treated as white-noise processes and adjusted to remove their effects on the solutions. Also adjusted were the zenith tropospheric delays at all ground sites, which were treated as random-walk parameters to model the temporal variation. Such models have proven to be effective in removing tropospheric errors without heavily depleting the data strength [2]. The same network of six tracking sites was also used to assess Strategy 2. The estimated quantities were the GPS satellite states, the coordinates of all six sites except for the fixed longitude of the reference site, white-noise clocks, random-walk troposphere parameters, and carrier phase biases.

Data arcs of various lengths were used in the covariance analysis. In Strategy 1, at the end of 34 hours the formal error in origin offset from geocenter is 4.0 cm (rms of all three components). The dominating error is due to the assumed error in the fiducial baselines. Any improvement in the baseline estimates will therefore directly benefit the geocenter determination. As the arc length of the data is increased, the error due to data noise is reduced. On the other hand, the systematic error due to the fiducial baseline errors persists. The situation in Strategy 2, however, is different. Here, under the assumptions of the study, data noise is the primary error source, which can be reduced by increasing the data arc length. In reality there will be other errors, such as multipath, troposphere mismodeling, and higher order ionospheric effects. The origin offset error from the geocenter—as predicted by the covariance analysis using this strategy—was 2.1 cm at the end of the 34 hours [1].

III. Methodology for Processing Casa Uno Data

The first Central and South America (Casa Uno) GPS experiment was carried out from January 18 to February 5, 1988. This experiment was the first civilian effort at implementing an extended international (15 nations) GPS satellite tracking network [3]. Twelve globally distributed sites were selected (Fig. 1) to provide improved global coverage for the geocenter study.

The collected data are from seven GPS satellites in the constellation, distributed in two orbital planes with a separation of approximately 120 deg in the right ascension of the ascending nodes. Both strategies are applied to determine the geocenter location. The analyses present the achievable accuracy based upon the available suboptimal tracking conditions during the Casa Uno experiment. For example, the baselines known to higher accuracies are concentrated only within the continental United States and are relatively short compared to the extensive area covered by the tracking network; the seven satellites distributed in only two orbital planes do not provide a good global coverage; and the pseudorange data are not of high quality due to the antennas and receivers used in this experiment.

For Strategy 1 the Owens Valley Radio Observatory (OVRO)-Haystack baseline is held fixed. The other ground station locations, the geocenter location, and the GPS satellite states are adjusted with respect to the baseline reference point OVRO. Thus the coordinate system has the scale and orientation as defined by this fixed baseline, and the adjustment to the geocenter location gives the offset of the coordinate frame origin from the geocenter. Table 1 gives the a priori uncertainties adopted for Strategy 1. In Strategy 2, the longitude of the reference site OVRO is held fixed, and all the other site coordinates along with the GPS satellite states are adjusted simultaneously. Table 2 lists the a priori uncertainties adopted in Strategy 2, which differ from those in Strategy 1 (Table 1).

The coordinate systems defined by Strategy 1 and Strategy 2 have fundamental differences. In Strategy 1 the geocenter offset is determined directly while fixing one baseline; in other words, the estimated correction to the geocenter location represents the coordinate frame offset implied by the two ends of the fixed baseline. In Strategy 2, the coordinate offset is not directly determined. Instead, the corrections to all the tracking site coordinates, except for the reference site longitude, are estimated. The geocenter offset is inferred from these estimates through a constrained seven parameter coordinate transformation. The seven parameters are solved by treating filter estimates of corrections to tracking site coordinates as the measurements, whereas the measurement covariance matrix is represented by the corresponding filter covariance plus the a priori covariance of tracking sites. The arbitrarily fixed reference longitude introduces a small rotation R_z about the Z axis; this rotation is taken into account by solving for R_z while the scale factor and the rotations about the X and Y axes are kept fixed. The geocenter offset is represented by three translational parameters which are assigned a priori uncertainty of 10 m. The Appendix gives a full account of the method of seven parameter coordinate transformation.

Since the same tracking network was used for both strategies, they have similar a priori conditions. For example, in both strategies, the a priori geocentric reference frame, in which the nominal station coordinates are expressed, is derived from Goddard global VLBI coordinates for those same sites and is rotated and translated using the results from SLR data for geocentricity.

IV. Results

Although the Casa Uno ground network was designed to collect a maximum amount of data with the available seven GPS satellites, it provides only a semiglobal coverage. In order to compensate for this suboptimal circumstance, the data strength was increased by using a 5-day data arc, and the geometrical strength of the network was improved by constraining the well-known sites with appropriate a priori uncertainties. The a priori uncertainties assumed for Strategy 1 in this experiment are listed in Table 1. In the covariance analysis presented in [1] the GPS Block I ROCK 4 model for solar radiation pressure was used, where the three parameters G_x, G_y, and G_z were considered with 10-percent error. The constant acceleration in the y axis G_y is called the y-bias parameter [4]. In the Casa Uno multiday analysis a new approach was adopted in which two constant solar pressure parameters G_y and G_{xz} were estimated along with two tightly constrained process noise parameters G_x and G_z [2]. The parameter G_{xz} is a combined effect of $(G_x + G_z)$.

Figure 2 shows the formal rms error in the geocenter solution using Strategy 1. The error associated with the geocenter offset estimation due to data noise is 31 cm and that due to baseline uncertainty is 7.1 cm, resulting in a total formal rms error in the geocenter offset solution of 31.8 cm. The effect due to data noise dominates the error, showing the poor quality of data; however, the error due to the fixed baseline is also significant. These formal errors are significantly higher than the 4 cm predicted by the earlier covariance study because the ground tracking sites were not uniformly distributed, the fixed baseline was not long enough to provide a global control, and only one-third of the GPS constellation was in place during the experiment. The origin offsets estimated with the Casa Uno GPS data, with respect to the SLR derived geocenter, are −142, −33, and −43 cm along the X, Y, and Z axes (Fig. 3).

Table 2 lists the variation in the a priori uncertainties from Table 1 as applied to Strategy 2. The longitude at OVRO was fixed and all the other stations had large a priori sigmas. The same tracking station network was used in both cases. The estimated parameters include all the ground station coordinates and the GPS satellite states. The formal rms error in the geocenter using this strategy is found to be 21.5 cm, which is due only to the data noise. The estimated origin offsets from the nominal geocenter are −96, −97, and −51 cm along the X, Y, and Z axes (Fig. 3).

The origin solutions from the two strategies differ from each other along X, Y, and Z axes by 46, 65, and 8 cm respectively. The rms of these differences is 45.7 cm, which is the mean origin offset in each component between results from Strategy 1 and Strategy 2. The geocenter estimates from Strategy 1 and Strategy 2 agree at the 2-sigma level. The apparent disagreement in X and Y components between the GPS and SLR solution is puzzling. It may be that the baselines between European and U.S. sites are known less accurately than the 10 cm assumed here. A recent independent (but inconclusive) experiment gave a baseline adjustment of up to 50 cm between the station at Wettzell and the U.S. sites.

The anticipated improvement in data quality, better GPS constellation, and even distribution of the tracking stations should in the future improve the accuracy of geocenter estimation to the few centimeters predicted by the earlier covariance studies.

V. Summary and Conclusion

The Casa Uno 5-day GPS data arc, despite the uneven data quality, limited global coverage, and partial satellite constellation, can recover the geocenter position to an estimated accuracy of better than 35 cm using either of two strategies. The two coordinate frames differ from one another by three translational parameters and one rotational parameter about the Z axis. The transformation parameters between the two coordinate frames are very sensitive to the a priori values and constraints applied to the participating sites. The discrepancy between the results from Strategy 1 and Strategy 2 falls within their 2-sigma error.

With future superior receiver data quality, an evenly distributed global network including longer fiducial baselines, and an increased number of satellites distributed in more orbital planes, the results are expected to improve to an accuracy of a few centimeters as indicated by previous covariance studies. Inclusion of the Deep Space Network (DSN) sites will provide the well determined global baselines that are lacking in the Casa Uno network. Longer baseline ties to the well-known sites in North America from Europe and the southern hemisphere will provide a much stronger network geometry for geocenter determination.

References

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Table 1. The a priori assumptions for geocenter study using Casa Uno data (one baseline fixed)

Reference site:	OVRO
Other fiducial sites:	Haystack
Nonfiducial sites:	Blackbirch, Canberra, Kokee, Samoa, Cocos, Albrook, Mojave, Ft. Davis, Wettzell, and Onsala
GPS constellation:	GPS 11, GPS 3, GPS 4, GPS 6, GPS 8, GPS 10, and GPS 9 distributed in 2 orbital planes
Cutoff elevation:	15 deg
Data span:	up to 5 days
Data interval:	6 min
Data noise:	175 cm pseudorange; 1 cm carrier phase
GPS epoch state:	20 km and 20 m/sec (adjusted)
Geocenter position: (each comp.)	10 m (adjusted)
Baseline coordinates: (relative to OVRO, each comp.)	Haystack 4 cm (fixed); Ft. Davis and Mojave—4 cm (adjusted); Wettzell and Onsala—10 cm (adjusted); Others—1 km each comp. (adjusted)
Solar pressure:	G_y and G_{xz} (adjusted)

Table 2. Variation of assumptions from Table 1 for Strategy 2 (one longitude fixed)

Reference site:	OVRO
Reference site coordinates:	10 m (latitude) 0 m (longitude) 10 m (height)
Other site coordinates:	10 m each component

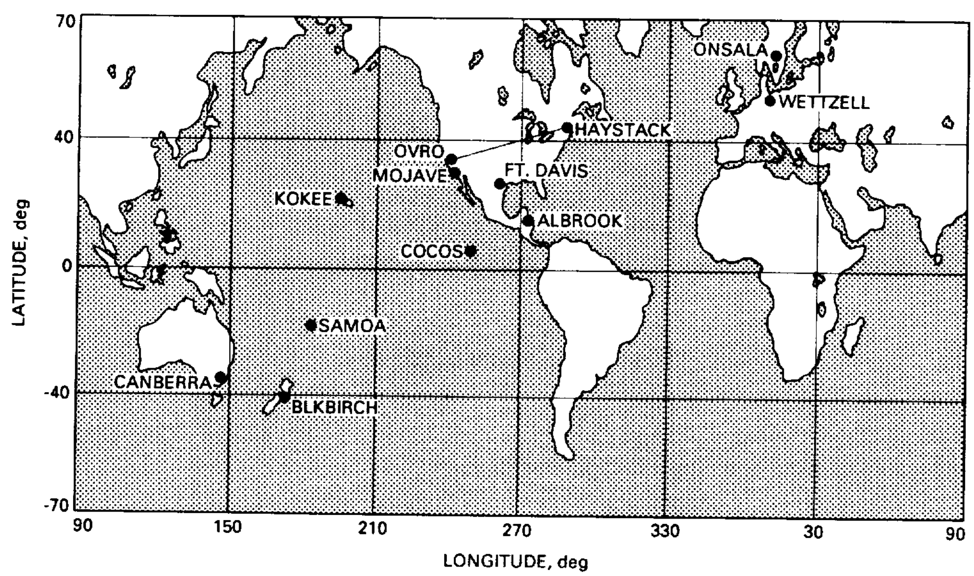

Fig. 1. The Casa Uno tracking sites for geocenter studies.

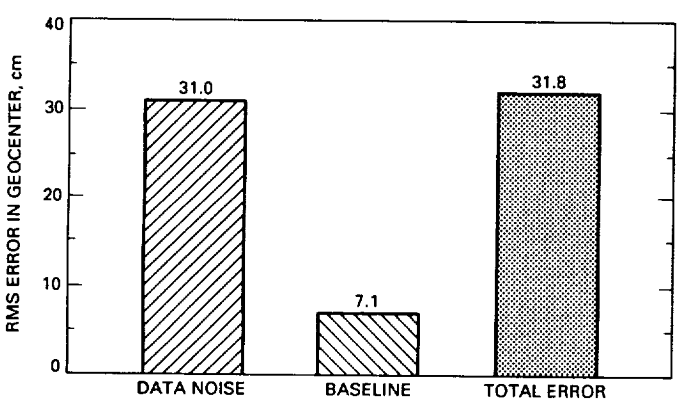

Fig. 2. Geocenter error (1σ) from covariance study for Casa Uno 5-day arc data using Strategy 1.

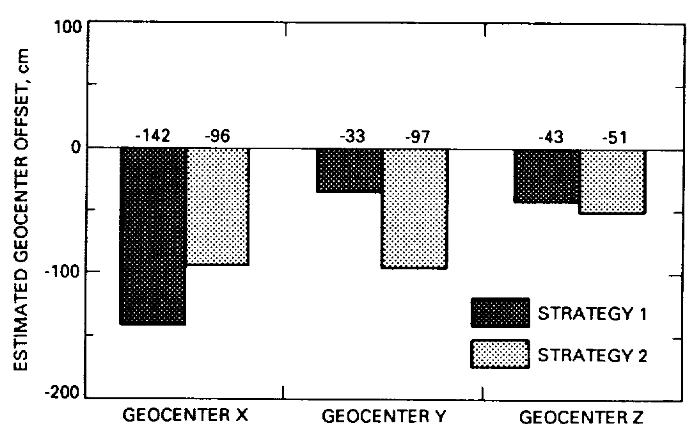

Fig. 3. Geocenter estimates from Casa Uno data using Strategies 1 and 2.

Appendix

Seven Parameter Coordinate Transformation

The coordinates of a point in a given Cartesian reference frame can be expressed with respect to any other Cartesian reference frame as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{Ref1}} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1 + \Delta L)\ \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{Ref2}} \tag{A-1}$$

where ΔX, ΔY, and ΔZ are the three translational parameters, and ΔL is the scale difference between reference frames Ref1 and Ref2. The rotation matrix $\mathbf{R}$ represents the rotation required about the three axes in order to align the two reference frames

$$\mathbf{R} = \mathbf{R}(R_y)\mathbf{R}(R_x)\mathbf{R}(R_z)$$

where $\mathbf{R}(R_i)$ is the matrix representing a right-handed rotation about the ith axis through an angle R_i. The rotation matrix $\mathbf{R}$ is obtained by multiplying the three matrices in sequence, i.e.,

$$\mathbf{R} = \begin{bmatrix} \cos R_z \cos R_y - \sin R_z \sin R_x \sin R_y & \sin R_z \cos R_y + \cos R_z \sin R_x \sin R_y & -\cos R_x \sin R_y \\ -\sin R_z \cos R_x & \cos R_z \cos R_x & \sin R_x \\ \cos R_z \sin R_y - \sin R_z \sin R_x \cos R_y & \sin R_z \sin R_y - \cos R_z \sin R_x \cos R_y & \cos R_x \cos R_y \end{bmatrix} \tag{A-2}$$

For small angle rotations and neglecting products of small angles:

$$\mathbf{R} = \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \tag{A-3}$$

Substituting Eq. (A-3) into (A-1) and rearranging the terms gives:

$$\begin{aligned} \Delta X - R_y Z_{\mathrm{Ref2}} + R_z Y_{\mathrm{Ref2}} + (X_{\mathrm{Ref2}} + R_z Y_{\mathrm{Ref2}} - R_y Z_{\mathrm{Ref2}})\ \Delta L + (X_{\mathrm{Ref2}} - X_{\mathrm{Ref1}}) &= 0 \\ \Delta Y + R_x Z_{\mathrm{Ref2}} - R_z X_{\mathrm{Ref2}} + (Y_{\mathrm{Ref2}} - R_z X_{\mathrm{Ref2}} - R_x Z_{\mathrm{Ref2}})\ \Delta L + (Y_{\mathrm{Ref2}} - Y_{\mathrm{Ref1}}) &= 0 \\ \Delta Z - R_x Y_{\mathrm{Ref2}} + R_y X_{\mathrm{Ref2}} + (Z_{\mathrm{Ref2}} + R_y X_{\mathrm{Ref2}} - R_x Y_{\mathrm{Ref2}})\ \Delta L + (Z_{\mathrm{Ref2}} - Z_{\mathrm{Ref1}}) &= 0 \end{aligned} \tag{A-4}$$

This set of equations represents the relationship between two closely oriented, closely scaled orthonormal Cartesian reference frames. Every point appearing in both frames will generate three observation equations, where the seven transformation parameters (ΔX, ΔY, ΔZ, R_x, R_y, R_z, ΔL) relating the two frames are to be uniquely estimated. A unique solution requires two stations with their coordinates known in both reference frames and only one component of a third station known in both frames. In practice there will be more than three stations participating and the weighted least squares solution gives the solution for the parameters.

Let L_b represent the observations with corresponding variance-covariance matrix Σ_L, so that the weight matrix $\mathbf{P} = (\Sigma_L)^{-1}$. Each observed quantity is expressed as a function of the parameters in the model

$$L_a = F(X_a) \tag{A-5}$$

Here L_a represents the theoretical values of the observed quantities and X_a the theoretical values of the parameters. The Taylor series linearization gives the observation equation as

$$V = Ax + (L_o - L_b) = Ax + L$$

where V is the residuals; $A = \partial F/\partial X_a|_{X_a=X_o}$ is the matrix of the partials evaluated with respect to the a priori value X_o of the parameters; $L = L_o - L_b$, where $L_o = F(X_o)$; and $x = X_a - X_o$. The least squares estimate of x is

$$\hat{x} = -(A^T PA)^{-1} A^T PL \tag{A-6}$$

with the corresponding covariance matrix

$$\Sigma_x = (A^T PA)^{-1} \tag{A-7}$$

If a priori knowledge of the covariance matrix corresponding to the parameters exists, then a constrained least squares solution of the parameters is given by

$$\hat{x} = -(P_{ox} + A^T PA)^{-1} A^T PL \tag{A-8}$$

and

$$\Sigma_x = (P_{ox} + A^T PA)^{-1} \tag{A-9}$$

where the parameters are constrained by their a priori weight P_{ox}.

The partial matrix **A** of Eq. (A-3) with respect to the seven transformation parameters, when evaluated with an a priori estimate of the parameters as zero, would become

$$A_{(3n,7)} = \begin{bmatrix} 1\ 0\ 0 & 0 & -Z_{1Ref2} & Y_{1Ref2} & X_{1Ref2} \\ 0\ 1\ 0 & Z_{1Ref2} & 0 & -X_{1Ref2} & Y_{1Ref2} \\ 0\ 0\ 1 & -Y_{1Ref2} & X_{1Ref2} & 0 & Z_{1Ref2} \\ \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots \\ \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots \\ \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots & \cdots\ \cdots \\ 1\ 0\ 0 & 0 & -Z_{nRef2} & Y_{nRef2} & X_{nRef2} \\ 0\ 1\ 0 & Z_{nRef2} & 0 & -X_{nRef2} & Y_{nRef2} \\ 0\ 0\ 1 & -Y_{nRef2} & X_{nRef2} & 0 & Z_{nRef2} \end{bmatrix} \tag{A-10}$$

Matrix **A** will have dimension ($3n \times 7$) for n participating stations. The weight matrix **P** of the observation is a full ($3n \times 3n$) matrix. The vector $\vec{\mathbf{L}}$ is of the form

$$\begin{aligned} L_{(3n,1)} = {} & [(X_{1Ref2} - X_{1Ref1}), (Y_{1Ref2} - Y_{1Ref1}), (Z_{1Ref2} - Z_{1Ref1}), \\ & \cdots \qquad \cdots \qquad \cdots \\ & (X_{nRef2} - X_{nRef1}), (Y_{nRef2} - Y_{nRef1}), (Z_{nRef2} - Z_{nRef1})]^T \end{aligned} \tag{A-11}$$

The least squares solution is obtained for the parameter vector

$$x_{(7,1)} = [\Delta X, \Delta Y, \Delta Z, R_x, R_y, R_z, \Delta L]^T \tag{A-12}$$