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LARGE-EDDY SIMULATION OF TRANSITIONAL CHANNEL FLOW

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ABSTRACT

A large-eddy simulation (LES) of transition in plane channel flow has been carried out. The LES results have been compared with those of a fine direct numerical simulation (DNS), and with those of a coarse DNS that uses the same mesh as the LES, but no residual stress model. While at the early stages of transition LES and coarse DNS give the same results, the presence of the residual stress model was found to be necessary to predict accurately mean velocity and Reynolds stress profiles during the late stages of transition (after the second spike stage). The evolution of single Fourier modes is also predicted more accurately by the LES than by the DNS. As small scales are generated, the dissipative character of the residual stress starts to reproduce correctly the energy cascade; as transition progresses, then, and the flow approaches its fully developed turbulent state, the subgrid scales tend towards equilibrium and the model becomes more accurate.

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1. Introduction

In large-eddy simulations (LES) of the Navier-Stokes equations the large, energy-carrying scales of motion are accurately captured, while the effect of the small scales is modeled. The models used to parameterize the effect of the subgrid scales (known as residual stress models or subgrid-scale stress models) are simpler and more universal than the turbulence models used for the Reynolds-averaged Navier-Stokes equations, given that the subgrid scales tend to be more isotropic and homogeneous than the large ones, and less affected by the boundary conditions. Since only the large structures are accurately resolved by the grid, LES requires less computational effort than the direct numerical simulation (DNS) of the Navier-Stokes equations, in which all scales of motion are accurately resolved, at the expense of the empiricism introduced by the modeling assumptions. Thus, one can perform large-eddy simulations of a given flow at a fraction of the expense required by the DNS, or, conversely, one can study higher Reynolds number flows by LES than one can by DNS.

Large-eddy simulations have been successfully applied to a variety of turbulent, wall-bounded flows such as plane channel flow [1, 2, 3], boundary layers [4] and channel flow with transpiration [5], but only recently efforts have been made to study transition to turbulence using LES [6, 7, 8, 9]. While earlier work [6, 7] was characterized by the application of well-established residual stress models to the simulation of laminar-turbulent transition, Piomelli and co-workers [8] were the first to use the databases generated by the direct simulation of transition in a plane channel to study the behavior of the residual stress tensor during transition. They observed that during the nonlinear stages, and in particular during the second-spike stage, the subgrid scale dissipation (*i.e.*, the energy transfer from large to small scales) is significantly smaller than in turbulent flow. They devised a modification of the Smagorinsky [10] model which allowed accurate prediction of the early stages of transition in a flat-plate boundary layer. Subsequently, in [9], they again calculated a transitional boundary layer flow using a residual stress model based on the Renormalization Group (RNG) Theory [11] which predicts zero eddy-viscosity as long as the magnitude of the strain rate tensor is less than some threshold value.

The latter results [8, 9], however, indicate that, up to the early stages of the laminar breakdown, the effect of the small scales on the resolved ones is negligible; a coarse direct simulation, then, appears to be sufficient to give results in satisfactory agreement with those of finely resolved direct simulations [12], as far as engineering statistics are concerned (Reynolds stresses and mean velocity profiles, for example). For this reason the present article will focus on the late stages of transition, and apply the modified Smagorinsky model [8] to the study of transition in a plane channel. The present calculation will extend through the entire transition regime and into turbulent flow. Large-eddy simulation results will be compared with those of a finely resolved direct simulation and with those of a coarse direct simulation to distinguish modeling from resolution errors, and ascertain whether or not the presence of a residual stress model is required for accuracy.

In Section 2 the governing equations will be shown. In Section 3, the results of the large-eddy simulation of transitional channel flow will be discussed. Conclusions and recommendations for future work will be made in the last section.

2. Numerical formulation

In large-eddy simulations we decompose the flow variables into a large scale (or resolved) component, denoted by an overbar, and a subgrid scale component. The large scale component is defined by the filtering operation:

$$\overline{f}(x_1, x_2, x_3) = \int_D \prod_{i=1}^{3} G_i(x_i, x_i') f(x_1', x_2', x_3') dx_1' dx_2' dx_3' \tag{1}$$

where the integral is extended over the entire domain D and G_i is the filter function in the ith direction [in the present work x_1 or x is the streamwise direction, x_2 or y is the direction normal to the walls, which are located at $y = \pm 1$, and x_3 or z is the spanwise direction; u, v and w (or u_1, u_2 and u_3) are the velocity components in the coordinate directions].

The filtered Navier-Stokes and continuity equations, which describe the evolution of the large, energy-carrying eddies, can be obtained by applying the filtering operation to the incompressible Navier-Stokes and continuity equations to yield

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u}_i \overline{u}_j) = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} \tag{2}$$

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0, \tag{3}$$

in which a reference length and velocity scale are used to make $\overline{u}_i$, $\overline{p}$, x_i and t dimensionless. The effect of the subgrid scales appears in the residual stress, $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$, which, in the present work, was parameterized by the modified Smagorinsky model [8]:

$$\tau_{ij} = -2\nu_T \overline{S}_{ij} + \delta_{ij} q_{sgs}^2/3 \tag{4}$$

where δ_{ij} is Kronecker delta, and $q_{sgs}^2 = \tau_{kk}$ is the subgrid scale energy (which is added to the pressure), $\overline{S}_{ij} = (\partial \overline{u}_i/\partial x_j + \partial \overline{u}_j/\partial x_i)/2$ is the large-scale strain-rate tensor, and ν_T is an eddy viscosity given by

$$\nu_T = \left[\gamma C_S (\Delta x \Delta y \Delta z)^{1/3} \left(1 - e^{-y^+/25}\right)\right]^2 \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}, \tag{5}$$

where a superscript $+$ indicates a quantity made dimensionless by the kinematic viscosity ν and the shear velocity $u_\tau = (\tau_w/\rho)^{1/2}$, where τ_w is the wall shear stress and ρ the fluid density. The scaling factor $\gamma = (H_l - H)/(H_l - H_t)$ (in which $H = \delta^*/\theta$ is the shape factor, δ^* is the displacement thickness, θ the momentum thickness, and the subscripts l and t refer respectively to laminar and fully developed turbulent flow) was introduced to decrease the dissipation by the subgrid scales during the early stages of transition [8].

The scaled Smagorinsky model described above was applied to the simulation of transition in a plane channel at $Re = 8{,}000$ (based on the channel half-width δ and on the centerline velocity in laminar flow, U_c). Initial conditions consisted of the parabolic mean flow, on which a 2D Tollmien-Schlichting (TS) mode of 2% amplitude and a 3D TS mode of 0.02% amplitude were superimposed. The initial conditions and Reynolds number matched those of the direct simulation described in [12]. The governing equations (2–3) were integrated in time using a Fourier-Chebyshev collocation method [13]. The sharp Fourier cutoff filter

was employed [8]. The model constant C_s was set equal to 0.1, and the values 5/2 and 1.7 were used for H_l and H_t. The numerical results, at least in the fully developed turbulent regime, are not expected to depend much on the values of H_l and H_t: turbulent statistics are insensitive to changes in C_s of the order of 20% [2]. A mesh using up to 48×64×48 grid points was used. The large-eddy simulation results will be compared with those of a coarse DNS which used the same number of points as the LES, but no model, and with the finely resolved DNS [12], which used 216×216×192 grid points. Although the LES and the DNS codes were essentially the same, and thus the CPU time per point and per step was equal, the smaller number of points used in the LES resulted in savings, in terms of CPU time required to calculate the full transition, of over 90%.

3. Results and discussion

The time-development of the wall shear is shown in Figure 1a (in the following, time is normalized by δ/U_c, velocities by U_c, and lengths by δ; moreover, $< . >$ denotes plane averaging, $U_i = < \overline{u}_i >$, and the resolved fluctuations are defined as $u_i'' = \overline{u}_i - U_i$). During the linear stages of the growth of the perturbation ($t < 160$) all simulations agree very well. The two coarse simulations, however, predict an earlier occurrence of the second spike stage than the direct simulation. Little difference can be observed between the coarse simulations for $t < 180$, since up to that time the mean velocity has been affected little by the perturbation. It should be remarked here that up to $t = 165$, 16×16×48 grid points or fewer are necessary to resolve the velocity field. For $t > 165$, however, the mesh is refined in stages, as increased resolution is required, to its final value. During laminar breakdown, moreover, very large velocity fluctuations are observed; to satisfy the CFL condition, therefore, the timestep must be significantly decreased. As a result of these two constraints, the first 165 time units required only a small fraction of the CPU time required for the entire simulation: on a Cray YMP, 3600 CPU seconds were used to integrate the Navier-Stokes equations from $t = 0$ to $t = 165$, while 26,500 CPU seconds were expended to advance the solution from $t = 165$ to $t = 250$.

Although up to $t = 175$ the perturbation energy level is small enough that the interaction of the perturbations with the mean flow is negligible, so that the shape factor (Figure 1b) is close to its laminar value and the eddy viscosity ν_T is essentially zero (Figure 2), after that time they start affecting the mean flow, as indicated by the rapid rise in the wall shear and in the decrease in the shape factor. As the shape factor is altered, the eddy viscosity begins to increase; at $t = 220$ it is approximately 40% larger than in fully-developed turbulent flow. This is due to the large velocity fluctuations that occur during laminar breakdown (see below), which cause oscillations of the large-scale strain rate tensor of greater amplitude than in fully-developed turbulent flow. The discrepancy observed around $t = 170$ between coarse and fine simulations appears to be caused mostly by insufficient energy transfer to the small scales and viscous dissipation. Although in this flow the scaling factor γ causes underprediction of the energy transfer from large to small scales, for the flat-plate boundary layer the opposite effect was observed [8], perhaps because of deficiencies in the form of γ chosen in the present work.

As the flow undergoes laminar breakdown, more small scales are generated, and the dissipation provided by the increased eddy viscosity becomes significant, and the LES results

become more accurate. The coarse DNS, on the other hand, further loses accuracy because the viscous stresses cannot provide sufficient dissipation. In particular, the overshoot in the wall shear is captured accurately by the LES, the shape factor development is well predicted, and the mean velocity profiles agree well with the fine DNS results (Figure 3). The peak wall shear computed with the LES is within 3% of the DNS prediction, while that calculated with the coarse DNS is in error by approximately 12%. While at $t = 176$ the Reynolds stresses are overpredicted by the LES because of the accelerated development of the disturbance (Figures 4 and 5), during laminar breakdown ($t > 185$) better agreement between LES and DNS is observed; in those cases also the LES results are more accurate than those obtained from the coarse DNS. Time-averaged velocity and Reynolds stress profiles in fully-developed turbulent flow (obtained from the large-eddy simulation) are also shown for comparison. During the late stages of transition the Reynolds stresses can be several times larger than their counterparts in turbulent flow, perhaps due to the highly intermittent character of the late stages of the transition process, which results in large velocity gradients and increased turbulent kinetic energy production. The evolution of the energy content of selected harmonics, shown in Figure 6, exhibits similar trends to those described above: for $t > 165$ the presence of the subgrid scale model gives improved results. The large-eddy simulation was carried out into the turbulent regime, and its results were found to compare well experimental and numerical results [14, 15].

4. Conclusions

The present study indicates that, although a coarse direct simulation is capable of reproducing the entire transition process from laminar to turbulent flow, during the late stages of laminar-turbulent transition the presence of a residual stress model is beneficial, and yields more accurate evolution of the perturbations, as evidenced by the comparison of various quantities of engineering interest such as mean velocity profiles, Reynolds stresses, wall shear and shape factor. The evolution of single harmonics is also predicted more accurately when a residual stress model is used than when none is employed.

The predictions of the scaled Smagorinsky model [8] are not very accurate at the early nonlinear stages of transition, but the model becomes more accurate as small scales evolve towards local equilibrium (in which they receive energy from the large scales and transfer it entirely to smaller scales until viscous dissipation becomes significant). Since the Smagorinsky model is based on the local equilibrium assumption for the small scales, it is not surprising that the model fares better at the late stages of transition than at the early nonlinear ones. Presumably, for engineering flows in which a high level of disturbances is present, this type of model could give satisfactory results. It appears, however, desirable to develop better ways to relate the length and velocity scales present in (5) to the energy content of the unresolved scales.

The fact that in the boundary layer case the scaling factor γ produced a slowed down perturbation growth, while in this flow accelerated growth is observed, indicates that the present formulation cannot fully account for the very complex flow physics. More accurate scaling or intermittency factors must be developed, which depend strongly on the local state of the flow. This local character would facilitate large-eddy simulation of natural transition, which usually takes place through the development of localized regions of turbulence (spots),

in which the velocity statistics resemble turbulent ones, surrounded by relatively quiescent areas. A model which exhibits this behavior is the RNG model used in [9]; this model is still, however, based on local equilibrium assumptions; its results, moreover, were found to be very grid-dependent, indicating that a more accurate length scale than that employed in [9] is necessary. Alternatively, use of a transport equation for the velocity scale valid during transition could also result in more accurate prediction of the residual stresses during transition. These approaches are presently under investigation.

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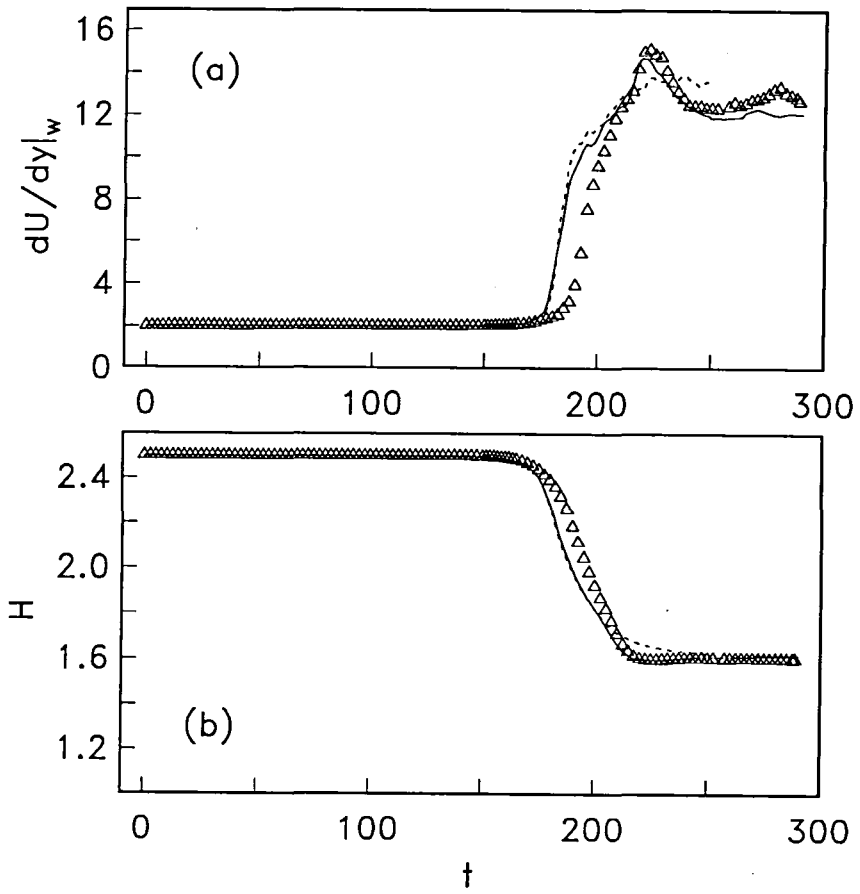

Figure 1: Time evolution of the wall shear and shape factor. ····· Coarse direct simulation; —— large-eddy simulation; △ fine direct simulation [12]. (a) Wall shear; (b) shape factor.

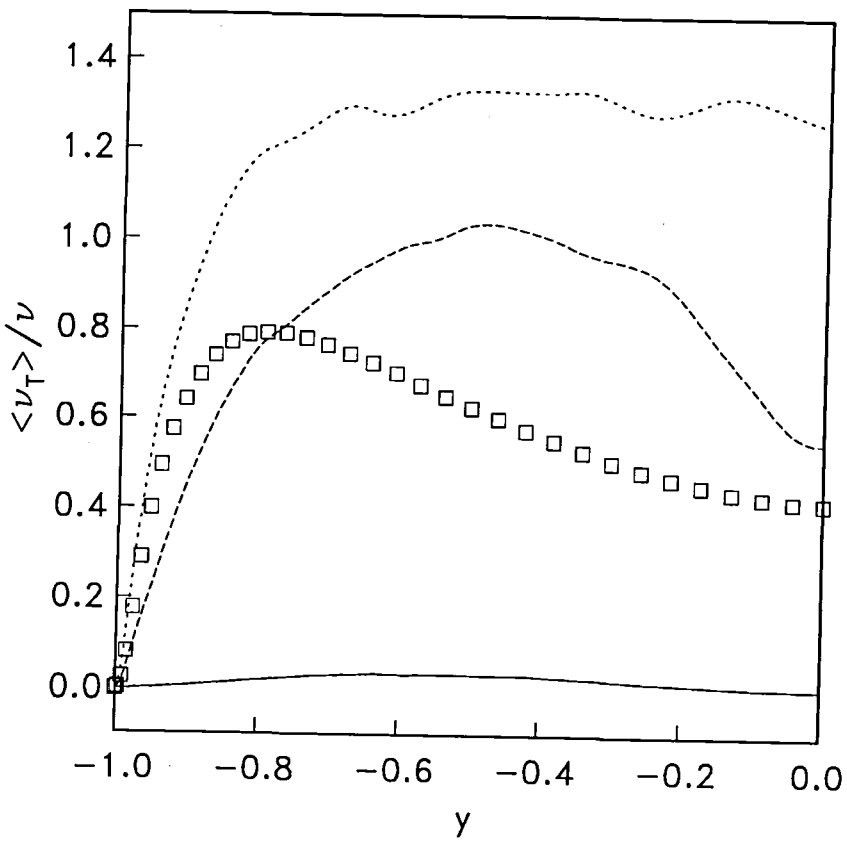

Figure 2: Ratio of the plane-averaged eddy viscosity, $< \nu_T >$, to the molecular viscosity ν. —— $t = 176$; --- $t = 200$; ····· $t = 220$, ▫ fully-developed turbulent flow.

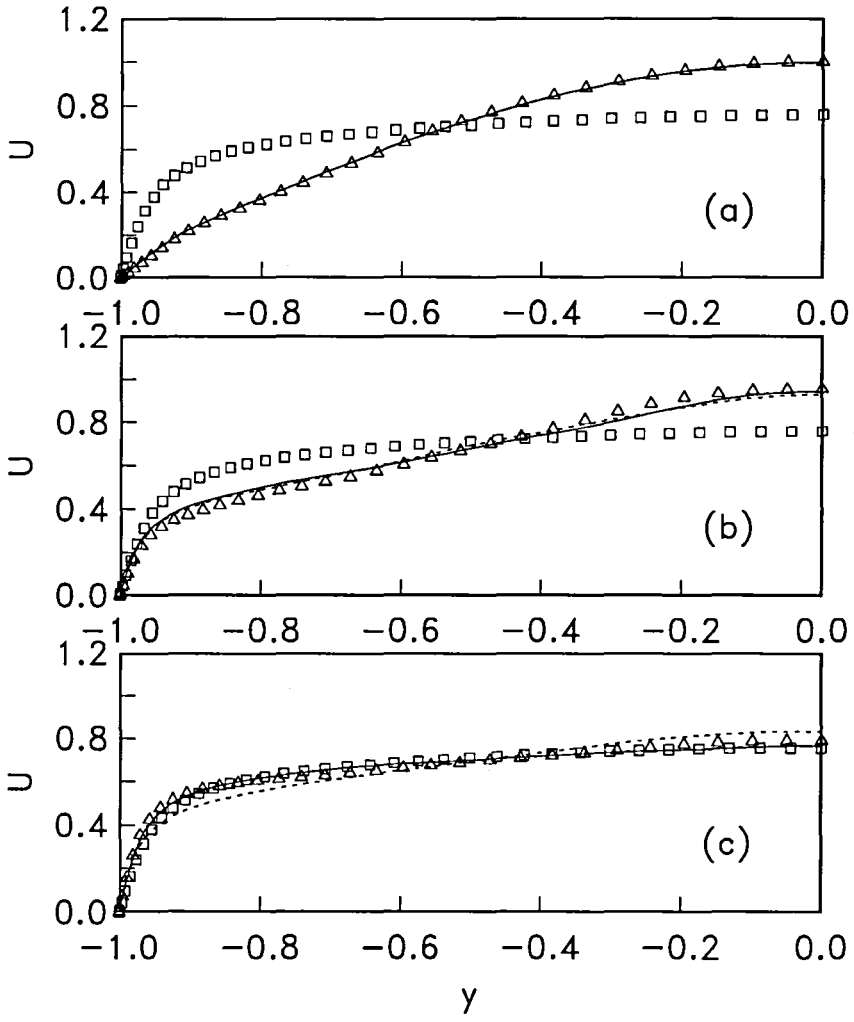

Figure 3: Plane-averaged mean velocity. ····· Coarse direct simulation; —— large-eddy simulation; △ fine direct simulation [12], ▫ fully-developed turbulent flow. (a) $t = 176$; (b) $t = 200$; (c) $t = 220$.

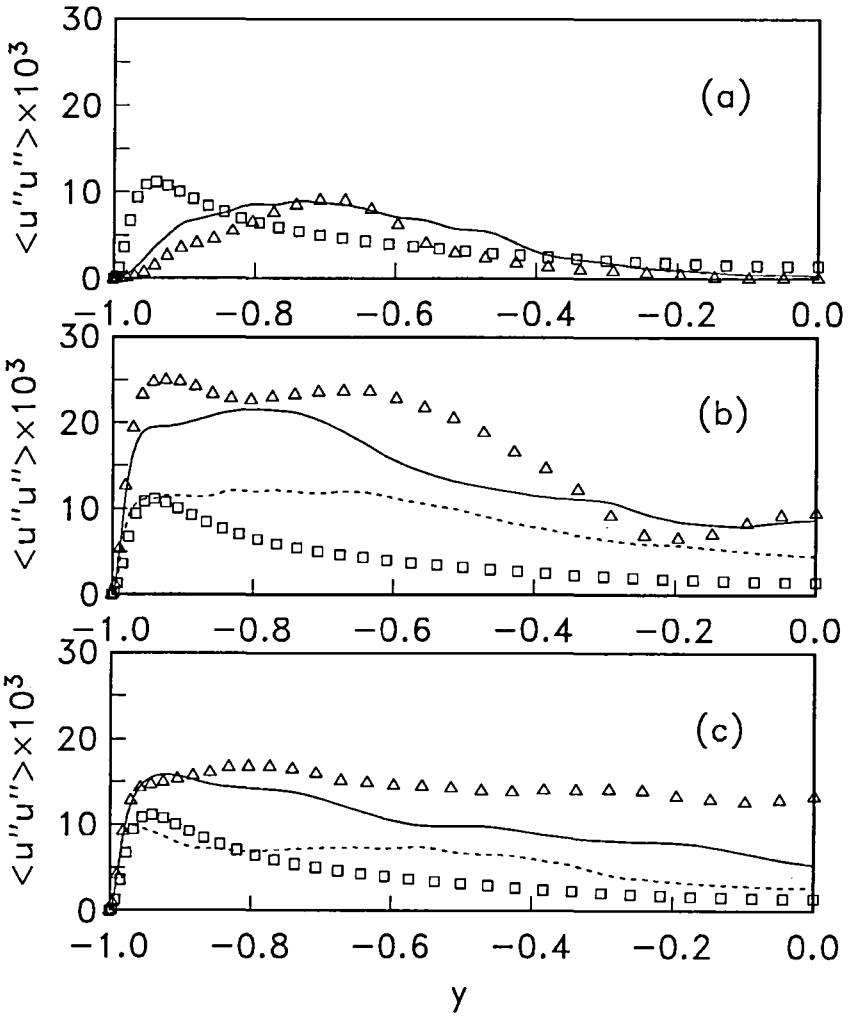

Figure 4: Plane-averaged normal Reynolds stress $< u''u'' >$. ····· Coarse direct simulation; —— large-eddy simulation; △ fine direct simulation [12], ▫ fully-developed turbulent flow. (a) $t = 176$; (b) $t = 200$; (c) $t = 220$.

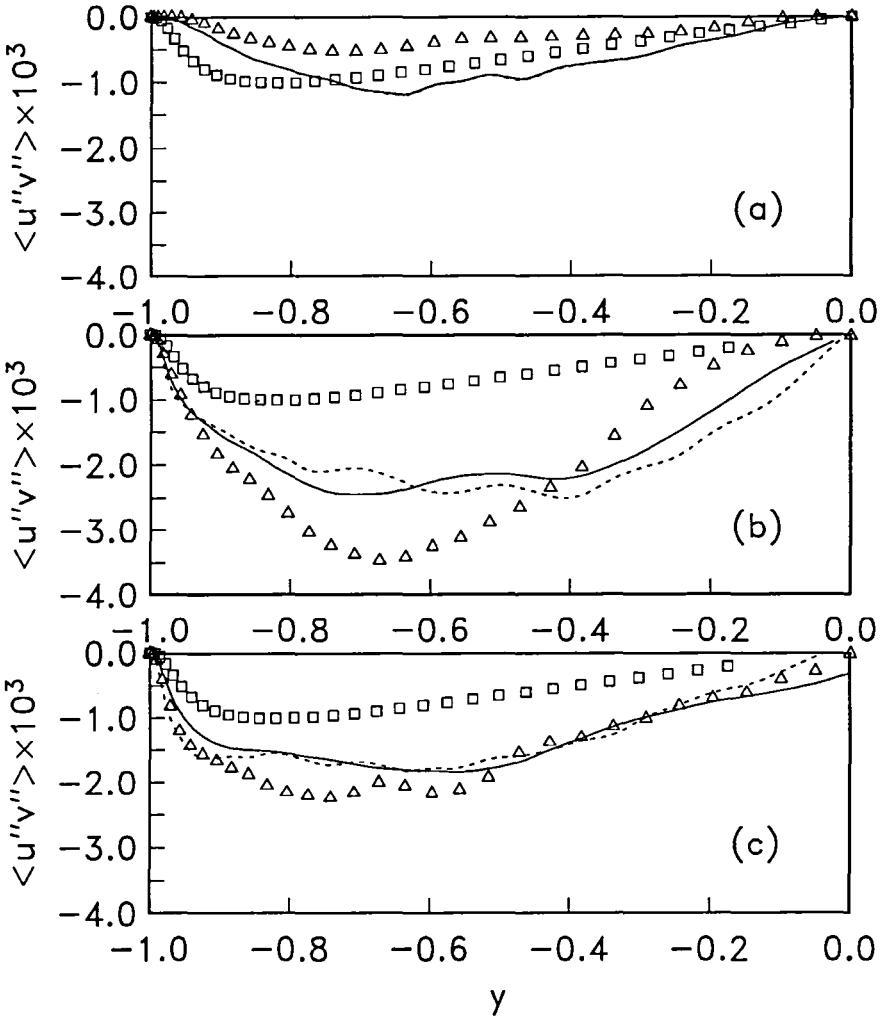

Figure 5: Plane-averaged shear Reynolds stress $< u''v'' >$. ····· Coarse direct simulation; —— large-eddy simulation; △ fine direct simulation [12], ▫ fully-developed turbulent flow. (a) $t = 176$; (b) $t = 200$; (c) $t = 220$.

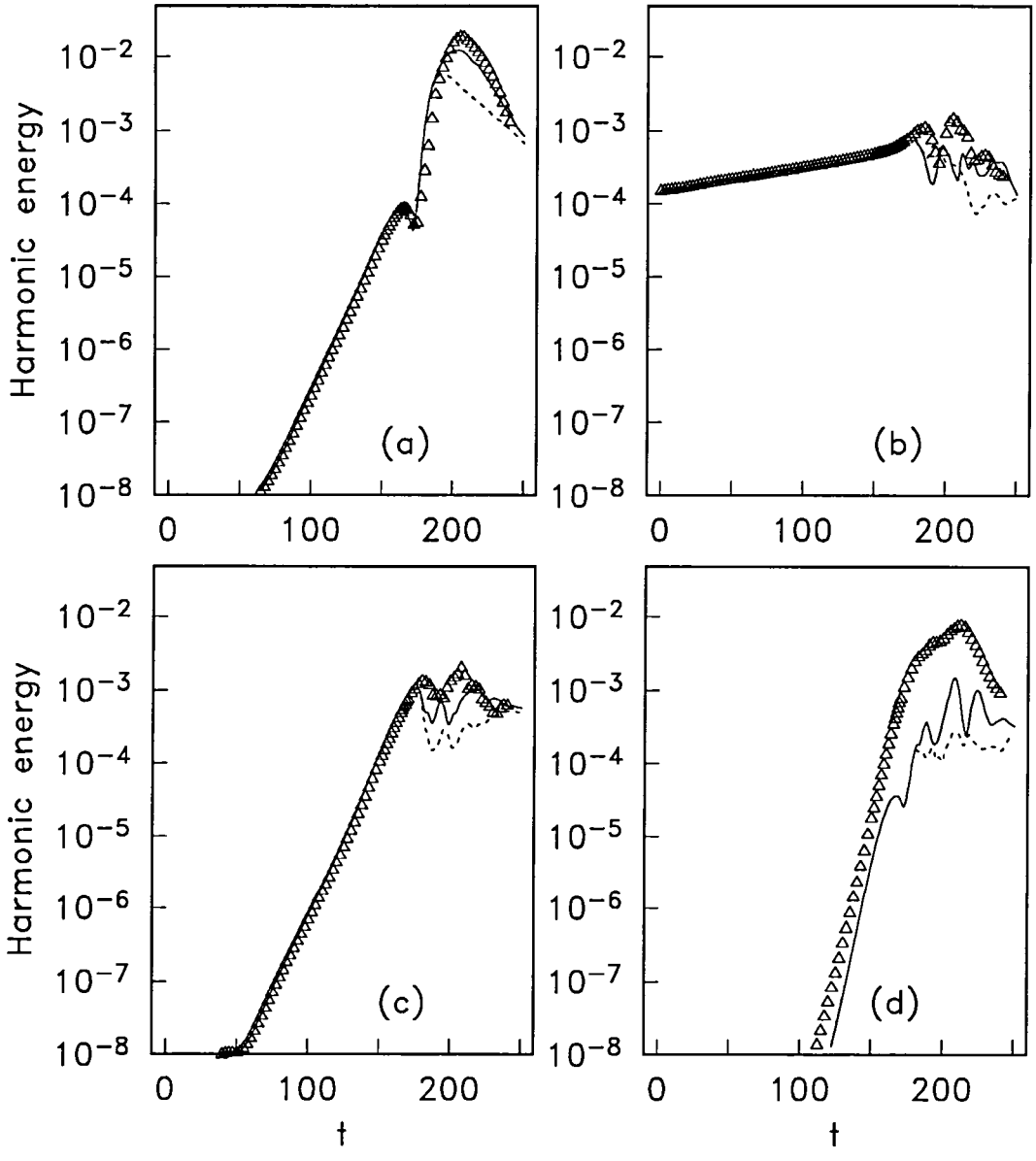

Figure 6: Evolution of selected harmonics. ····· Coarse direct simulation; —— large-eddy simulation; △ fine direct simulation [12]. (a) (0,1) mode; (b) (1,0) mode; (c) (1,1) mode; (d) (2,2) mode.

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16 Abstract

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