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A REVIEW OF NONLINEAR CONSTITUTIVE MODELS FOR METALS

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ABSTRACT

Over the past two decades a number of thermomechanical constitutive theories have been proposed for viscoplastic metals. These models are in most cases similar in that they utilize a set of internal state variables which provide locally averaged representations of microphysical phenomena such as dislocation rearrangement and grain boundary sliding. The state of development of several of these models is now at the point where accurate theoretical solutions can be obtained for a wide variety of structural problems at elevated temperatures.

The purpose of this paper is threefold. First, the fundamentals of viscoplasticity are briefly reviewed and a general framework is outlined. Second, several of the more prominent models are reviewed in some detail. And third, predictions from models are compared to experimental results.

INTRODUCTION

Since World War II there have been an increasing number of applications in which structural materials are required to operate at very high temperatures. Perhaps the first large scale example of this occurred in the nuclear power industry, wherein temperatures in excess of 2000°F are common. Recently, interest in the National Aerospace Plane, wherein hypersonic shock interaction causes predicted temperatures in excess of 3000°F, has enhanced interest in this subject. The quest for more efficient gas turbines has also forced operating temperatures to increase. Since experimentation in such hostile environments is extremely expensive, it is desirable to produce accurate theoretical models for the structural analysis of components constructed from viscoplastic metals.

In all of these cases the structural materials commonly in use exhibit a substantial amount of inelastic constitutive behavior. Indeed, they are loading history, temperature, and strain rate dependent, as well as highly nonlinear. Hence, it is clear that any successful modelling attempt will be extremely complex in nature.

The most recent advances in constitutive theories to predict the inelastic behavior of structural materials have been the incorporation of the effects of temperature and rate dependence into the stress-strain relationships. The ability to predict the temperature and rate dependence of structural materials used in elevated temperature applications is especially important to the aerospace industry wherein substantial weight savings can be accomplished if safety factors can be reduced by the use of accurate analytical models. Most metals become viscoplastic, i.e., exhibit rate dependent inelasticity at temperatures above about four-tenths of their melting temperature. The models to describe this material behavior are more intricate than elastic-plastic models since the inclusion of rate dependence represents a significant increase in complexity of the mathematical model required to describe the observed material behavior. This is evident because in the classical rate-independent plasticity theory of metals the only parameter required to characterize the plastic strain is $\dot{\lambda}$, a history dependent scalar material property that relates inelastic strain rate to stress through the flow rule, which may be obtained experimentally from a single phenomenological uniaxial stress-strain curve. However, when the material becomes significantly rate dependent the uniaxial monotonic stress-strain curve is no longer unique. Therefore, it becomes necessary to construct a mathematical equation governing $\dot{\lambda}$. This equation can only be constructed by obtaining considerable experimental information about the response of the material to changes in the independent variables such as strain, strain rate, and temperature. The experiments required to obtain this information are usually cumbersome and expensive.

Historically, there have been two distinct approaches to the modelling of inelastic materials: 1) the functional theory [1], in which all dependent state variables are assumed to depend on the entire history of the specified observable state variables; and 2) the internal state variable (ISV) approach [2], wherein history dependence is postulated to appear explicitly in a set of ISV's. Lubliner [3] has shown that in most circumstances ISV models can be considered to be special cases of functional models. Because the internal state variables are readily identifiable in metals, most models currently under development are of the ISV type. This form has the added benefit that it is also usually more computationally tractable than the functional form.

This article will focus on several of the ISV models which have shown promise for predicting the complex stress-strain response of metals at elevated temperature. After establishing the general framework for a constitutive model using the ISV formulation, several state-of-the-art thermoviscoplastic models will be reviewed along with examples of the model predictions compared to experimental results.

SYMBOLS

σ	uniaxial stress
σ_{ij}	stress tensor
$D_{ijk\ell}$	elastic modulus tensor
E	Young's modulus
ε	uniaxial strain
ε^{I}	uniaxial inelastic strain
ε^{T}	uniaxial thermal strain
$\varepsilon_{k\ell}$	strain tensor
$\varepsilon_{k\ell}^{C}$	creep strain tensor
$\varepsilon_{k\ell}^{I}$	inelastic strain tensor
$\varepsilon_{k\ell}^{P}$	plastic strain tensor
$\varepsilon_{k\ell}^{T}$	thermal strain tensor
α_2	drag stress
α_3	uniaxial back stress
α_{3ij}	back stress tensor
$\alpha_{k\ell}^{\mu}$	general set of internal state variables
h_2, h_3	hardening parameters
r_2, r_3	recovery parameters
$\dot{\lambda}$	inelastic flow parameter
sgn	sign
t	time

T	temperature
α_{4ij}	damage tensor
σ'_{ij}	deviatoric stress tensor
α'_{3ij}	deviatoric back stress tensor
J_2	second deviatoric stress invariant
$\dot{W}_p$	rate of inelastic work = $\sigma_{ij} \dot{\varepsilon}^{I}_{ij}$

GENERAL THERMOVISCOPLASTIC CONSTITUTIVE MODEL FRAMEWORK

The concept of ISV's, sometimes called hidden variables, was apparently first utilized in thermodynamics by Onsager [4,5], and numerous applications have been recorded in the literature over the last forty years [2,6-14]. A general framework for an ISV formulation of a thermoviscoplastic constitutive model can be developed by following the thermodynamic approach described by Coleman and Gurtin [2]. Historically, attempts to model rate dependence began with extensions of rate-independent classical plasticity theory. In these attempts the inelastic strain was "uncoupled" into rate-independent plastic and rate-dependent creep components to obtain

$$\sigma_{ij} = D_{ijk\ell} \left(\varepsilon_{k\ell} - \varepsilon^{P}_{k\ell} - \varepsilon^{C}_{k\ell} - \varepsilon^{T}_{k\ell}\right) \tag{1}$$

where the superscripts P, C, and T refer to the plastic, creep and temperature components of strain, respectively. Ultimately, these attempts failed due to the fact that rate-independent and rate-dependent inelastic deformations are caused by the same microphysical mechanism, predominately dislocation movement. Whereas plasticity is controlled by dislocation glide, viscoplasticity is driven by thermally assisted diffusion in the form of dislocation climb and cross-slip, which may in turn contribute to further dislocation glide. Thus, a more salient approach evolved using an approach in which the plastic strain and creep strain are "unified" into a single inelastic strain, $\varepsilon^I_{k\ell}$. The general form of the model for a metal is thus described by the following stress-strain equation of state:

$$\sigma_{ij} = D_{ijk\ell}(\varepsilon_{k\ell} - \varepsilon^I_{k\ell} - \varepsilon^T_{k\ell}) \tag{2}$$

Although the total elastic strain, $\varepsilon_{k\ell}$, and the thermal strain, $\varepsilon^T_{k\ell}$, are normally specifiable, the inelastic strain tensor, representing a locally averaged measure of the distance traversed by dislocations, is not. Therefore, equation (2) must be augmented by an ISV evolution law (also sometimes called the flow law) of the form:

$$\dot{\varepsilon}^I_{ij} = \dot{\lambda}\ (\sigma'_{ij} - \alpha'_{3ij}) \tag{3}$$

where $\dot{\lambda}$ is a complicated history dependent function of state. For example, the Prandtl-Reuss equations [15,16] utilized in rate independent applications may be obtained as a special case by differentiating (2) in time, substituting (3) into this result, and setting α'_{3ij} to zero.

For rate dependent circumstances, however, the equations must be further augmented by additional ISV evolution laws to account for the diffusive nature of dislocation mechanisms at elevated temperatures. These are of the form:

$$\dot{\alpha}_2 = h_2\ (\varepsilon_{k\ell}, T, \alpha^{\mu}_{k\ell}) - r_2\ (\varepsilon_{k\ell}, T, \alpha^{\mu}_{k\ell}) \tag{4}$$

$$\dot{\alpha}_{3ij} = h_3\ (\varepsilon_{k\ell}, T, \alpha^{\mu}_{k\ell}) - r_3\ (\varepsilon_{k\ell}, T, \alpha^{\mu}_{k\ell}) \tag{5}$$

where the drag stress, α_2, is an ISV related to the number or density of dislocations and the backstress, α_{3ij}, is an ISV related to the residual stresses at the microstructural level produced by the dislocation arrangement. The functions h_2 and h_3 represent the hardening terms in the drag stress and backstress, respectively, due to loss of dislocation mobility. The functions r_2 and r_3 represent the recovery terms in the drag stress and backstress, respectively, due to recovery of dislocation mobility. In some applications it may be necessary to append an additional internal variable, α_{4ij}, called a damage parameter and representing the effects of grain boundary sliding and microfracture [17-20].

The mathematical expressions for the ISV's and the flow rule, equations 3 through 5, are typically determined phenomelogically by curve fitting data obtained from a prescribed set of complicated experiments to this form. The precise experiments required to obtain the models depend on the theory being utilized. However, these experiments are typically complex in nature [21-23]. Since they are normally performed at temperatures in excess of 1000°F, they require the use of sophisticated test apparatus such as that shown in Fig. 1. In addition, many of the models require that cyclic tension-compression tests be performed to obtain data such as shown in Fig. 2, so that a highly aligned testing machine and support fixtures are required in order to avoid buckling of the specimens.

REVIEW OF CURRENT MODELS

In this section several of the more prominent unified models will be reviewed; because the uncoupled models possess limited modelling ability, they will not be covered. The first concerted attempt to model the inelastic strain rate in a rate-dependent framework appears to have been due to Bodner and co-workers [17,18,24-36]*, and an indication of the complexity of this problem is that they are still actively pursuing this model. Since 1975, there has been a veritable explosion of models such as Hart [36,37], Miller and co-workers [19,38-48], Valanis [49,50], Robinson [51-57], Walker and co-workers [20,32,35,36,58,59,60], Krempl and co-workers [61-66], Krieg and co-workers [36,67], as well as others [68-78]. Doubtless there are numerous efforts we have overlooked, and the authors apologize for any oversight on our part.

In a paper of this limited scope it is unrealistic to expect that an in-depth review can be provided for each of the models. Therefore, we have chosen what we hope is a reasonable and expedient dissemination method. First, we will discuss each of the models briefly, and we have encapsulated a summary of each of the models mentioned above (in uniaxial form) in Table I. Because many of the models have appeared in several forms, in this table we have chosen a relatively simple version of each of the models. Second, we have summarized the capabilities of the models in Table II, and reviewed the experimental requirements in Table III. Finally, we will discuss recent advances and review in somewhat greater detail the models of Bodner, Miller, and Walker.

Because the scope of this paper is limited, we are unable to pursue all of the important issues regarding this subject. Readers who are interested in further study on this subject will find a far more detailed discussion of recent advances in viscoplasticity in reference 36, as well as in the bibliography at the end of this paper.

*Although a promising model proposed by Valanis had been previously reported, it was rate-independent at the time of Bodner's work.

In this discussion the models are reviewed only in uniaxial form because in virtually all cases they are converted to multiaxial form by using J_2 theory or in conjunction with Drucker's postulate [79]. We should also point out that we have used a common terminology since each author uses different notation.

Probably the simplest model to date was proposed by Krempl and co-workers [61-66]. Because this model does not contain evolution laws for the back stress and drag stress, it is best used for monotonic loadings.

The model proposed by Valanis [49,50] is built on a single integral framework which makes it quite different in form from equations (2) and (3). However, as pointed out by Schapery [80], when this so-called "endochronic" theory is used with an exponential kernel function, the Prandtl-Reuss [15,16] equations can be recovered. Although Valanis' model is actually capable of producing much more general results, a single exponential is usually used, so that it reduces to equations (2) and (3) in the endochronic time scale after a Laplacian transformation.

An interesting and potentially very useful model has been proposed by Krieg, et al. [67]. The model appears to have been one of the first to include both drag stress and back stress terms. However, the authors moved on to other things and the model was not improved for about a decade. Recently, a second generation of the model has been proposed [36].

Robinson [51-57] has proposed one of the most complex and advanced models to date. His model is distinguished from the other current models both in that it possesses a yield criterion similar to that used in classical plasticity, and that it has been proposed in multiaxial form for orthotropic media such as metal-matrix composites.

Hart's model [37] is distinguished by the fact that the drag stress is assumed to be a constant, and it possesses an ISV called hardness which affects the back stress evolution law. Recent advances in this model have also been reported in reference 36.

Bodner's Model

As mentioned earlier, Bodner's model [17,18,24-36] appears to have been the first viable unified model proposed for viscoplastic metals. Although early versions of the model were somewhat primitive, it has remained at the forefront of technology via timely modifications. The initial model did not contain a back stress, α_{3ij}, and although the current version does include one, it is included in a significantly different way from other current models. Bodner calls α_2 the isotropic hardening variable, and α_{3ij} the directional hardening tensor. He interprets α_2 as the nonrecoverable and isotropic (scalar) resistance to plastic flow due to the microstructural stress fields associated with dislocation density, whereas α_{3ij} is regarded as the potentially recoverable part of the resistance to plastic flow that can be caused by changes in stress direction (tensorial). The resulting evolution laws are:

$$\dot{\varepsilon}^I = \frac{2}{\sqrt{3}} D_0 \exp \{- \frac{1}{2} |\frac{Z}{\sigma}|^{2n}\} \operatorname{sgn} \sigma \tag{6}$$

$$Z \equiv \alpha_2 + \alpha_3 \operatorname{sgn} \sigma \tag{7}$$

$$\dot{\alpha}_2 = m_1 [Z_1 - \alpha_2] \dot{W}_p - A_1 Z_1 [\frac{\alpha_2 - Z_2}{Z_1}]^{r_1} \tag{8}$$

$$\dot{\alpha}_3 = m_2 [Z_3 \operatorname{sgn} \sigma - \alpha_3] \dot{W}_p - A_2 Z_1 [\frac{|\alpha_3|}{Z_1}]^{r_2} \operatorname{sgn} \alpha_3 \tag{9a}$$

$$m_2 = \frac{\bar{m}_2}{2} (1 + \exp\{-m_3\alpha_3 \operatorname{sgn}(\sigma)\}) \qquad (9b)$$

where D_0, n, m_1, Z_1, Z_2, A_1, r_1, $\bar{m}_2$, m_2, Z_3, A_2, m_3, and r_2 are material constants, and $\dot{W}_p \equiv \sigma\dot{\varepsilon}^I$.

The flow law is exponentially based as seen in equation (6). The model gives a limiting strain rate in shear of D_0. The term $-m_1\alpha_2 \dot{W}_p$ is a dynamic recovery term for α_2 in the isotropic growth law (8) and $- A_1 Z_1 [(\alpha_2 - Z_2) Z_1^{-1}]^{r_1}$ is a static thermal recovery term. α_3 is a uniaxial representation of a second order tensor in the multiaxial state which models directional hardening. Equation (7) shows that Z can experience large changes in magnitude due to the sgn σ function as the stress changes sign. The evolution law for α_3 has the same components as the evolution law for D.

Bodner's model is seen to use the rate of plastic work, $\dot{W}_p$, instead of inelastic strain rate as the measure of work hardening. This is designed to allow for better modelling of strain rate jump tests. The modification used to account for the strain aging effects was patterned after Schmidt and Miller's solute strengthening correction [43,45]. The constant Z_3 in the α_3 evolution law was written in the following form:

$$Z_3 = Z_4 + Z_5 f\,(\dot{\varepsilon}^I) \qquad (10)$$

$$f(\dot{\varepsilon}^I) = F \exp\{- [\frac{\log(|\dot{\varepsilon}^I|) - \log(J)}{\beta}]^2\} \qquad (11)$$

where F is the maximum correction, J is the strain rate of maximum correction, and β is the width of correction.

Miller's Model

Miller's model [19,36,39-48] is probably the most complex model available at the time of this writing. It is capable of accounting for a wide range of physical phenomena, including solute strengthening and cyclic strain softening.

Schmidt and Miller's evolution laws have the following form:

$$\dot{\varepsilon}^I = B' \{\sinh (\frac{\alpha_3}{\alpha_2 + F_{sol}})^{1.5}\}^n \operatorname{sgn} (\sigma - \alpha_3) \quad (12)$$

$$\dot{\alpha}_3 = H_1 \dot{\varepsilon}^I - H_1 B' \{\sinh (A_1 |\alpha_3|)\}^n \operatorname{sgn} (\alpha_3) \quad (13)$$

$$\dot{\alpha}_2 = H_2 \,|\,|\dot{\varepsilon}^I|(C_2 + |\alpha_3| - \frac{A_2}{A_1} \alpha_3) - H_2 C_2 B' \{\sinh (A_2 \alpha_2^3)\}^n \quad (14)$$

$$F_{sol} = F \exp \{ - (\frac{\log (|\dot{\varepsilon}^I|) - \log (J)}{\beta})^2\} \quad (15)$$

where B', n, H_1, A_1, H_2, C_2, A_2, F, J, and β are material constants. F_{sol} is the noninteractive solute strengthening correction parameter.

The flow law has the form of a hyperbolic sine. This form was chosen to model creep response better. The same form is found in the static thermal recovery terms of the backstress and drag stress evolution laws. The drag stress hardening term contains a hardening term, a dynamic recovery term, and a term which couples drag stress hardening to backstress magnitude. These three terms provide the proper cyclic, hardening, softening and saturation behavior.

Walker's Exponential Model

The growth laws for Walker's model [20,32,35,36,58,59,60] have the following form:

$$\dot{\varepsilon}^I = \frac{\exp\left(\frac{\sigma - \alpha_3}{\alpha_2}\right)}{\beta} \operatorname{sgn}(\sigma - \alpha_3) \tag{16}$$

$$\dot{\alpha}_3 = n_2 - B \{[n_3 + n_4 \exp(-n_5 |\log(\frac{|\dot{R}|}{\dot{R}_0})|)] \dot{R} + n_6\} \tag{17}$$

$$\alpha_2 = D_1 + D_2 \exp(-n_7 R) \tag{18}$$

$$\dot{R} = |\dot{\varepsilon}^I| \tag{19}$$

where β, n_2, n_3, n_4, n_5, $\dot{R}_0$, n_6, D_1, D_2, and n_7 are material constants.

This version of Walker's flow law is based on an exponential function. The term $n_2 \dot{\varepsilon}^I$ is a work hardening term in the back stress growth law. The term $\alpha_3 [n_3 + n_4 \exp(-n_5|\log(|\dot{R}|/\dot{R}_0|)|] \dot{R}$ is a dynamic recovery term. Negative strain rate sensitivity effects can be modelled with the term $n_4 \exp(-n_5|\log(|\dot{R}|/\dot{R}_0)|)$. Back stress thermal recovery is handled by the $\alpha_3 n_6$ term. Drag stress hardening is modelled through the $D_2 \exp(-n_7 R)$ term. No provision is made for drag stress recovery in this model.

COMPARISON OF MODEL PREDICTIONS TO EXPERIMENTAL RESULTS

In most cases, the models are described by a set of ordinary differential equations in time which are mathematically "stiff". The definition of mathematical stiffness is that if the solution is expanded in an exponential series in time, at least two of the eigenvalues will differ by many orders of magnitude [81]. A characteristic of stiff differential equations is that they cannot be accurately integrated in time by standard integration schemes such as Runge-Kutta methods. Numerous intricate algorithms have been developed for integrating equations (3) through (5) in time [82-87]. It is often most efficient to use a simple Euler forward or backward time marching integration scheme, where accuracy is achieved by taking very small time steps, as shown in Fig. 3 [82]. When solving boundary value problems using the finite element method, it is normally possible to obtain convergence on each displacement increment by subincrementing the Euler integration at each integration point.

Many of the models mentioned in the previous section have been compared both qualitatively and quantitatively to one another as well as to experimental results for a variety of materials [88-93]. The accuracy of several of the models is demonstrated for INCONEL 718 under two constant strain rate conditions at 1100°F (593°C) in Figs. 4 and 5 [93]. A complex load history is demonstrated in Figs. 6 through 8 [93]. In this example INCONEL 718 is subjected to the strain history shown at the bottom right hand corner of each figure [93].

CONCLUSION

The complex task of predicting the response of viscoplastic metals has now reached a state where reliable structural analysis is sometimes possible [94]. However, the accuracy of predictions still depends on a number of complicated factors such as material type, loading conditions, thermal environment, numerical accuracy, and the constitutive model being utilized. Although this area of research has produced results, it has not yet reached a high degree of maturity.

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TABLE I. COMPARISON OF UNIAXIAL MODELS

Theory	Stress-Strain Relation	Internal State Variable Evolution Laws	Comments	Material Parameters
Cernocky and Krempl [21,22,61-66]	$\sigma=E\,(\varepsilon-\varepsilon^{I}-\varepsilon^{T})$	$\dot{\varepsilon}^{I}=\frac{\sigma-G}{Ek}$	1. $G=G(\varepsilon,T)$ from extrapolation of relaxation data. 2. k is curve-fit to $k=R_o e^{R_1 e^{-\left(\frac{\lvert\sigma-G\rvert}{R_2}\right)R_3}}$	E,R_o,R_1 R_2,R_3
Krieg, Swearengen, and Rohde [36,67]	$\sigma=E\,(\varepsilon-\varepsilon^{I}-\varepsilon^{T})$	$\dot{\varepsilon}^{I}=C_1\left(\frac{\lvert\sigma-\alpha_3\rvert}{\alpha_2}\right)^{C_2}\operatorname{sgn}(\sigma-\alpha_3)$ $\dot{\alpha}_2=C_6\,\lvert\dot{\varepsilon}^{I}\rvert-C_7\,(\alpha_2-\alpha_{2_o})^{n}$ $\dot{\alpha}_3=C_3\dot{\varepsilon}^{I}-C_4\alpha_3^2\ \exp[C_5\alpha_3^2-1]\operatorname{sgn}(\sigma)$		$E,C_1,C_2,C_3,C_4,$ $C_5,C_6,C_7,\ \alpha_{2_o},n$
Bodner <u>et al.</u> [17,18,24-36]	$\sigma=E\,(\varepsilon-\varepsilon^{I}-\varepsilon^{T})$	$\dot{\varepsilon}^{I}=\frac{2}{\sqrt{3}}D_o\exp\{-\frac{1}{2}\lvert\frac{Z}{\sigma}\rvert^{2n}\}\operatorname{sgn}(\sigma)$ $Z\equiv\alpha_2+\alpha_3\operatorname{sgn}\sigma$ $\dot{\alpha}_2=m_1\,[Z_1-\alpha_2]\,\dot{W}_p-A_1Z_1\left[\frac{\lvert\alpha_2-Z_2\rvert}{Z_1}\right]^{r_1}$ $\dot{\alpha}_3=m_2\,[Z_3\operatorname{sgn}\sigma-\alpha_3]\,\dot{W}_p-A_2\,Z_1\left[\frac{\lvert\alpha_3\rvert}{Z_1}\right]^{r_2}\operatorname{sgn}\alpha_3$ $m_2=\frac{\bar{m}_2}{2}\ (1+\exp\{-m_3\alpha_3\operatorname{sgn}(\sigma)\})$	1. $\dot{W}_p=\sigma\,\dot{\varepsilon}^{I}$	E,D_o,n,m_1 $\bar{m}_2,m_3,Z_1,$ Z_2,Z_3,A_1,A_2 r_1,r_2

Walker [20,36,58-60]	$\sigma=E\,(\varepsilon-\varepsilon^{I}-\varepsilon^{T})$	$\dot{\varepsilon}^{I} = \dfrac{\exp\left(\dfrac{\lvert\sigma-\alpha_3\rvert}{\alpha_2}\right)}{\beta}\ \mathrm{sgn}\,(\sigma-\alpha_3)$ $\alpha_2 = D_1 + D_2 \exp\,(-n_7 R)$ $\dot{\alpha}_3 = n_2\dot{\varepsilon}^{I} - \alpha_3 \{[n_3 + n_4 \exp\,(-n_5\lvert\log\,(\frac{\lvert\dot{R}\rvert}{\dot{R}_0})\rvert]\,\dot{R} + n_6\}$ $R = \int_0^t \lvert\frac{\partial\varepsilon^{I}}{\partial\tau}\rvert\,d\tau$ $\dot{R} = \dot{\varepsilon}^{I}$	1. R is the cumulative inelastic strain:	E, n_2, α_0 n_3, n_4, n_5, n_6, n_7 $q, \dot{R}_0, \beta$
Miller [19,36,38-48]	$\sigma=E\,(\varepsilon-\varepsilon^{I}-\varepsilon^{T})$	$\dot{\varepsilon}^{I} = B'\,\{\sinh\,(\frac{\lvert\frac{\sigma}{\varepsilon} - \alpha_3\rvert}{\sqrt{\alpha_2} + F_{sol}})^{1.5}\}^{n}\ \mathrm{sgn}\,(\frac{\sigma}{\varepsilon} - \alpha_3)$ $\dot{\alpha}_2 = H_2\,\lvert\dot{\varepsilon}^{I}\rvert(C_2 + \lvert\alpha_3\rvert - \frac{A_2}{A_1}\alpha_2^3) - H_2C_2B'\,\{\sinh\,(A_2\,\alpha_2^3)\}^{n}$ $\dot{\alpha}_3 = H_1\dot{\varepsilon}^{I} - H_1B'\{\sinh\,(A_1\lvert\alpha_3\rvert)\}^{n}\ \mathrm{sgn}\,(\alpha_3)$ $F_{sol} = F \exp\,\{-(\frac{\log\,(\lvert\dot{\varepsilon}^{I}\rvert) - \log\,(J)}{\beta})^2\}$		$E, B', n, H_1, A_1, H_2, C_2, A_2$ F, J, β H_2, C_2, A_2 F, J, β

Hart [36,37]	$\sigma=E\ (\varepsilon-\varepsilon^I-\varepsilon^T)$	$\dot{\varepsilon}^I=\dot{\alpha}^*(\frac{2}{3})^{\frac{M}{2}}(\frac{\lvert\sigma-\sigma_3\rvert}{\mu})^M \operatorname{sgn}(\sigma-\alpha_3)$ $\dot{\alpha}_3=\frac{3}{2}\mu\dot{\varepsilon}^I-\frac{\mu(\frac{\alpha_2'}{G})^m e^{-\frac{Q}{RT}}}{[\ln\frac{3\alpha_3'}{2\lvert\alpha\rvert}]^{1/\lambda}}\operatorname{sgn}(\alpha_3)$ $\dot{\alpha}_2=c(\frac{2}{3})^{\frac{k}{2}}fe^{-\frac{Q}{RT}}(\frac{\sigma}{\alpha_2'})^k\frac{\alpha_2'}{[\ln\frac{\alpha_2'}{\sqrt{\frac{2}{3}}\alpha_3}]^{1/\lambda}}$	1. The drag stress is taken to be a constant. μ Hence there is no α_3 as in other models. There is, however, a third internal state variable termed α_3'. 2. R is the gas constant. T is the absolute temp.	$E,\dot{\alpha}^*,M,$ $\mu,G,m,f,$ Q,k,λ,c
Robinson [23,51-57]	$\sigma=E(\varepsilon-\varepsilon^I-\varepsilon^T)$	$\dot{\varepsilon}^I=\frac{1}{2\mu}(\frac{1}{\sqrt{3}}\lvert\frac{\sigma-\alpha_3}{K}\rvert)^{n-1}(\sigma-\alpha_3)$ $\dot{\alpha}_3=\frac{2\mu H}{(\frac{1}{\sqrt{3}}\lvert\frac{\alpha_3}{K}\rvert)^{\beta}}\dot{\varepsilon}^I-R(\frac{1}{\sqrt{3}}\lvert\frac{\alpha_3}{K}\rvert)^{n-\beta-1}\alpha_3$	1. G_o is the initial value of $\frac{\alpha_3^2}{3K^2}$	$E,\mu,K,m,$ β,H,R,c
Valanis [11,49-50]	$\sigma=E\ (\varepsilon-\varepsilon^I-\varepsilon^T)$	$\dot{\varepsilon}^I=k_1f_1(\varepsilon,\sigma)\varepsilon+k_2f_2(\varepsilon,\sigma)\ \alpha_1$	1. Represents simplified form of Valanis' model.	E,k_1,k_2 f_1,f_2

TABLE II. ABILITY TO MODEL CERTAIN PHENOMENA

Theory	Unified	History Dependence	Bauschinger Effect	Temperature Dependence	Anelasticity	Multiaxial Representation
Cernocky and Krempl	X			X		X
Krieg, Swearengen, and Rhode	X	X	X	X	X	X
Bodner et.al	X	X	X	X	X	X
Walker	X	X	X	X	X	X
Miller	X	X	X	X	X	X
Hart	X	X	X	X	X	X
Robinson	X	X	X		X	X
Valanis	X	X		X		X

TABLE III. REQUIRED MATERIAL PARAMETER CHARACTERIZATION

Cernocky and Krempl	Constant Strain Rate Tensile Tests with Intermittent Hold Times Relaxations Tests
Krieg, Swearengen, and Rohde	Stress Drop Tests Constant Strain Rate Tensile Tests
Bodner *et al.*	Constant Strain Rate Tensile Tests* Creep Tests
Walker	Constant Strain Rate Cyclic Tests Constant Strain Rate Tensile Tests
Miller	Creep Tests Constant Strain Rate Cyclic Tests Constant Strain Rate Tensile Test
Hart	Relaxation Tests
Robinson	Stress Drop Tests
Valanis	Constant Strain Rate Tensile Test*

*Represents Simplest Form of the Model

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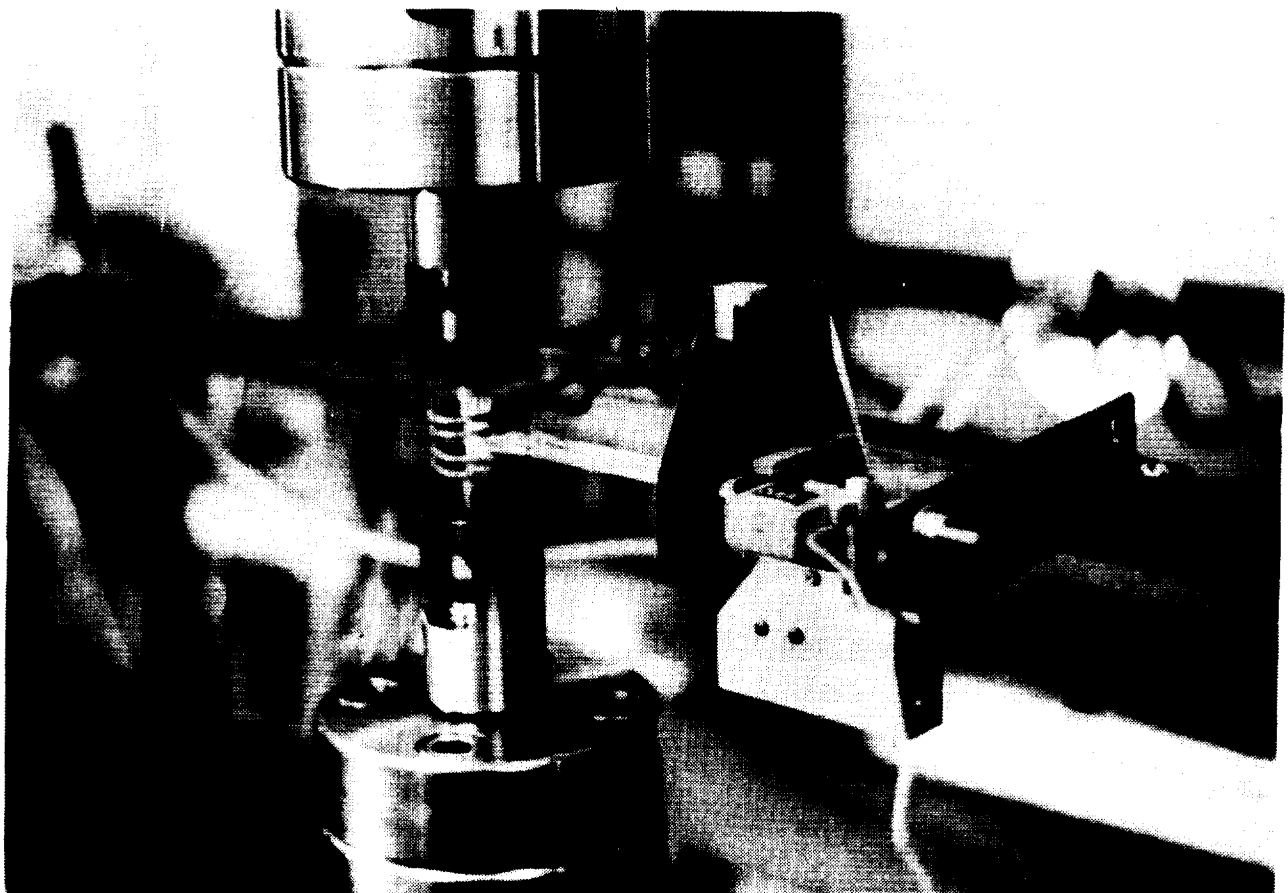

Fig. 1. Inconel 100 Speciman Tested at 1100°F in MTS-810 110 Kip Load Frame with MTS Quartz Rod Diametral Extensometer and Lepel Induction Heating Furnace

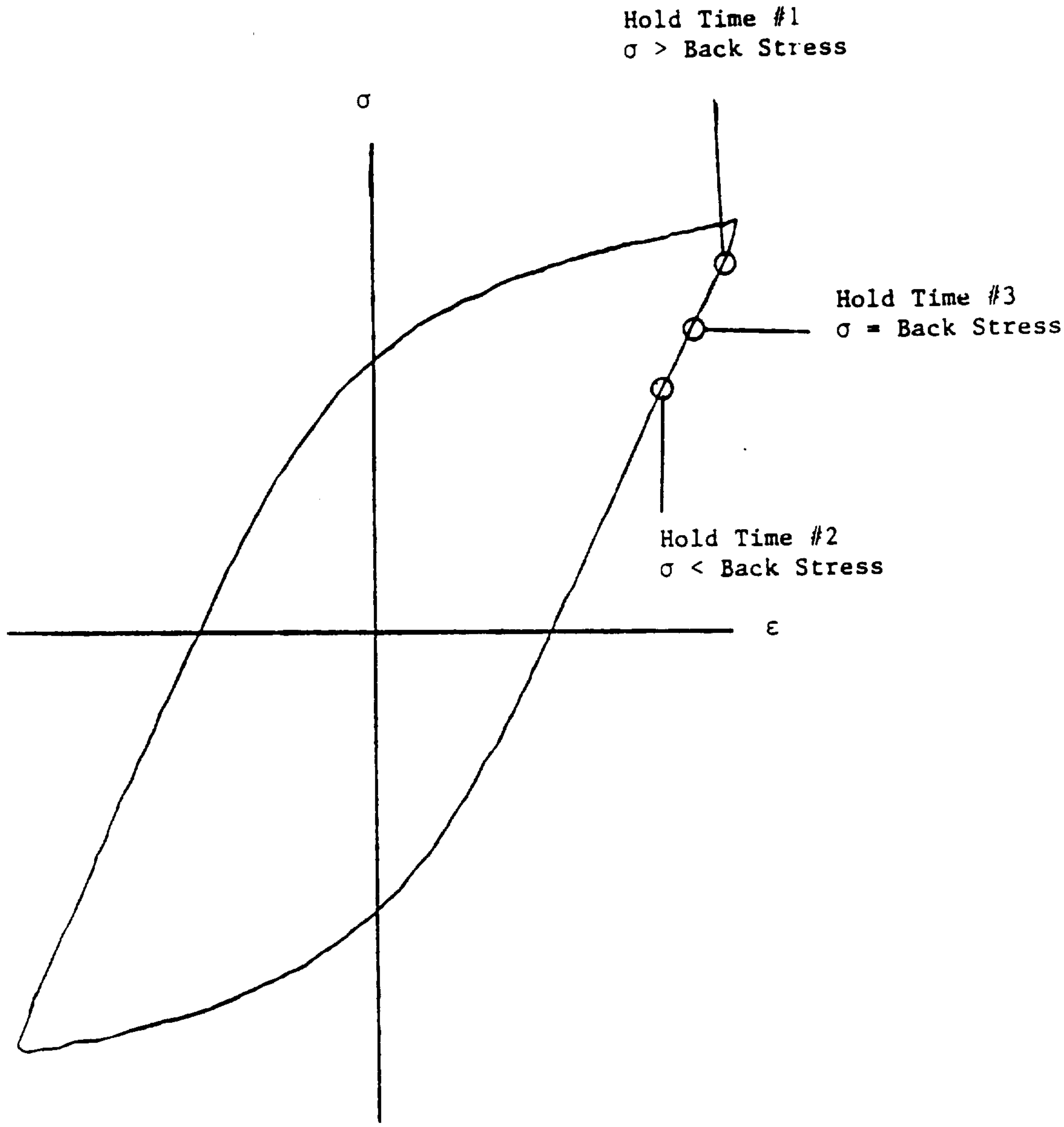

Fig. 2 Cyclic Hysteresis Loop With Hold Times

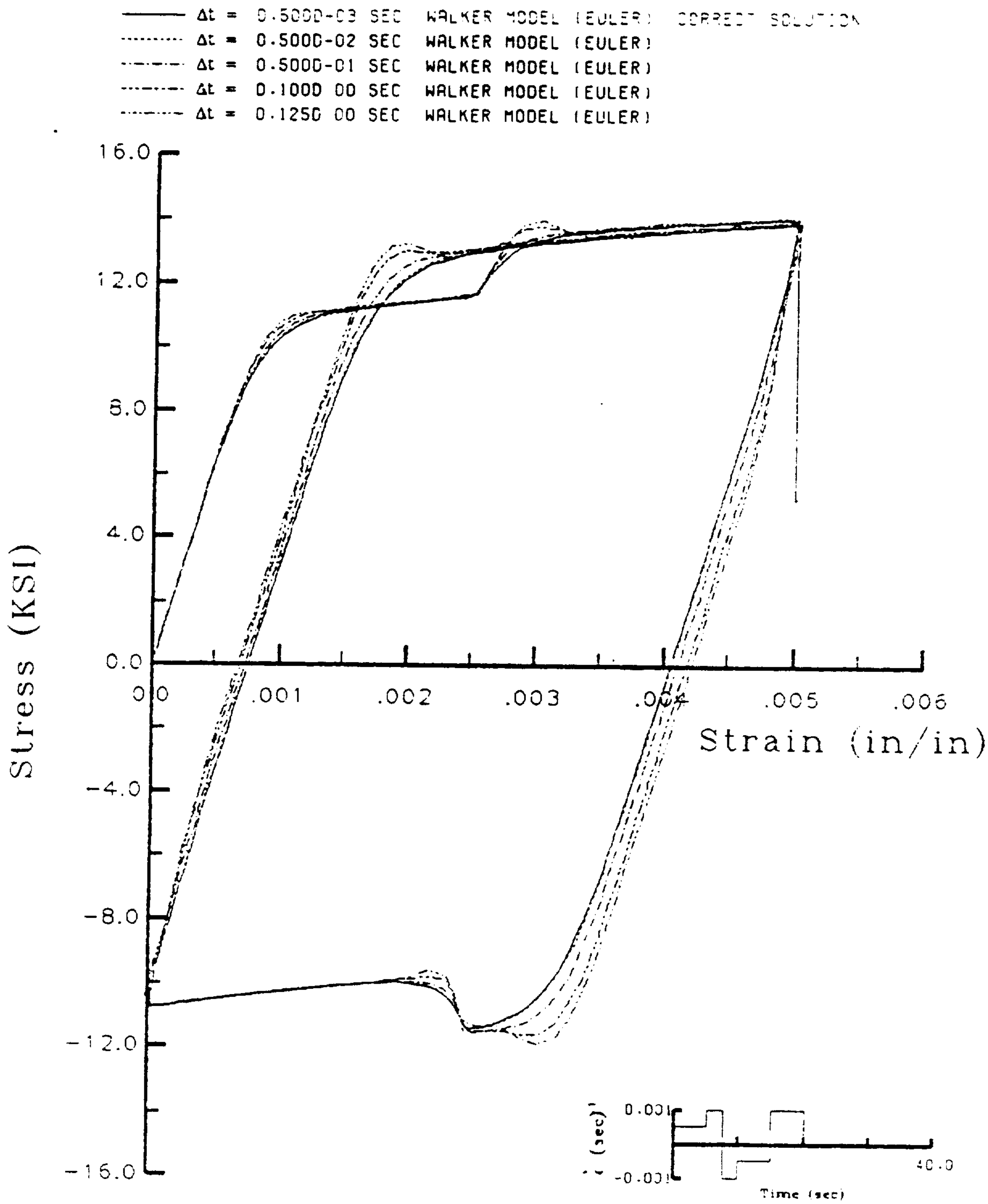

Fig. 3 Stability and Accuracy of Euler Integration for Walker's Model Using Various Step Sizes (Hastelloy-X at 1800°F)

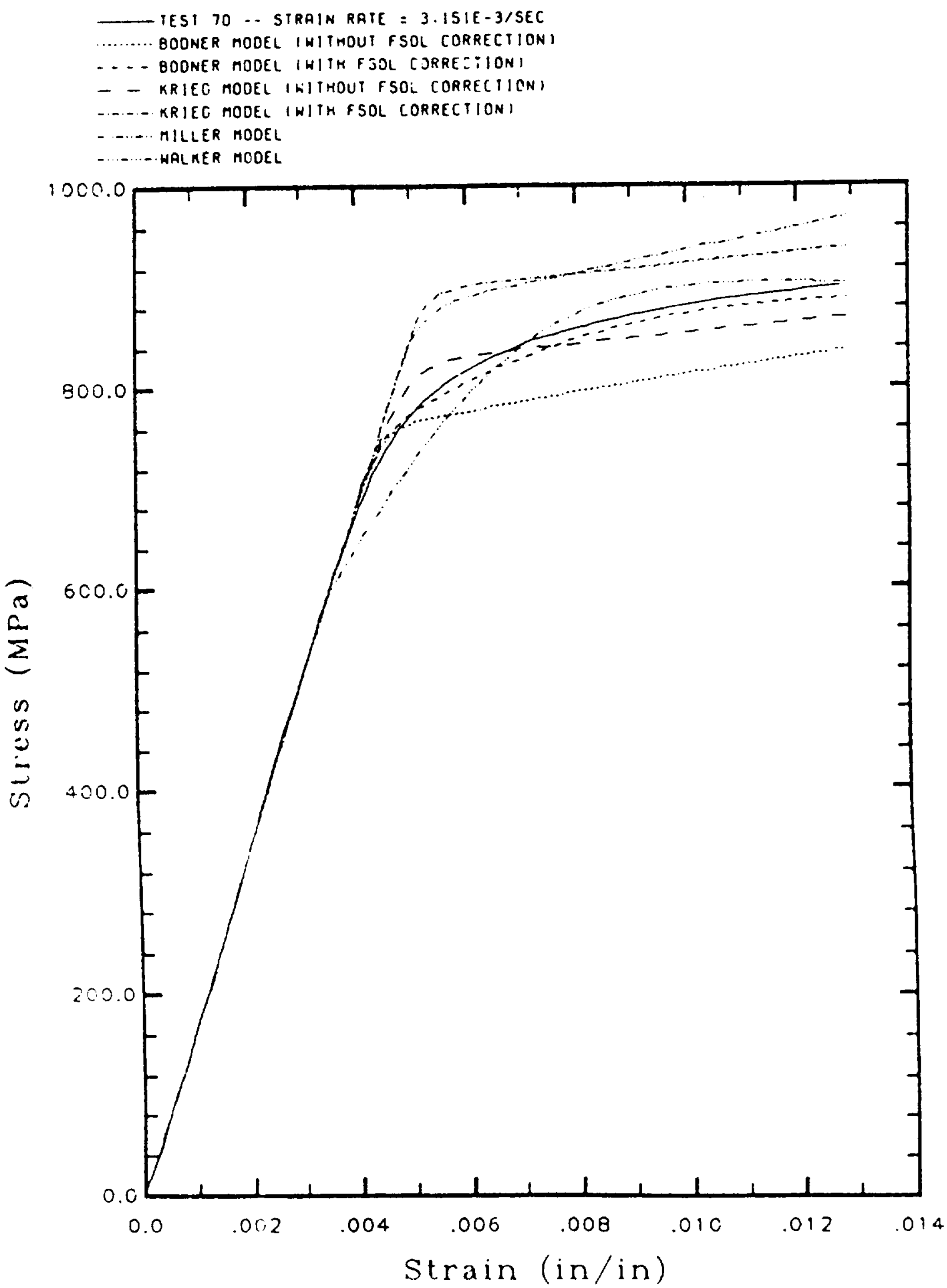

Fig. 4. Model Response for Inconel 718 at 1100°F at Constant Strain Rate $\varepsilon = 3.15 \times 10^{-3}$/sec (Courtesy American Society of Mechanical Engineers)

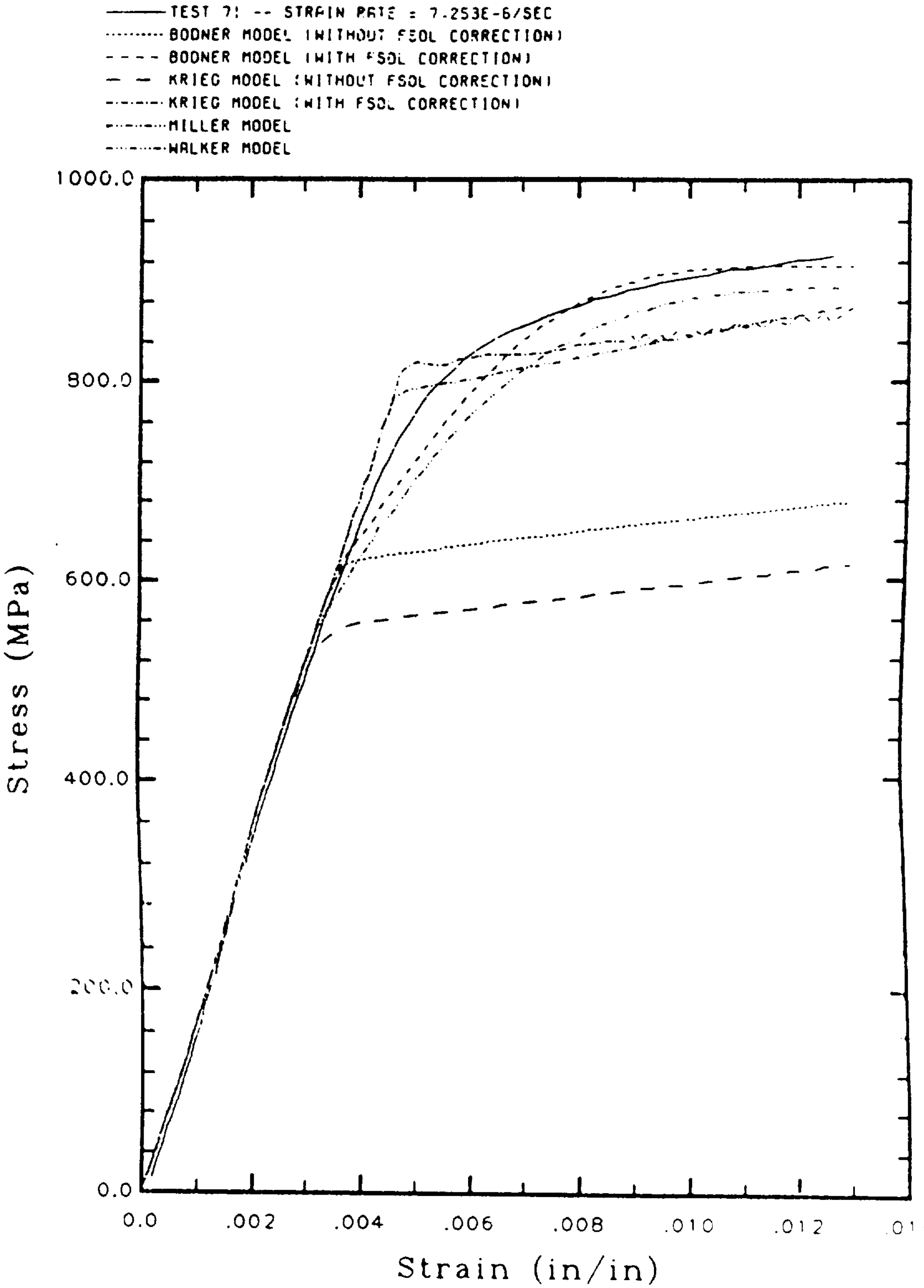

Fig. 5 Model Response for Inconel 718 at 1100°F at Constant Strain Rate $\dot{\varepsilon} = 7.293 \times 10^{-6}$ / sec (Courtesy American Society of Mechanical Engineers)

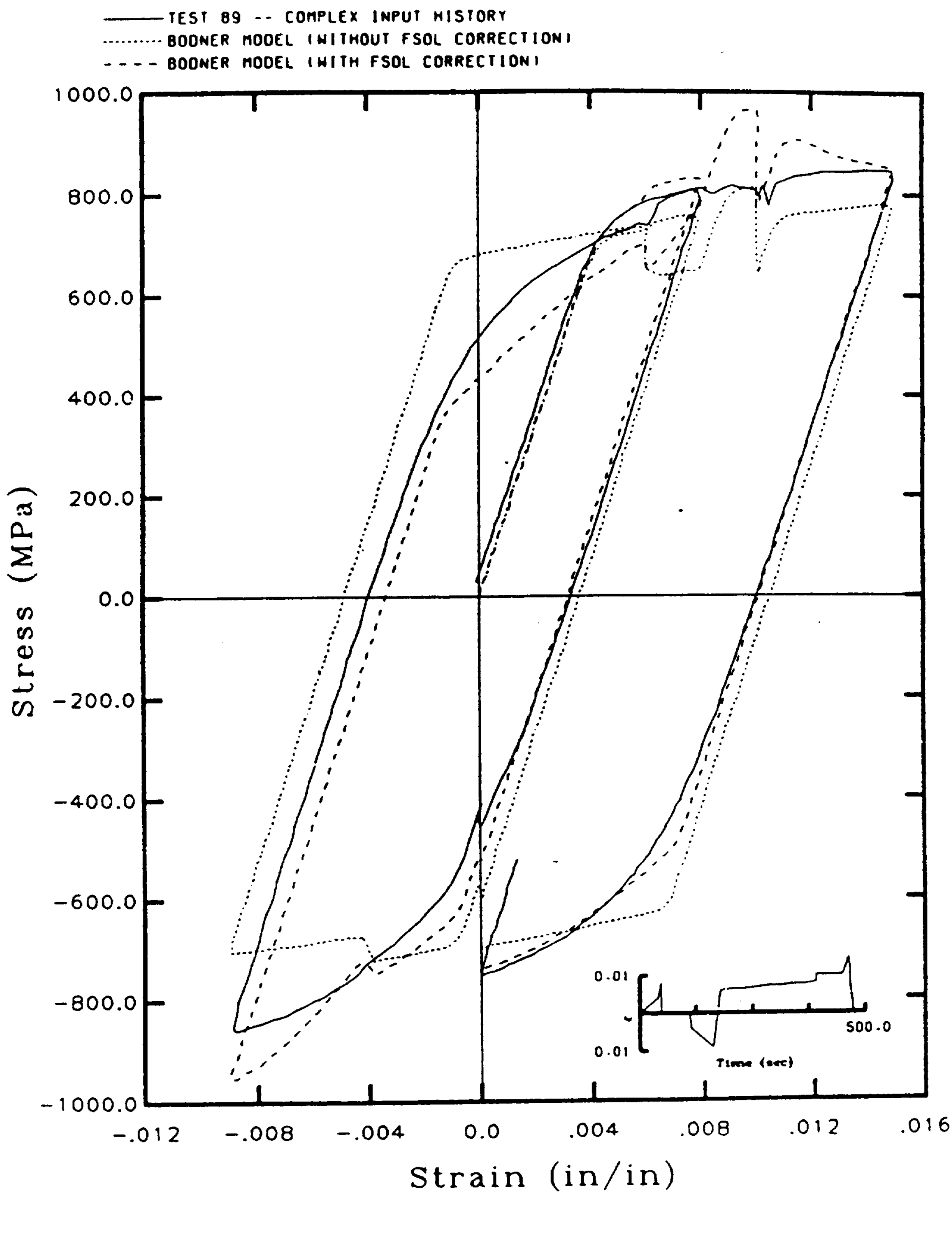

Fig. 6 Model Prediction Versus Experiment for the Complex Strain History Shown Above on Inconel 718 at 1100°F (Courtesy American Society of Mechanical Engineers)

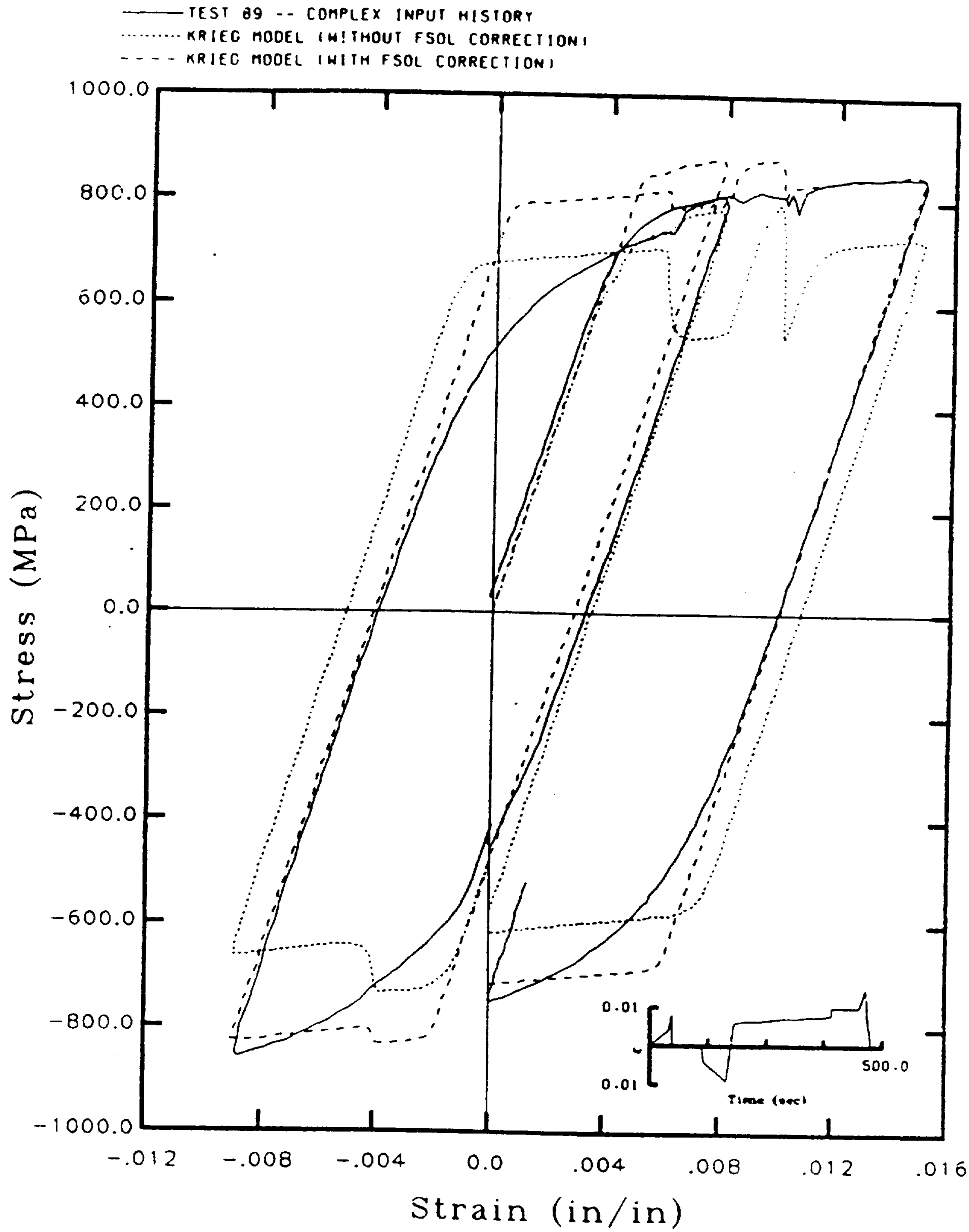

Fig. 7 Model Prediction Versus Experiment for the Complex Strain History Shown Above on Inconel 718 at 1100°F (Courtesy American Society of Mechanical Engineers)

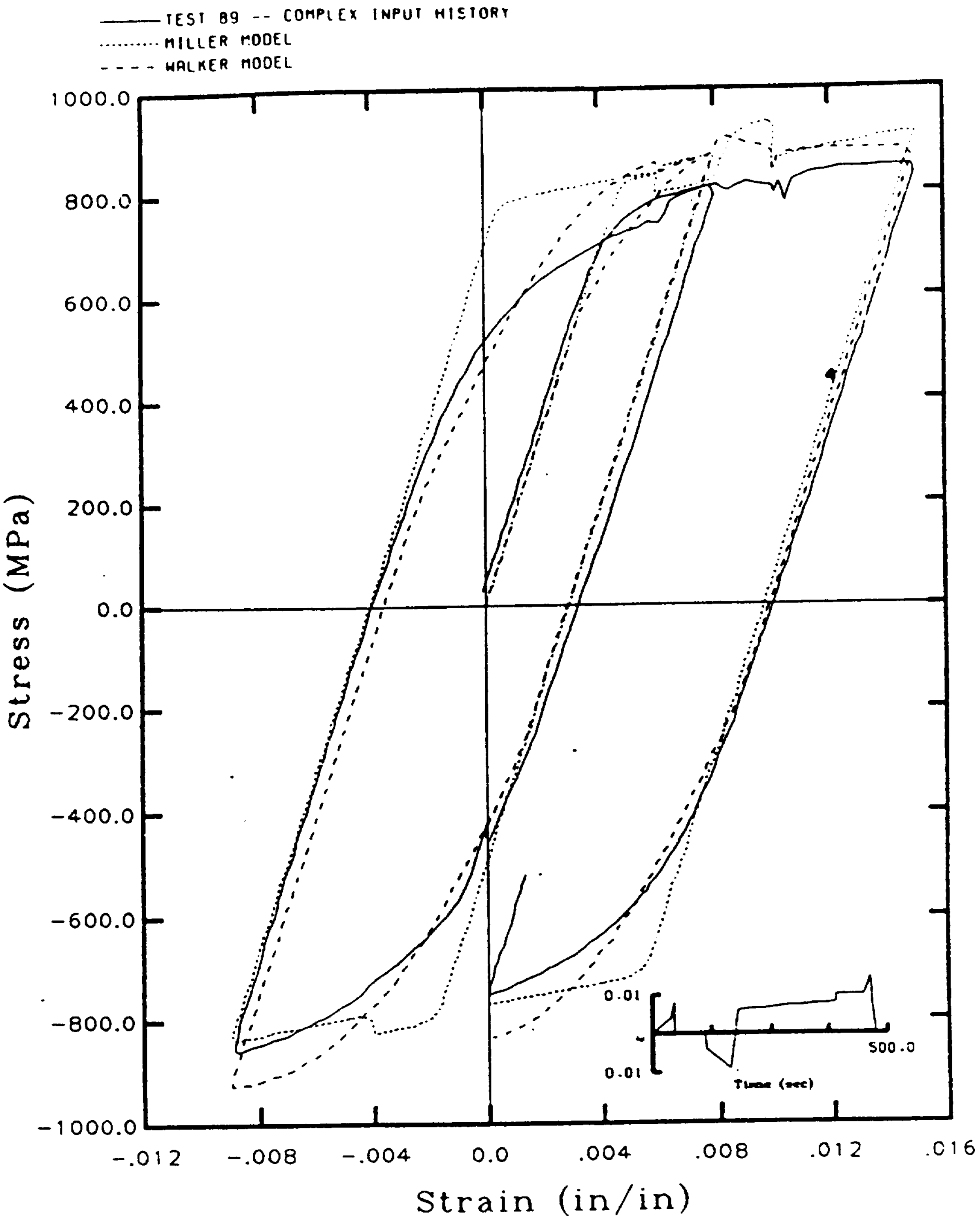

Fig. 8 Model Predictions Versus Experiment for the Complex Strain History Shown Above on Inconel 718 at 1100°F (Courtesy American Society of Mechanical Engineers)

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16. Abstract

Over the past two decades a number of thermomechanical constitutive theories have been proposed for viscoplastic metals. These models are in most cases similar in that they utilize a set of internal state variables which are locally averaged representors of microphysical phenomena such as dislocation rearrangement and grain boundary sliding. The state of development of several of these models is now at the point where accurate theoretical solutions can be obained for a wide variety of structural applications at elevated temperatures.

The purpose of this paper is threefold. First, the fundamentals of viscoplasticity are briefly reviewed and a general framework is outlined. Second, several of the more prominent models are reviewed in some detail. And third, predictions from models are compared to experimental results.

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