NASA Technical Memorandum 103277

Probabilistic Structural Analysis of a Truss Typical for Space Station

Shantaram S. Pai
Lewis Research Center
Cleveland, Ohio

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PROBABILISTIC STRUCTURAL ANALYSIS OF A TRUSS TYPICAL FOR SPACE STATION

Shantaram S. Pai
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

A three-bay, space, cantilever truss is probabilistically evaluated using the computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) to identify and quantify the uncertainties and respective sensitivities associated with corresponding uncertainties in the primitive variables (structural, material, and loads parameters) that defines the truss. The distribution of each of these primitive variables is described in terms of one of several available distributions, such as the Weibull, exponential, normal, log-normal, etc. The CDF's for the response functions considered and sensitivities associated with the primitive variables for given response are investigated. These sensitivities help in determining the dominating primitive variables for that response.

INTRODUCTION

It is a common practice to evaluate the structural integrity of trusses with the aid of deterministic analysis techniques and appropriate safety factors. Traditionally, these factors are an outcome of many years of analytical as well as experimental experience in the area of structural mechanics design. The load factors are used to take into account many different operating conditions and also to ensure that the maximum operating load has been considered. The safety factors are used to account for unknown effects in analysis and fabrication.

As an alternative to the deterministic approach, probabilistic structural analysis methods (PSAM) have been developed at NASA Lewis Research Center (ref. 1) which consider various uncertainties in a more structured manner. PSAM takes into account the scatter in the resistance (strength) and parameters such as geometric configuration, loadings, structural properties, etc., herein referred to as primitive variables. Furthermore, the primitive variables are not considered as either single values or upper and lower bound values. Instead, the actual probability distribution of the primitive variables is represented. Thus, the probabilistic methodologies have taken a prominent role in designing complex structural components which have experienced failures using a deterministic analysis approach (ref. 2).

The NESSUS Computer Code developed at NASA Lewis (refs. 3 and 4) contains PSAM techniques and provides a choice of algorithms for the solution of static, dynamic, buckling, and nonlinear analysis problems. It also includes a number of innovative analyses to evaluate the sensitivity of response variables to small variations in one or more user-defined primitive variables. Recently, NESSUS has been used for the analysis of Space Shuttle Main Engine (SSME) components, for example, to obtain a probabilistic assessment of a mistuned bladed disk assembly (ref. 5) and to evaluate the reliability and risk of a turbine blade under complex service environments (ref. 6).

The objective of this paper is to demonstrate the use of the NESSUS Computer Code to probabilistically evaluate a three-dimensional, three-bay, cantilever truss

typical for space station structures with the aim to identify and quantify the sensitivities associated with uncertainties in primitive variables. For a space truss the primitive variables such as spatial truss geometry, stiffness parameters, strength parameters, and applied loads or moments continuously vary due to changes in the environment. The distribution of each of these primitive variables is described in terms of any one of several available distribution functions, such as the Weibull, exponential, normal, log-normal, etc. These distributions have significant impact on the scatter of the response variables such as nodal displacements, member forces, vibration frequencies, etc. The specific PSAM technique used in the NESSUS computer code evaluates the scatter of the response variables from the scatter in the primitive variables.

PROGRAM CAPABILITY AND DESCRIPTION

The NESSUS code consists of three major modules namely; NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI. It is important to note that each of these modules can be used independently. NESSUS/FEM and NESSUS/FPI are combined into a single computer code called NESSUS/PFEM which performs the entire probabilistic finite element analysis including perturbations of the primitive variables. In general, the primitive variables are specified with their mean values (μ), standard deviation (σ), and the type of distribution.

The NESSUS/PRE module is a pre-processor used to obtain the description of a partially correlated Gaussian field in terms of a set of uncorrelated random vectors. The NESSUS/FEM module is a finite element analysis code with the capability to generate perturbed solutions about a deterministic state. It contains an efficient perturbation technique such that the perturbation of each variable is done rapidly. Each perturbation corresponds to a prescribed deviation from the deterministic model. The NESSUS/FPI module contains several advanced reliability methods including Monte-Carlo simulation. The fast probability integration (FPI) (ref. 6) techniques are one or several orders of magnitude more efficient than the Monte Carlo simulation methods. The module extracts the database of perturbed solutions from NESSUS/FEM to calculate the probability distribution functions of the response variables.

PROBABILISTIC FINITE ELEMENT ANALYSIS

Dias et al. (ref. 3) discuss the developments in probabilistic finite element analysis which are based upon perturbation methods for computing the sensitivity of the response of the random variables present. In general, the finite element equation for motion is written as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = F(t) \tag{1}$$

where [M], [C] and [K] denote the mass, damping, and stiffness matrices, respectively. It is important to note that these matrices are calculated probabilistically in the NESSUS code. Furthermore, $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are the acceleration, velocity and displacement vectors at each node, respectively. The forcing function vector, $\{F(t)\}$, is time dependent at each node.

In this paper, the static case is considered by setting the mass and damping matrices to zero and considering the forcing function being independent of time in equation (1) such that

$$[K]\{u\} = \{F\} \tag{2}$$

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Furthermore, the eigenvalue analysis is also carried by setting only the damping matrix to zero and using the following equation:

$$[K] - w^2[M] \{u\} = 0 \tag{3}$$

where w denote eigenvalues and $\{u\}$ are corresponding eigenvectors.

FINITE ELEMENT MODEL

A three-dimensional, three-bay cantilever truss is simulated using a linear isoparametric beam element based on the Timoshenko beam equations. The element is idealized as a two-noded line segment in three-dimensional space. The cantilever truss is assumed to be made from 44 hollow circular pipe type beam elements (see fig. 1). The pipes are made up of wrought Aluminum alloy (616-w) with modulus of elasticity (E) equal to 10 mpsi. The outer and inner radii (r_0 and r_i) of the tube, are 0.5 and 0.4375 in., respectively. All 6 degrees of freedom are restrained at the fixed end (left side) nodes. The truss is analyzed twice, once using beam elements and then using pseudo-truss elements. The beam element is converted into a pseudo-truss element by suppressing the effective shear areas in the principal planes (A_{xx} and A_{yy}), the two principal moments of inertias for the tube cross-section, (I_{xx} and I_{yy}) and torsional constant, J. In the case of truss elements, 3 rotational degrees of freedom at each node and 3 translational degrees of freedom at support nodes are restrained.

Each bay of the truss is 5 ft wide, 8 ft long, and 6 ft high (fig. 1). The overall length of the truss is 24 ft. Six vertical and two longitudinal loads are applied. Twisting moments are applied at the truss-end top nodes with beam elements. However, an equivalent couple is applied at the truss-end nodes for truss elements. The directions of the forces and moments are shown in figure 1 and the mean values are given in table I.

PROBABILISTIC MODEL

The following primitive variables are considered in perturbation analysis:

(1) Nodal Coordinates (X,Y,Z)
(2) Modulus of elasticity (E)
(3) Outer radius of the tube (r_0)
(4) Inner radius of the tube (r_i)
(5) Vertical loads (V)
(6) Longitudinal loads (H)
(7) Truss-end moments (M)
(8) Truss-end coupling forces (P)

It is possible that the above design variables will vary continuously and simultaneously due to extreme changes in the environment when such trusses are used in upper earth orbit for space station type structures. The normal distribution is used to represent the scatter in E, r_0, r_i, and X,Y,Z coordinates. The applied loads, moments and coupling forces are selected to represent anticipated loading conditions for a typical space truss. These are represented by log-normal distributions. Initially, the deterministic finite element analysis takes into consideration the mean value of these primitive variables. In the probabilistic analysis each variable is perturbed equidistant from the mean value. However, each variable is perturbed independently and by a different amount. Usually, the perturbed value

of the design variable is obtained by certain factor of the standard deviation at either side of the mean value. Finally, the NESSUS/FPI module extracts response variable values (one deterministic and two times the number of primitive variables) to calculate a probability distribution function of the response variable considered. The mean, distribution type and percentage variation for different primitive variables are given in table I.

DISCUSSION OF RESULTS

The three-bay cantilever truss is probabilistically analyzed and the cumulative probability distributions for the truss end displacements, member forces and vibration frequencies are plotted. The sensitivities of the primitive variables on the scatter in the truss structural responses (truss free end displacements, member axial forces and vibration frequencies) are quantified in table II. Figures 2 to 4 depict the probabilistic displacement of the truss free end nodes (top and bottom) in X, Y, and Z directions, respectively, using the truss element. The large differences in CDF's between top and bottom nodes as seen in figures 3 and 4 are, respectively, due to longitudinal loads at top nodes and vertical loads at bottom nodes. However, the results obtained using beam elements show a lower magnitude of the probabilistic displacement in Z direction as seen from figure 5. This is because of the fact that the beam elements increase the over all stiffness of the truss. It is noted from the table II that the perturbations in Y coordinates have a major impact on the scatter in the displacements. In addition, the perturbations in Z coordinates have an impact on truss end displacements in the Z direction when beam elements are used. The CDF of the axial forces in the Top Front Longeron (TFL), Bottom Batten (BB), Front Vertical (FV), and Rear Diagonal (RD) of the first bay from the support are shown in figures 6 to 9. The changes in axial forces, from compression to tension, in Bottom Batten (fig. 7) and Front Vertical (fig. 8) are due to reversal in loading or due to loading from either side of the mean value. The CDF's of frequencies of modes 1 and 2 using truss elements are plotted in figures 10 and 11, whereas, figures 12 and 13 are those for the truss with beam elements. It is important to note from table II that the cross sectional area (primitive variables r_o and r_i) has a significant impact on the probabilistic distribution of the vibration frequencies. Furthermore, the truss with beam elements has, higher frequencies due to inclusion of bending stiffness. Finally, the deterministic value of the response variable may be estimated from the 50 percent cumulative probability level.

SUMMARY

The application of the PSAM code to model and probabilistically evaluate a cantilever truss typical for space station is demonstrated using beam and truss elements. The probabilistic structural responses are predicted and plotted. The sensitivities associated with uncertainties in the primitive variables are quantified. The results indicate substantial scatter in frequency and some member axial force distributions. However, there is relatively less scatter in displacement distributions.

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TABLE I. - PRIMITIVE VARIABLES AND UNCERTAINTIES FOR

PROBABILISTIC STRUCTURAL ANALYSIS OF A SPACE TRUSS

[Random Input Data.]

Primitive variables		Distribution type	Mean value	Scatter, ±percentage
Geometry	Width	Normal	60 in.	6.0
	Length	Normal	96 in. 192 in. 288 in.	6.3 7.5 6.3
	Height	Normal	72 in.	7.5
Loads	Vertical	Log-normal	200 lb	6.3
	Longitudinal	Log-normal	200 lb	2.5
	Couple	Log-normal	0.7 lb	6.3
	End moment[a]	Log-normal	50 lb/in.	6.3
Material property	Modulus	Normal	10 Mpsi	7.5
Tube radii	Outer radius	Normal	0.5 in.	7.5
	Inner radius	Normal	0.44 in.	7.5

[a]For beam elements only.

TABLE II. - SENSITIVITIES OF PRIMITIVE VARIABLES UNCERTAINTIES OF TRUSS STRUCTURAL RESPONSE

Response type	Sensitivity factors									
	Geometry			Loads				Material property	Tube radii	
	Width	Length	Height	Vertical	Longitudinal	End moment	Couple	Modulus	Outer	Inner
Displacement:										
X-direction	0.35	0.66	(b)	0.22	(b)	(b)	(b)	(b)	(b)	(b)
Y-direction	.18	.69	(b)	.36	0.16	↓	↓	↓	↓	↓
Z-direction	.12	.73	(b)	.28	(b)	↓	↓	↓	↓	↓
Z-direction[a]	(b)	.76	0.34	.34	(b)	↓	↓	↓	↓	↓
Axial force:										
Top front longeron	0.24	0.44	(b)	0.35	(b)	(b)	(b)	(b)	(b)	(b)
Bottom batten	.39	.28	0.14	(b)	↓	↓	↓	(b)	0.56	0.44
Front vertical	(b)	(b)	(b)	(b)	↓	↓	↓	0.12	.76	.63
Rear diagonal	.55	.39	(b)	.30	↓	↓	↓	(b)	(b)	(b)
Frequency:										
Mode-1	(b)	0.47	(b)	(b)	(b)	(b)	(b)	(b)	0.25	0.19
Mode-1[a]	0.23	.30	0.11	↓	↓	↓	↓	↓	.30	.23
Mode-2	.17	.39	(b)	↓	↓	↓	↓	↓	.32	.24
Mode-2[a]	(b)	.27	.18	↓	↓	↓	↓	↓	.33	.25

[a]For beam element only.
[b]Sensitivity factors less than 10-percent.

FIGURE 1. - SOLAR ARRAY PANELS MAST - TYPICAL TRUSS.

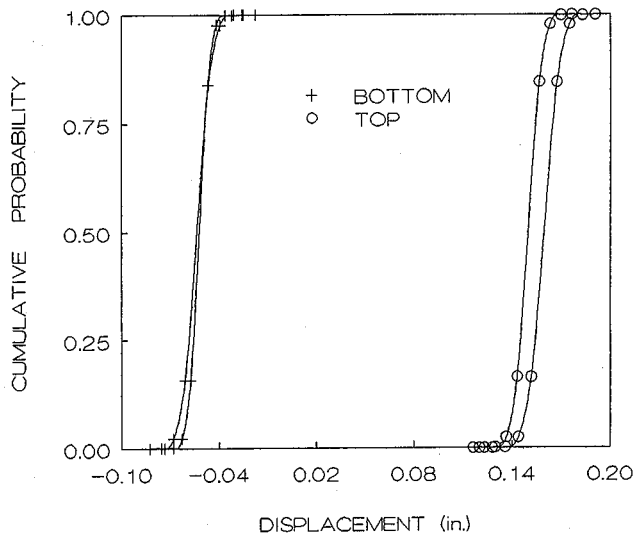FIGURE 2. - PROBABILISTIC DISPLACEMENT. TRUSS FREE END: X - DIRECTION (WIDTH).

FIGURE 3. – PROBABILISTIC DISPLACEMENT.
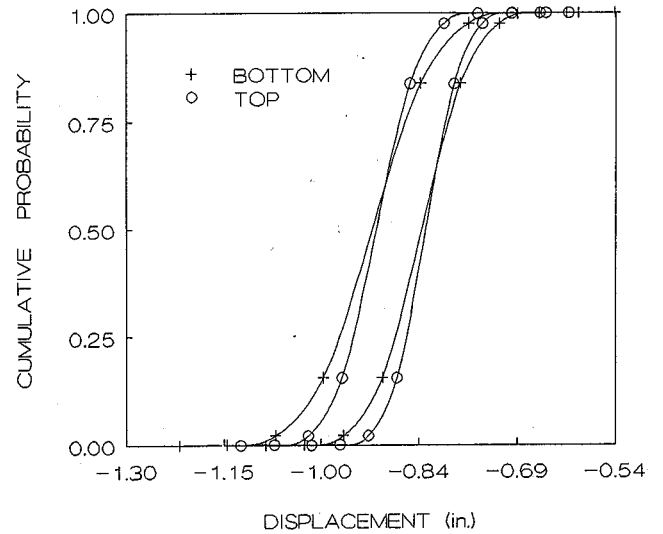TRUSS FREE END: Y – DIRECTION (LENGTH).

FIGURE 4. – PROBABILISTIC DISPLACEMENT.
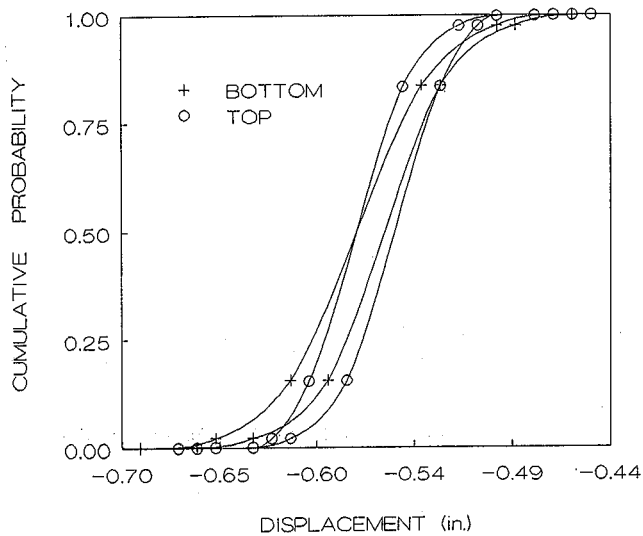TRUSS FREE END: Z – DIRECTION (HEIGHT).

FIGURE 5. – PROBABILISTIC DISPLACEMENT.
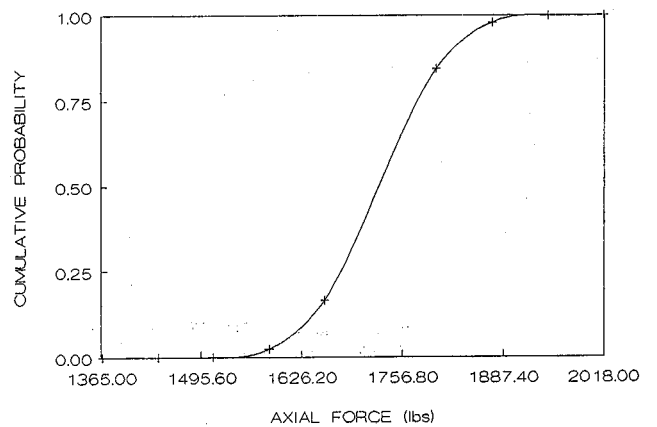TRUSS END: Z – DIRECTION (HEIGHT).

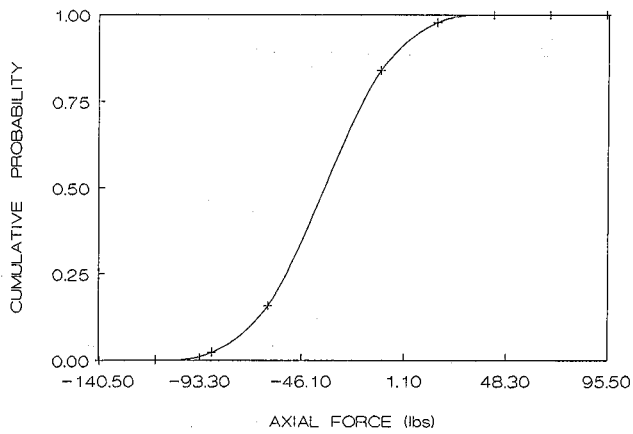FIGURE 6. – PROBABILISTIC MEMBER FORCE.
FIRST BAY TOP FRONT LONGERON.

FIGURE 7. - PROBABILISTIC MEMBER FORCE.
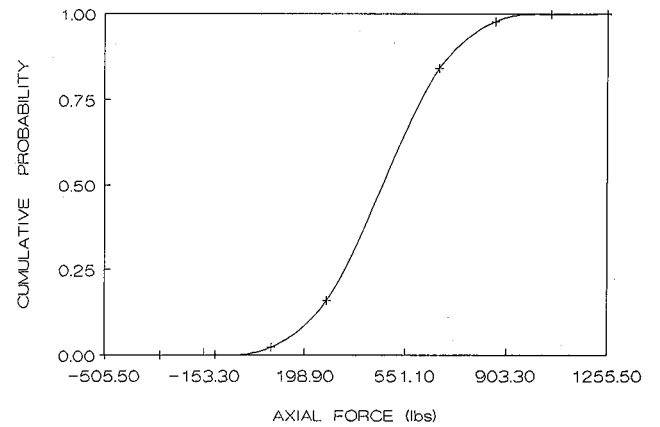FIRST BAY RIGHT BOTTOM BATTEN.

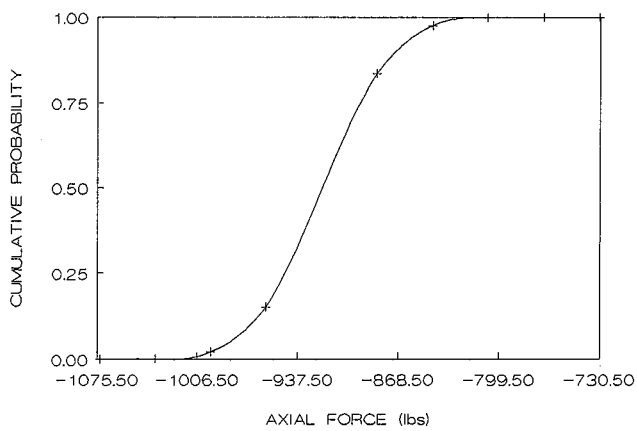FIGURE 8. - PROBABILISTIC MEMBER FORCE.
FIRST BAY RIGHT FRONT VERTICAL.

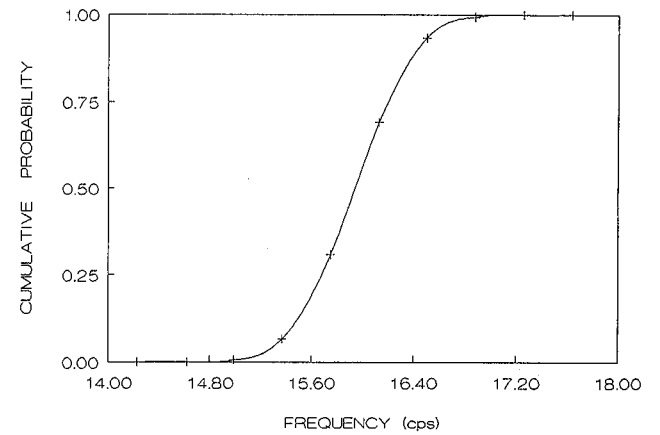FIGURE 9. - PROBABILISTIC MEMBER FORCE.
FIRST BAY REAR DIAGONAL.

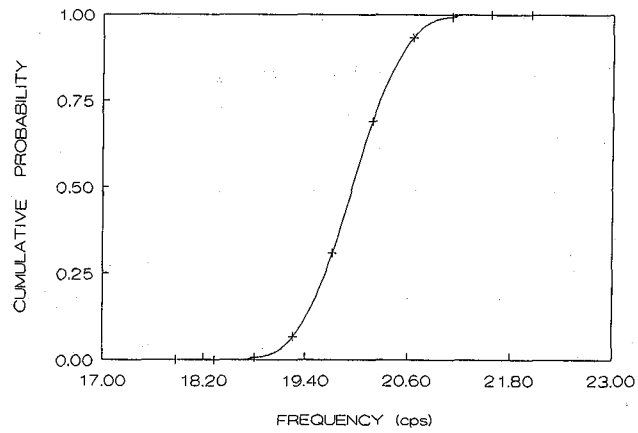FIGURE 10. - PROBABILISTIC FREQUENCY MODE 1.
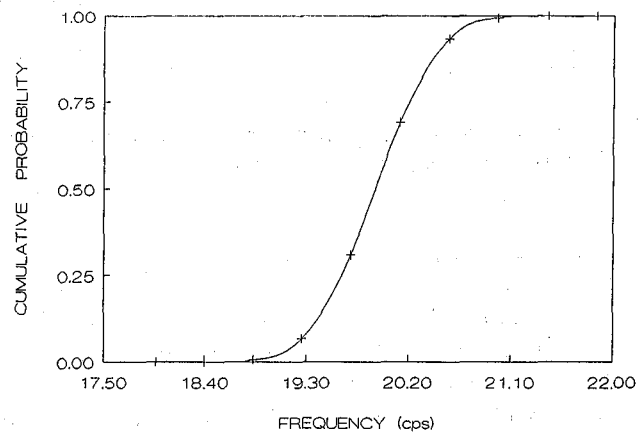
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FIGURE 11. - PROBABILISTIC FREQUENCY MODE 2.

FIGURE 12. - PROBABILISTIC FREQUENCY MODE 1.

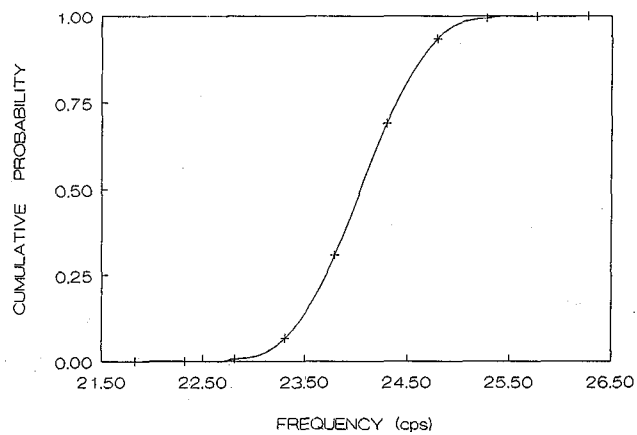FIGURE 13. - PROBABILISTIC FREQUENCY MODE 2.

Report Documentation Page

1. Report No. NASA TM-103277	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Probabilistic Structural Analysis of a Truss Typical for Space Station		5. Report Date
		6. Performing Organization Code
7. Author(s) Shantaram S. Pai		8. Performing Organization Report No. E-5725
		10. Work Unit No. 553-13-00
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the Third Air Force/NASA Symposium On Recent Advances in Multidisciplinary Analysis and Optimization, San Francisco, California, September 24-26, 1990.

16. Abstract

A three-bay, space, cantilever truss is probabilistically evaluated using the computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) to identify and quantify the uncertainties and respective sensitivities associated with corresponding uncertainties in the primitive variables (structural, material, and loads parameters) that defines the truss. The distribution of each of these primitive variables is described in terms of one of several available distributions such as the Weibull, exponential, normal, log-normal, etc. The CDF's for the response functions considered and sensitivities associated with the primitive variables for given response are investigated. These sensitivities help in determining the dominating primitive variables for that response.

17. Key Words (Suggested by Author(s)) Probabilistic; Random; Sensitivity; Damping; Mass; Frequency		18. Distribution Statement Unclassified – Unlimited Subject Category 39	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 10	22. Price* A02