

# Mutual Coupling between Circular Apertures on an Infinite Conducting Ground Plane and Radiating into a Finite Width Slab

by

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The problem of electromagnetic coupling between two horns is of interest for the Microwave Reflectometer Ionization Sensor (MRIS) that will be used in the Aeroassist Flight Experiment (AFE). Laboratory measurements of mutual coupling between conical horns (using a flat metallic reflector to simulate a critically dense plasma outside) have shown a strong dependence on the finite dimensions of the shuttle tile over the apertures. Since both, the dielectric tile and the plasma outside the tile reflect microwaves, a study should be done to isolate the two mechanisms so that the MRIS reentry flight data can be interpreted correctly. Once the coupling due to the tile itself is determined then the location of the critical electron number density layers can be determined.

As a first attempt to tackle this problem the *Geometrical Theory of Diffraction* was used to "modify" the existing solution [1] to mutual coupling between apertures with infinite dielectric sheets. Figure 1 depicts the main rays that contribute to coupling between the two horns.

The mutual admittance for two apertures in a infinite ground plane and radiating into a finite width dielectric slab can be written as [2]:

$$Y_{12} = Y_{12}^o + \sum_{n=1}^N Y_{12}^n \quad (1)$$

where  $Y_{12}^o$  = mutual inductance for apertures in ground plane,  $N$  = the number of reflected rays, and

$$Y_{12}^n = \frac{1}{V_1 V_2} \int_0^{2\pi} \int_0^a [E_\rho^1(\rho, \phi) H_\phi^{r,d}(\rho, \phi) - E_\phi^1(\rho, \phi) H_\rho^{r,d}(\rho, \phi)] \rho d\rho d\phi \quad (2)$$

By using the equivalent current method, aperture theory to determine the radiated fields inside the dielectric tile, and ray tracing the following contributions to mutual coupling were determined:

## Coupling due to Reflection

$$Y_{12}^n = \left[ \frac{j(2/\pi) R_1 k_d a \cos^2 \theta_0}{J_1^2(x_{11}) (x_{11}^2 - 1) Z_d s_{01}} \frac{J_1'(k_d a \sin \theta_0)}{[1 - (\frac{k_d a \sin \theta_0}{x_{11}})^2]} \sqrt{\frac{s_{01}}{s_{02}(s_{02} + s_{01})}} \sqrt{\frac{s_{02}}{s_{03}(s_{03} + s_{02})}} e^{-jk_d(s_{01} + s_{02} + s_{03})} \int_0^{2\pi} \int_0^a [J_1'(x_{11} \rho/a) \cos^2 \phi (x_{11}/a) e^{-jk_d \rho \cos \phi \sin \theta_0} - \frac{J_1(x_{11} \rho/a)}{\rho} \sin^2 \phi e^{-jk_d \rho \cos \phi \sin \theta_0}] \rho d\rho d\phi \right] \quad (3)$$

where  $R_1$  is the reflection coefficient and  $a$  the aperture radius.

### Coupling due to Diffraction from Bottom Wedges

$$Y_{12}^n = -(4j/Z_d)(k_d)^{1/2} D_h \frac{1}{k_d s_o} \left( \frac{s_o'}{s_o} \right)^{1/2} \left( \frac{J_1^2(k_d a) \sin^2(\phi_0')}{(x_{11}^2 - 1) [k_d(s_o' + s_o)]^{1/2}} \right) e^{-jk_d(s_o' + s_o)} \quad (4)$$

where  $D_h$  is the diffraction coefficient.

### Coupling due to Diffraction from Top Wedges

$$\begin{aligned} Y_{12}^n = & B_1 \frac{2 J_1(x_{11}) J_1(k_d a \sin \theta_o)}{(x_{11}/a) (k_d \sin \theta_o)} \\ & + B_3 \frac{(k_d \sin \theta_o a) J_2(x_{11}) J_1(k_d \sin \theta_o a) - x_{11} J_1(x_{11}) J_2(k_d \sin \theta_o a)}{(x_{11}/a)^2 - (k_d \sin \theta_o)^2} \\ & + B_2 \frac{(k_d \sin \theta_o a) J_0(x_{11}) J_{-1}(k_d \sin \theta_o a) - x_{11} J_{-1}(x_{11}) J_0(k_d \sin \theta_o a)}{(x_{11}/a)^2 - (k_d \sin \theta_o)^2} \\ & + .5 B_2 (\cos 2\phi_o - \sin 2\phi_o) \frac{(k_d \sin \theta_o a) J_2(x_{11}) J_1(k_d \sin \theta_o a) - x_{11} J_1(x_{11}) J_2(k_d \sin \theta_o a)}{(x_{11}/a)^2 - (k_d \sin \theta_o)^2} \\ & + .5 B_2 (\cos 2\phi_o + \sin 2\phi_o) \cdot \int_0^a J_0(x_{11} \rho/a) J_2(k_d \sin \theta_o \rho) \rho d\rho \end{aligned} \quad (5)$$

where  $B_1, B_2, B_3$  are constants containing information about the incident fields and their phase.

The contribution  $Y_{11}^n$  to the self-admittance of one aperture due to diffraction is obtained by setting  $\phi_o = \pi/2$  in the above equations.

Results from two cases with different tile thicknesses have indicated that the main contribution to mutual coupling is due to diffraction from the bottom and top (back and front) wedges.

### References

1. M.C. Bailey : "Near Field Coupling Between Elements of a Finite Planar Array of Circular Apertures", Ph.D. Dissertation, VPI&SU, December 1972.
2. M.C. Bailey : "Mutual Coupling Between Circular Waveguide-Fed Apertures in a Rectangular Ground Plane", IEEE Trans. on Antennas and Propagation, July 1974, pp. 597-599.

- Reflected
- - - Diffraction from bottom edges
- ..... Diffraction from upper side edges
- - - Diffraction from upper (front & back) edges

