Task Title: A Finite Element Conjugate Gradient
FFT Method for Scattering

Investigators: Jeffery D. Collins, John Zapp, Chang-Yu Hsa and John L. Volakis

Period Covered: February 1990 - September 1990

ABSTRACT

An extension of the two-dimensional formulation developed last year is presented for a three dimensional body of revolution. With the introduction of a Fourier expansion of the vector electric and magnetic fields, a coupled two-dimenional system is generated and solved via the finite-element method. As before, an exact boundary condition is employed to terminate the mesh and the FFT is used to evaluate the boundary integrals for low O(n) memory demand when an iterative solution algorithm is used. Again, by virtue of the finite element method the algorithm is applicable to structures of arbitrary material composition.

Several improvements to our two-dimensional algorithm are also described. These include (1) modifications for terminating the mesh at circular boundaries without distorting the convolutionality of the boundary integrals, (2) the development of our own non-proprietery mesh generation routines for two-dimensional applications, (3) the development of preprocessors for interfacing SDRC IDEAS@ with the main algorithm, and (4) the development of post-processing algorithms based on the public domain package GRAFIC to generate 2D and 3D gray level and color field maps.

OBJECTIVE

The objective of this task is to develop innovative techniques and related software for scattering by three dimensional composite structures. The proposed analysis is a hybrid finite element-boundary integral method formulated to have an O(n) memory demand. This low storage is achieved by employing the FFT to evaluate all boundary integrals and resorting to an iterative solution algorithm. Particular emphasis in this task is the generation of software applicable to airborne vehicles and the validation of these by comparison with measured and other reference data. Because the approach is new, a step by step development procedure has been proposed over a three-year period. During the first year the technique was developed and implemented for two-dimensional composite structures. Support software for the two-dimenional analysis such as pre- and post-processor routines were developed during the second year and a formulation was also developed and implemented for three-dimensional bodies of revolution. Finally, during the third year, we will develop, implement, and test the method for arbitrary three dimensional structures.

BACKGROUND

Interest in three-dimensional (3-D) methods has increased in recent years, however, the associated demands in computation time and storage are often prohibitive for electrically large 3-D bodies. Vector and concurrent (i.e. hypercube, connection, etc.) computers are beginning to alleviate the first of these demands, but a minimization of the storage requirements is essential for treating large structures.

The traditional Conjugate Gradient Fast Fourier Transform (CGFFT) method [1]-[4] is one such frequency domain solution approach which requires O(n) storage for the solution of n equations. This method involves the use of FFTs whose dimension equals that of the structure under consideration [5]-[7] and, therefore, demands excessive computation time when used in an iterative algorithm. Also, the standard CGFFT requires uniform rectangular gridding that unnecessarily includes the impenetrable portions of the scatterer. With these issues in mind, a new solution approach is propsed for solving scattering problems. The proposed method will be referred to as the Finite Element-Conjugate Gradient Fast Fourier Transform (FE-CGFFT) method.

During last year's effort the FE-CGFFT method was developed for two-dimensional scatterers where the finite element mesh was terminated at a rectangular box. Inside the box boundaries, Helmholtz equation is solved via the finite element method and the boundary constraint is obtained by an appropriate integral equation which implicitly satisfies the radiation condition. Along the parallel sides of the box, this integral becomes a convolution and is, therefore, amenable to evaluation via the FFT. The dimension of the required FFT in this hybrid method is one less than the dimensionality of the structure thus, making it attractive for 3-D simulations. Also, because it incorporates the finite element method, the FE-CGFFT formulation remains valid regardless of the structure's geometry and material composition.

The proposed method described in the University of Michigan Report 025921-6-T (see also [8]) is similar to the moment method version developed by Jin [9]. Jin's method was in turn based on work published in the early 70's by McDonald and Wexler [10] who introduced an approach to solve unbounded field problems. The proposed method is also similar to other methods (a few of which will be mentioned here), neither of which provides a storage reduction comparable to the proposed FE-CGFFT method. The unimoment method [11] uses finite elements inside a fictitious circular boundary and an eigenfunction expansion to represent the field in the external region. The coefficients of the expansion are then determined by enforcing field continuity at the finite element (FE) mesh boundary. The coupled finite element-boundary element method [12] uses the finite

2

element method within the boundary and the boundary element method to provide the additional constraint at the termination of the mesh. Unlike the proposed method, the solution in [12] was accomplished by direct matrix inversion (as in [9]), and the outer mesh boundary is not rectangular to take advantage of the FFT for the evaluation of the boundary integrals.

PROGRESS

The proposed FE-CGFFT formulation was implemented last year (see Figs. 1 and 2) but as can be expected, the rectangular mesh boundary does not always lead to the most efficient formulation, particularly when dealing with structures whose outer boundary is not rectangular. Because of this, during this year we developed and implemented a formulation which permits mesh termination at circular (see Fig. 3) boundaries for the 2D case with the corresponding boundary enclosure being a pillbox for the 3D case (see Fig. 4). As before, these boundaries lead to convolutional integrals and do not therefore destroy the O(n) memory demand of the method. The FE-CGFFT formulation relating to circular (and ogival) boundary enclosures is described in the University of Michigan report 025921-11-T (see also [13]) and results based on its implementation are shown in Figures 5 and 6. Fig. 5 shows bistatic scattering patterns for a coated circular cylinder with a conductor radius of 3λ, 0.05λ coating thickness and material properties $\varepsilon_r = 3\text{-}j5$ and $\mu_r = 1.5 - j0.5$. The agreement with the series solution result is excellent. In Fig. 6 a backscatter pattern is shown for a $\lambda/2 \times 1\lambda$ conducting ogive (see Fig. 3). In comparison with the moment method results, the agreement is again excellent. Additional results are given in Figure 7 for a missle-like shape scatterer.

Pre- and Post-Processing Algorithms

The availability of pre- and post-processing algorithms is crucial for the generation of the geometry and display of results in a graphical form. Generally, it is desirable that these tasks be done with a graphical user interface (GUI) and possibly in an X-window setting. Part of this year's effort was therefore devoted to the development of such algorithms and/or interfaces for the more sophisticated commercial pre- and post-processing packages.

3

For the most part, there exist commercial geometry, mesh generation and post-processing packages which are highly interactive and graphical. Nevertheless, there is always a need for a suitable interface or data interpretor between the commercial packages and the solution algorithm described in the previous section. The specific package interfaced with the computational algorithm was SDRC IDEAS@ and the selection of this was based on its availability on the U-M Network, its versatility, graphical user interface, and capability to generate meshes for 2D and 3D structures. Furthermore, a new version of SDRC IDEAS@, to be released soon, will support X-windows. IDEAS was developed for mechanical design purposes, but its geometry and finite element mesh generation modules are particularly suited for our needs. The geometry is defined graphically using the area (for 2D) or solids (for 3D) modeling capability provided by the module Geomod. Alternatively, the user may choose to enter the geometry in terms of individual points, surve segments (for 2D) or surfaces (for 3D). Once the geometry is entered, mesh areas or regions are specified and either a mapped mesh or free mesh can be generated. Further, individual nodes and/or elements may be inputted as desired using the CREATE command. Once the mesh is generated, two files are created, one containing the nodes and their corresponding coordinates, and another specifying the nodes of each element. These files are then read by an interpretor which creates a new input file compatible with the format required by the computational modules.

Examples of two-dimensional meshes generated with SDRC IDEAS@ were shown in Figures 1, 2b and 7, and these are in a form suitable for the FE-CGFFT analysis. Some three-dimensional meshes are also displayed in Figure 8 for an ogive and missile-like structures. As seen, the 3D meshes are terminated at a cylindrical surface, tightly enclosing the scatterer which is the intended enclosure for the proposed FE-CGFFT method. A brief manual for geometry and mesh generation using SDRC IDEAS@ is currently being prepared.

SDRC IDEAS@ is a rather sophisticated package and its use is certainly preferable for 3D modeling and mesh generation. For 2D mesh generation, though, it is possible to construct a non-proprietory package without much effort, and which is also simpler without a serious sacrifice in versatility. Clearly, the primary reason for resorting to such an algorithm is to permit mesh generation at sites not having a license for SDRC IDEAS@. The specific geometry and mesh generation package developed for this purpose is based on the algorithm described in [14]. Examples of free meshes generated by this package are displayed in Figures 2a and 3. The package is interactive/menu driven and can be readily used without much preparation. The mesh can be displayed in the Apollo screens or a postscript file may be generated for display on other workstations. At present,

4

visualization cannot be done in an X-window but this capability is planned for early next year.

A variety of post-processing capabilities have also been exmployed for a graphical display of the output data. The output is either in the form of echowidth plots as a function of observation and/or incidence angle or in the form of gray level field maps. Color instead of gray level field maps can also be generated at those workstations which support this feature. Generally, all echowidth plots are generated and displayed using standard software, and each workstation provides its own selection. To generate and visualize the gray level and color field maps we employed the public domain package GRAFIC. An example of a gray level plot is shown in Figure 9. This is generated from a postscript file and can thus be displayed on other sponsor machines.

3D Algorithm for Bodies of Revolution

Before extending the presented 2D formulation to scattering by arbitrary 3D structures, it is instructive that we first consider its implementation for a restrictive class of 3D bodies. In particular, during this year an algorithm was developed for scattering by inhomogeneous bodies of revolution. Because of the symmetry of this structure, it is only necessary to discretize it in a single plane slicing the structure as shown in Figure 10. A knowledge of the fields over this cross-section is then sufficient to generate the fields everywhere by employing a Fourier expansion in the azimuthal direction. Clearly, the discretization can be accomplished using a 2D mesh generation routine and this is the primary reasons for considering this class of structures. Also, the storage requirements are comparable to that of the 2D formulation although, as expected, the computational intensity is much greater.

The mathematical details pertaining to the BOR formulation will be presented at sufficient detail in an upcoming technical report. Briefly, the method consists of the following steps

1) A Fourier expansion is used to expand the fields in terms of those over a single cross-section of the BOR.

2) The fields in the finite-element region are then formulated via the Coupled Azimuthal Potential (CAP) method as described in [15]. This results in a banded finite-element matrix in terms of the boundary fields.

5

3) The boundary fields are formulated via the usual Stratton-Chu equations which are then discretized via the boundary element method. As before, the boundary enclosure is chosen to yield convolutional integrals computed via the FFT.

4) The finite-element and boundary-element systems are coupled via the boundary fields and solved via the CGFFT method maintaining an $O(n)$ storage requirement, where n is the number of nodes over a single cross section of the BOR.

Presently, a code has been written based on the proposed formulation and is in the final stages of the validation process.

CONCLUSIONS

So far, we have formulated and implemented the FE-CGFFT method for a variety of 2D structures and we are now in the process of completing its implementation for BOR structures. The method was proposed because of its versatility, accuracy and low memory demand in comparison with other methodologies, and all of these attributes have been demonstrated in the testing and validation process. It is therefore a promising method for general 3D implementations to be considered in the following year.

TRANSITIONS

The validation of the 3D BOR formulation is expected to be completed by early Fall 1990. We will then begin the development and implementation of the formulation for arbitrary 3D structures. This implementation is expected to be much more involved than those considered earlier and the same if true for the geometry and mesh generation. It is therefore, likely that the proposed 3D implementation may not be completed by the end of the 3rd year. Also, because of the need to generate suitable pre-processing and post-processing algorithms additional man-hours are required during the third year of this effort. Most likely, a practical user-oriented validated and benchmarked code will not be available until the fourth year. As part of this effort it would also be desirable to design and develop a graphical user interface (GUI) compatible with the X-window platforms. The GUI is

particularly necessary for the 3D analysis package. Otherwise, the user would be faced with a long list of subprograms whose interfacing would likely be cumbersome.

The proposed FE-CGFFT formulation employs an exact boundary condition at the termination of the mesh. This eliminates a need to extend the mesh far from the scatterer leading to a substantial savings in storage requirements. However, this storage reduction and solution accuracy is achieved at the expense of computational complexity and intensity. In many cases, though, where accuracy is not of primary concern, one could resort to the use of non-exact (i.e. absorbing boundary conditions), for terminating the mesh. This leads to completely sparse matricies which can be solved more efficiently using special purpose algorithms. In the future, it is therefore desirable to include this formulation as an option to the user. Also, a new class of boundary conditions are currently being investigated for terminating the mesh.

REFERENCES

[1]   T.K. Sarkar, E. Arvas and S.M. Rao, "Application of FFT and the conjugate gradient method for the solution of electromagnetic radiation from electrically large and small conducting bodies," IEEE Trans. Antennas Propagat., Vol. AP-30, pp. 409-418, May 1986.

[2]   T.J. Peters and J.L. Volakis, "Scattering from thin planar layers of arbitrary composition," IEEE Trans. Antennas Propagat., AP-36, pp. 518-526, April 1988.

[3]   M.F. Cátedra, J.G. Cuevas and L. Nuño, "A scheme to analyze conducting plates of resonent size using the conjugate gradient method and the fast Fourier transform," IEEE Trans. Antennas Propagat., AP-36, pp. 1744-1752, Dec. 1988.

[4]   C.Y. Shen, K.J. Gloner, M.I. Sancer and A.D. Varvatsis, "The discrete Fourier tranform method of solving differential-integral equations in scattering theory," To appear in IEEE Trans. Antennas Propagat.

[5]   C.Y. Shen, "Application of the discrete Fourier transform method to thin dielectric structures," IEEE Trans. Antennas Propagat., Vol. AP-37, pp. 1277-1283, Oct. 1989.

[6] M.F. Cátedra, E. Gago, and L. Nuño, "A numerical scheme to obtain the RCS of three-dimensional bodies of resonant size using the conjugate gradient method and the fast Fourier transform," IEEE Trans. Antennas Propagat., AP-37, pp. 528-537, May 1989.

[7] J.E. Bussoletti, F.T. Johnson, K.W. Sidwell, B.L. Everson, D.P. Young, R.H. Burkart and S.S. Jamant, "EM-TRANAIR: Computer a program for the solution of Maxwell's equations in three dimensions: Volume I, theory manual," Report No. AFWAL-TR-87-3802, prepared by Boeing Military Airplane Co. for the U.S.A.F. Wright Aeronautical Labs, Sept. 1988.

[8] J.D. Collins, J.L. Volakis and J.M. Jin, "A Combined Finite Element-Boundary Integral Formulation for Solution of Two-Dimensional Scattering Problems via CGFFT," IEEE Trans. Antennas Propagat., Nov. 1990.

[9] J.M. Jin and V.V. Liepa, "Application of hybrid finite element method to electromagnetic scattering from coated cylinders," IEEE Trans. Antennas Propagat., Vol. AP-36, pp. 50-54, Jan. 1988.

[10] B.H. McDonald and A. Wexler, "Finite-element solution of unbounded field problems," IEEE Trans. Microwave Theory Tech., Vol. MTT-20, pp. 841-847, Dec. 1972.

[11] K.K. Mei, "Unimoment method of solving antenna and scattering problems," IEEE Trans. Antennas Propagat., Vol. AP-22, pp. 760-766, Nov. 1974.

[12] K.L. Wu, G.Y. Delisle, D.G. Fang and M. Lecours, PIER2: Finite Element and Finite Difference Methods in Electromagnetic Scattering, (Eds. J.A. Kong and M.A. Morgan), Elsevier: New York, Ch. 3, 1990.

[13] J.D. Collins, J.M. Jin and J.L. Volakis, "A combined finite element-boundary element formulation for solution of two-dimensional problems via CGFFT," Electromagnetics, 1990.

[14] J.C. Cavendish, "Automatic triangulation of arbitrary planar domains for the finite element method," Int. J. Numerical Methods Engineering, Vol. 8, pp. 679-696, 1974.

PUBLICATIONS RESULTING FROM THIS RESEARCH

Reports written (to date) related to this Task

- J.D. Collins and J.L. Volakis, "A Combined Finite Element and Boundary Integral Formulation for Solution via CGFFT of 2-Dimensional Scattering Problems." Univ. of Michigan Radiation Lab Technical Report 025921-6-T, August 1989.

- J.M. Jin and J.L. Volakis, "A Finite Element-Boundary Integral Formulation for Scattering by a Cavity-Backed Aperture in a Ground Plane," Univ. of Michigan Radiation Lab Technical Report 025921-10-T, Feb. 1990.

- J.D. Collins and J.L. Volakis, "A Combined Finite Element and Boundary Integral Formulation for Solution via the CGFFT," Univ. of Michigan Radiation Lab Technical Report 025921-11-T, Feb. 1990.

- Changyu Hua, "AUTOMESH-2D User's Manual," University of Michigan Radiation Laboratory Report 025921-15-T, August 1990.

- John Zapp, "User's Manual for Finite Element Mesh Generation Using SDRC IDEAS," University of Michigan Radiation Laboratory Report 025921-16-T.

Papers written (to date) related to this Task

- J.D. Collins, J.L. Volakis and J.M. Jin, "A Combined Finite Element and Boundary Integral Formulation for Solution via CGFFT of 2-Dimensional Scattering Problems," *IEEE Trans. Antennas Propagat.*, Oct. 1990.

- J.M. Jin and J.L. Volakis, "A New Technique for Characterizing Diffraction by an Inhomogeneously Filled Slot of Arbitrary Cross Section in a Thick Conducting Ground Plane," *IEE Electronics Letters*, Vol. 25, No. 17, pp. 1121-1123, 17th August 1989.

- J.M. Jin and J.L. Volakis, "TE Scattering by an Inhomogeneously Filled Aperture in a Thick Conducting Plane," *IEEE Trans. Antennas Propagat.*, 1990.

- J.M. Jin and J.L. Volakis, "TM Scattering by an Inhomogeneously Filled Aperture in a Thick Conducting Plane," *Proc. Inst. Elec. Eng.*, part H, Vol. 137, pp. 153-159, June 1990.

- J.D. Collins, J.M. Jin and J.L. Volakis, "A Combined Finite Element-Boundary Element Formulation for Solution of Two-Dimensional Problems via CGFFT," to appear in *Electromagnetics*.

- J.M. Jin and J.L. Volakis, "Electromagnetic Scattering and Transmission through a Three-Dimensional Slot in a Thick Conducting Plane," submitted to *IEEE Trans. Antennas Propagat.*

- J.M. Jin and J.L. Volakis, "A Finite Element-Boundary Element Method for Scattering by Two and Three Dimensional Structures," submitted to *Proceedings of the IEEE*.

- J.M. Jin and J.L. Volakis, "A Unified Approach to Integral Formulations for Electromagnetic Scattering," submitted to *IEEE Trans. Antennas Propagat.*

Conference Papers/Presentations (to date) related to this Task

- J.D. Collins and J.L. Volakis, "A Combined Finite Element and Boundary Integral Formulation for Solution via CGFFT of 2-Dimensional Scattering Problems," 1989 IEEE AP-S/URSI Symposium, San Jose, CA.

- J.M. Jin and J.L. Volakis, " A FEM/BEM Formulation for a CG-FFT Solution of 2-D Scattering by Grooves and Thick Slots," presented at the 1990 IEEE AP-S/URSI International Symposium, Dallas, TX, URSI Digest p. 260.

- J.M. Jin and J.L. Volakis, "A FEM/BEM Formulation for a CG-FFT Solution of 3-D Scattering by a Cavity-Backed Aperture in a Ground Plane," presented at the 1990 IEEE AP-S/URSI International Symposium, Dallas, TX, AP-S Digest, pp. 1726-1729.

- J.D. Collins and J.L. Volakis, "A Combined Boundary Integral Finite Element Formulation for Solution of Three Dimensional Scattering via Conjugate Gradient Fast Fourier Transform Method," 1990 IEEE AP-S/URSI Symposium, Dallas, TX, URSI Digest p. 263.

- J.M. Jin, J.L. Volakis and J.D. Collins, "A Finite Element Boundary Integral Formulation for Scattering by Two and Three Dimensional Structures," to be presented at the 1990 URSI General Assembly, Session B4, Prague, Chechoslovakia.

- J.M. Jin and L. Volakis, "A Finite Element-Boundary Integral Formulation for Scattering by a Three Dimensional Aperture in a Thick Conducting Plane," Submitted to the 4th Biennial IEEE Conference on Electromagnetic Field Computation, Oct. 1990, Toronto.
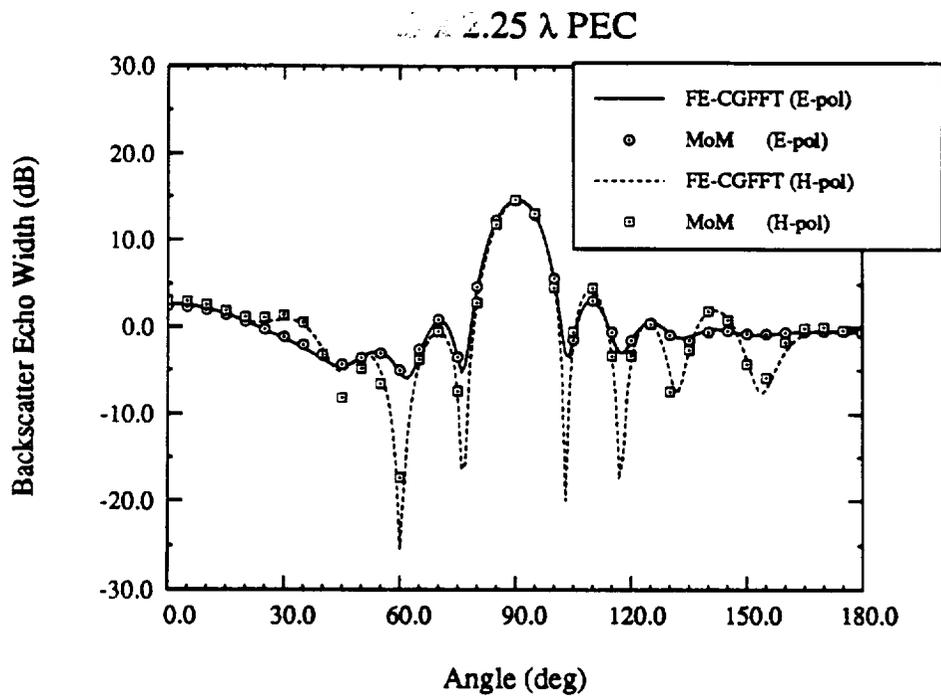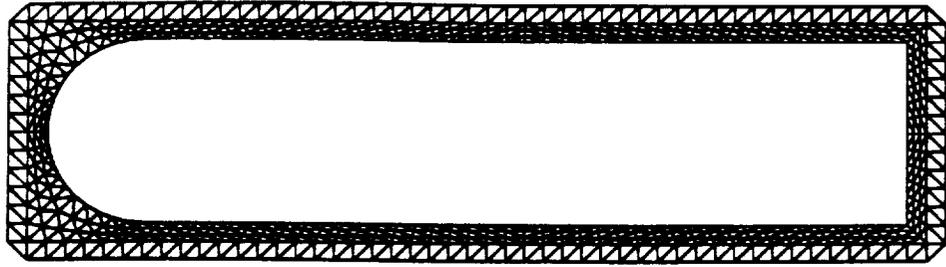
Figure 1: E_z and H_z backscatter echowidth patterns for the illustrated perfectly conducting cylinder.
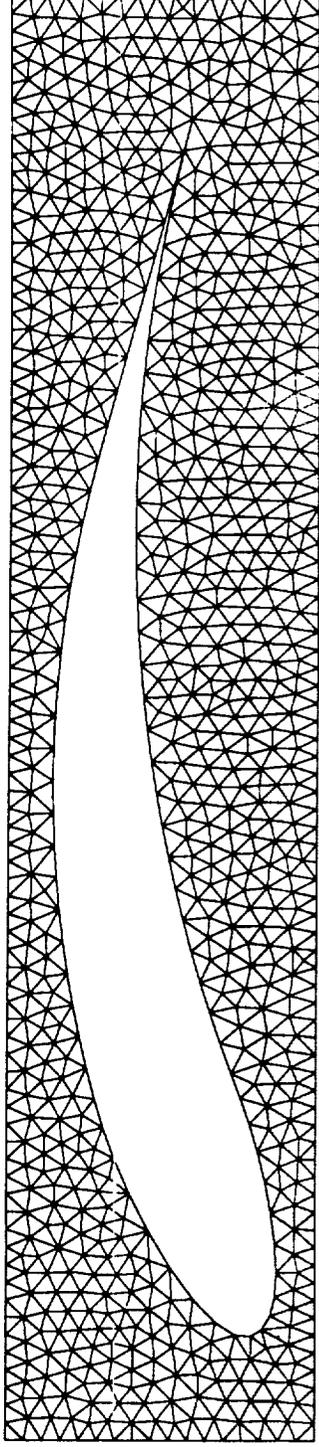
Figure 2(a):   Example of a rectangular mesh enclosing a wing cross-section.

COATED OGIVE

0.05λ coating $\left(\begin{array}{l} \epsilon_r = 5 - j3 \\ \mu_r = 1.5 - j0.5 \end{array} \right)$
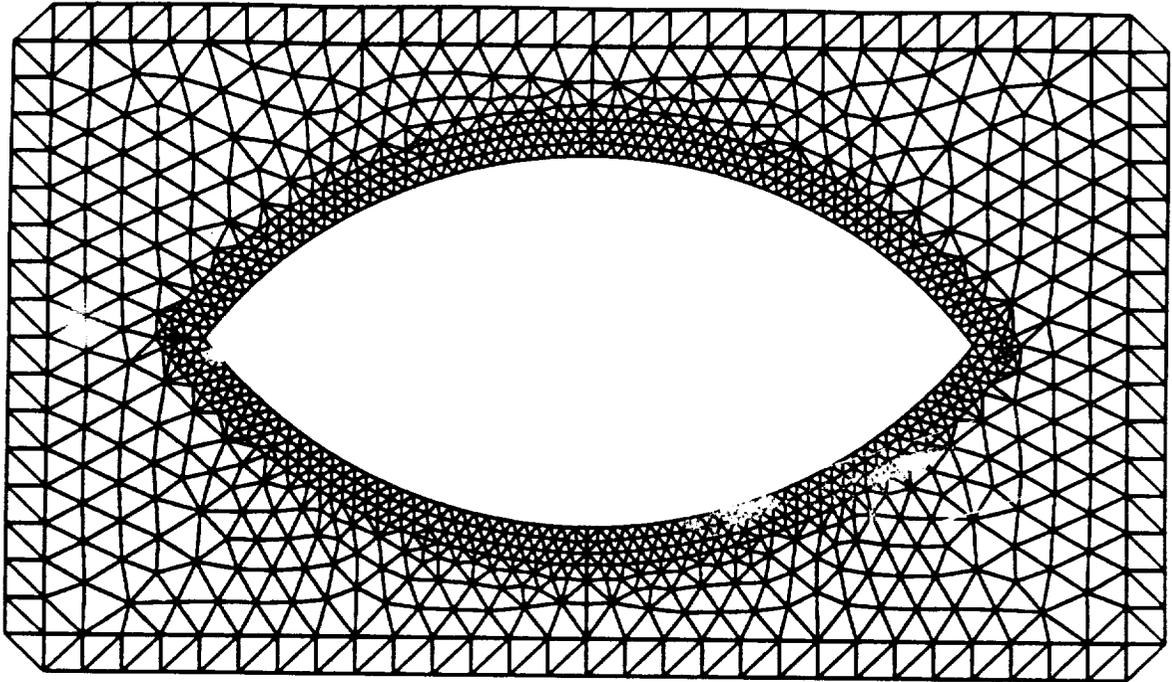
Figure 2(b):   Example of a rectangular mesh enclosing a coated ogive.

FILE: ogive_leo_out
CREATION DATE: 7 Aug 1990
STRUCTURE: 0.5 x 1 λ coated conductor
ENCLOSURE: rectangle

number of nodes: 1084
number of elements: 1936
nodes on pec boundary: 140
nodes on obs boundary: 92

nodes on unknowns (E-pol): 944
nodes on unknowns (H-pol): 1084
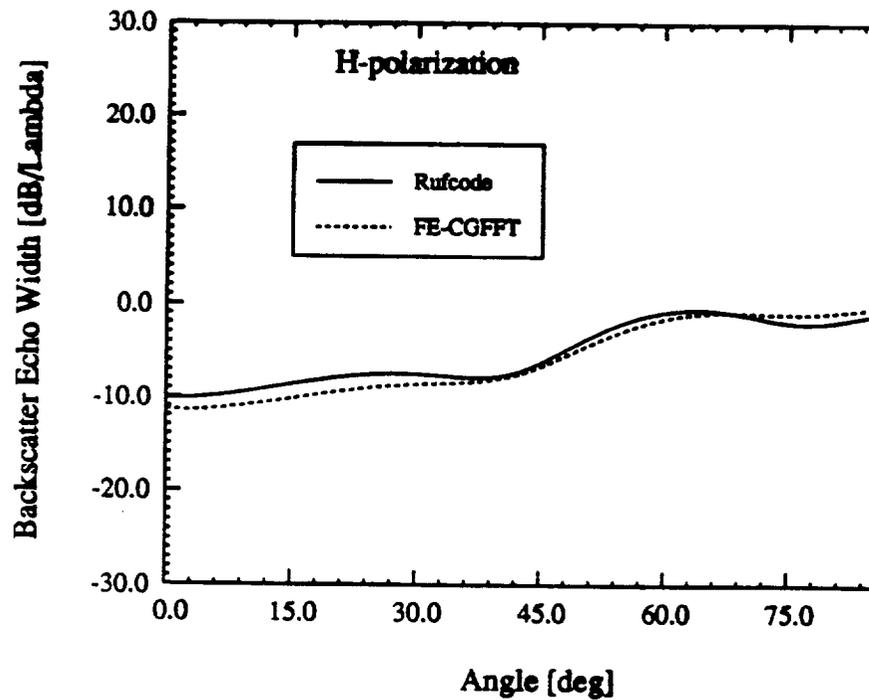
Figure 2(c):   H-polarization backscatter pattern for the coated ogive in
Fig. 2(b).
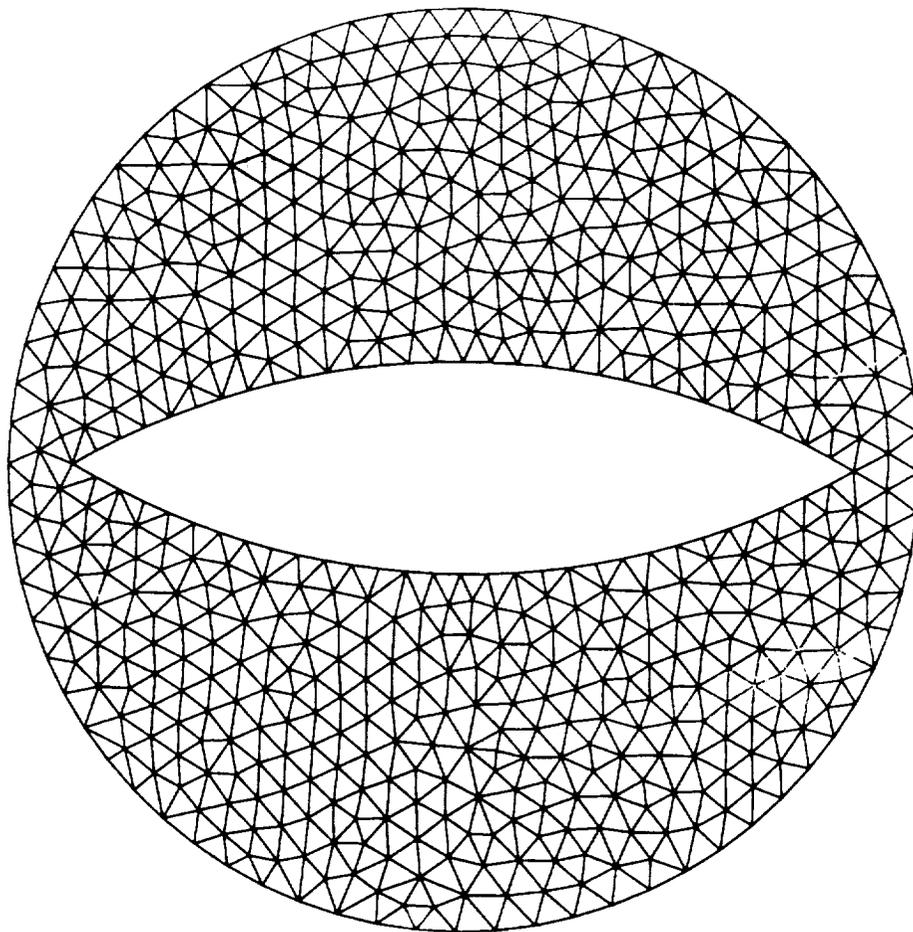
Figure 3: Example of a circular mesh enclosing an ogival cylinder.
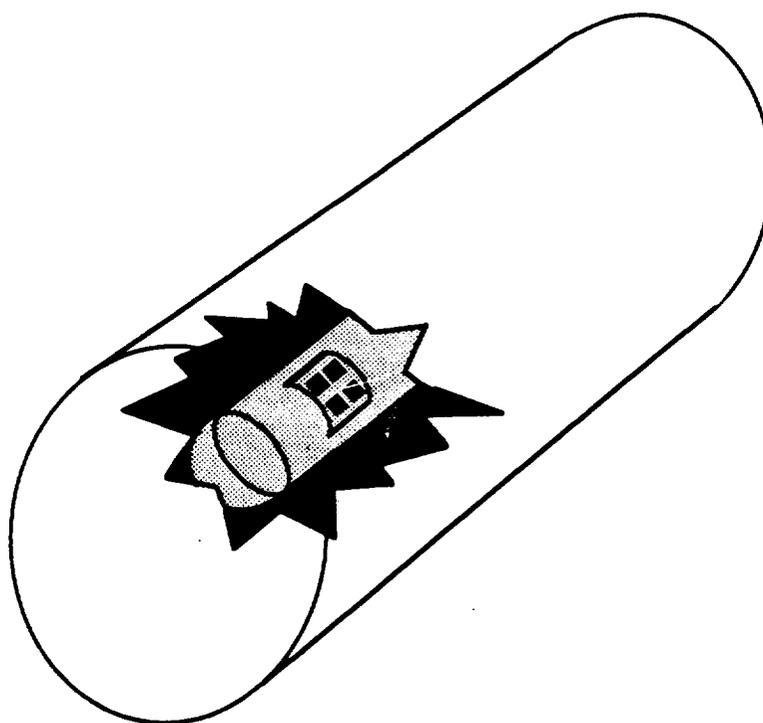
Figure 4: Three-dimensional finite-cylinder enclosure.

Coated Conducting Cylinder R=3 λ (E-pol)

Coated Conducting Cylinder R=3 λ (H-pol)
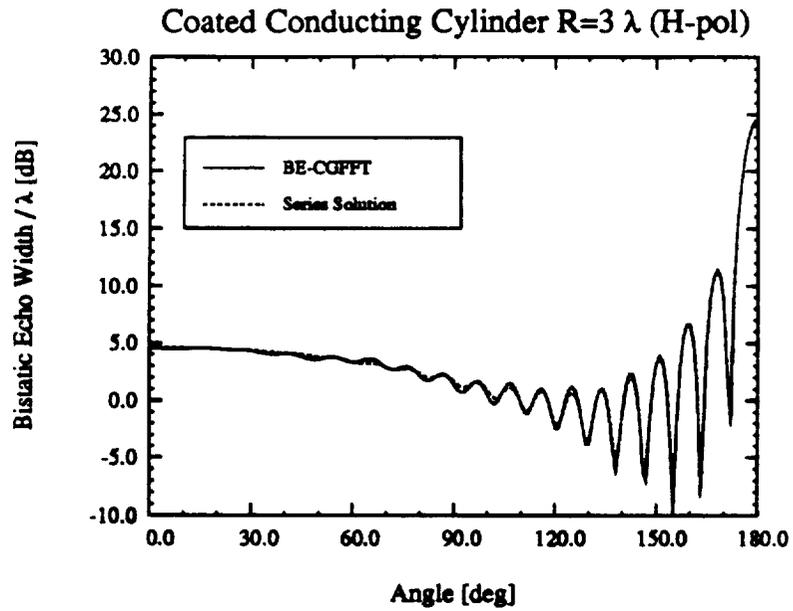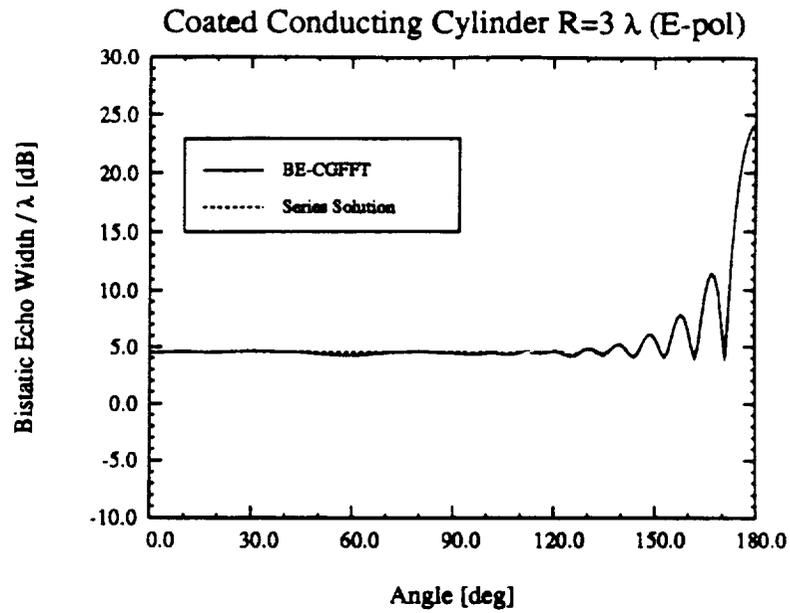
Figure 5: E_z and H_z bistatic echowidth from a coated circular cylinder with
a conductor radius of 3λ and coating thickness of 0.05λ with
material properites $\varepsilon_r = 5 - j5$, $\mu_r = 1.5 - j0.5$.

.5 x 1.λ PEC Ogive (E-pol)

.5 x 1.λ PEC Ogive (H-pol)
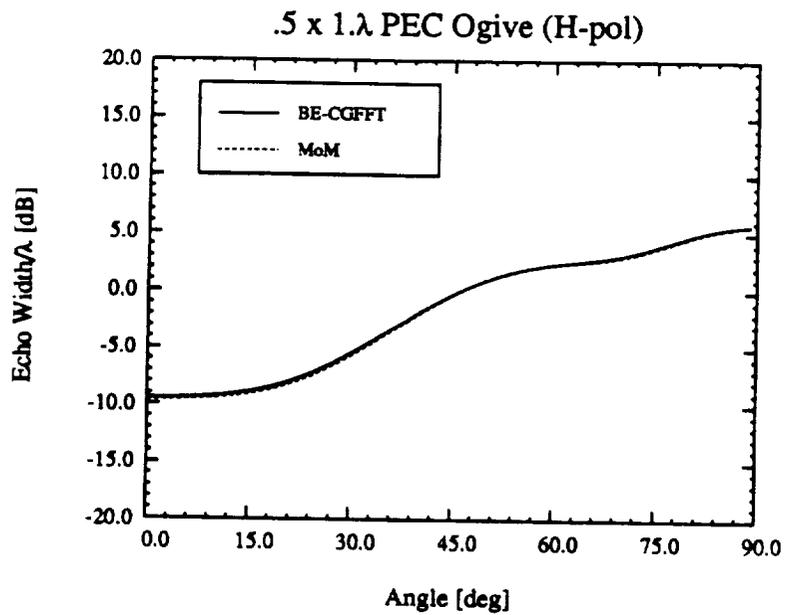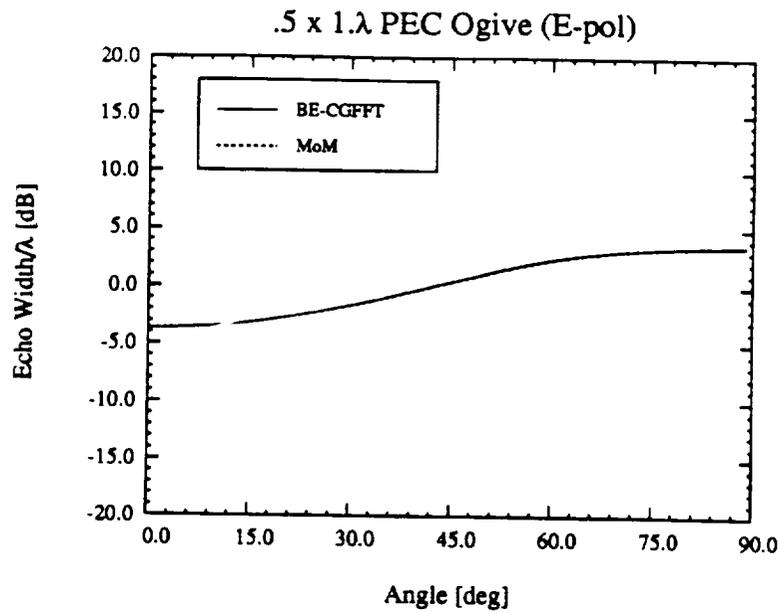
Figure 6: E_z and H_z backscatter echowidth from a $\lambda/2$ x 1λ perfectly conducting ogive.
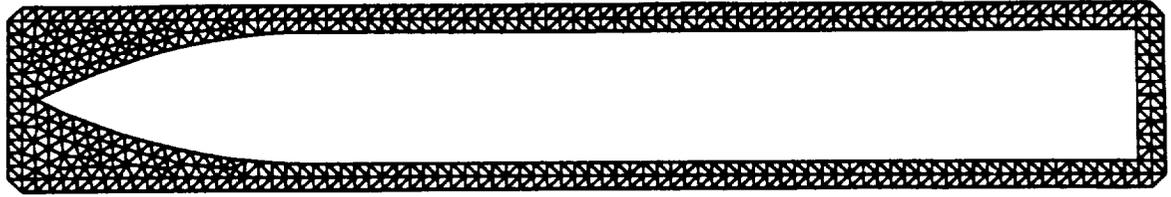
Missile Structure

FILE: missile_out
CREATION DATE: 1 Aug 1990
STRUCTURE: missile shape
ENCLOSURE: rectangle

number of nodes: 636
number of elements: 908
nodes on pec boundary: 172
nodes on obs boundary: 192

nodes on unknowns (E-pol): 464
nodes on unknowns (H-pol): 636
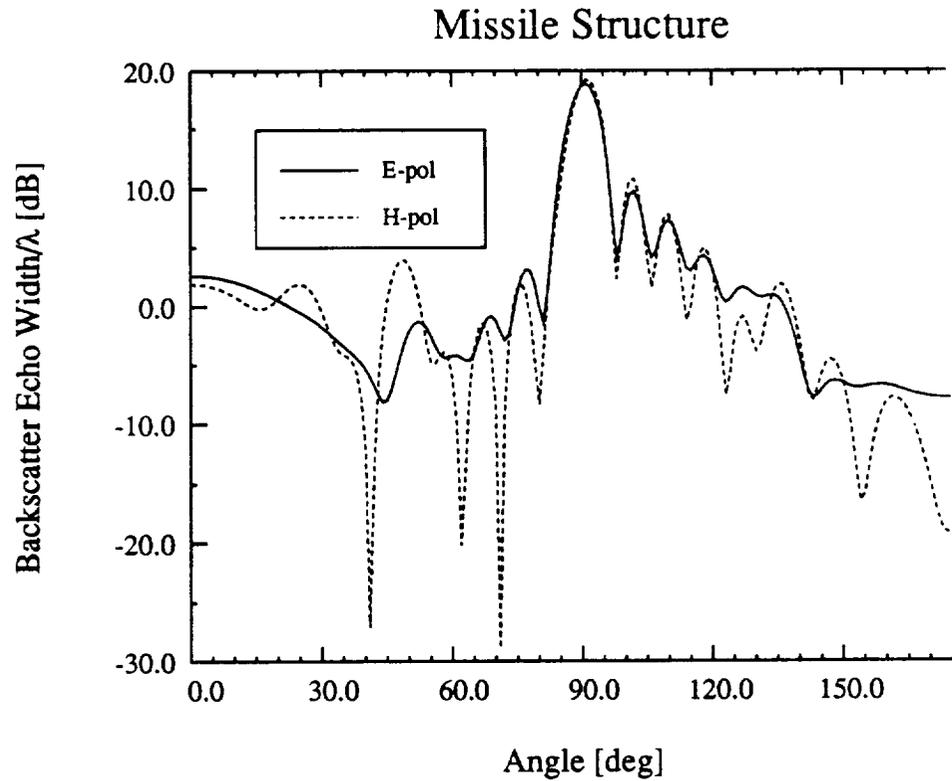
Figure 7:   Backscatter patterns for a missile-like perfectly conducting and coated cylinder.
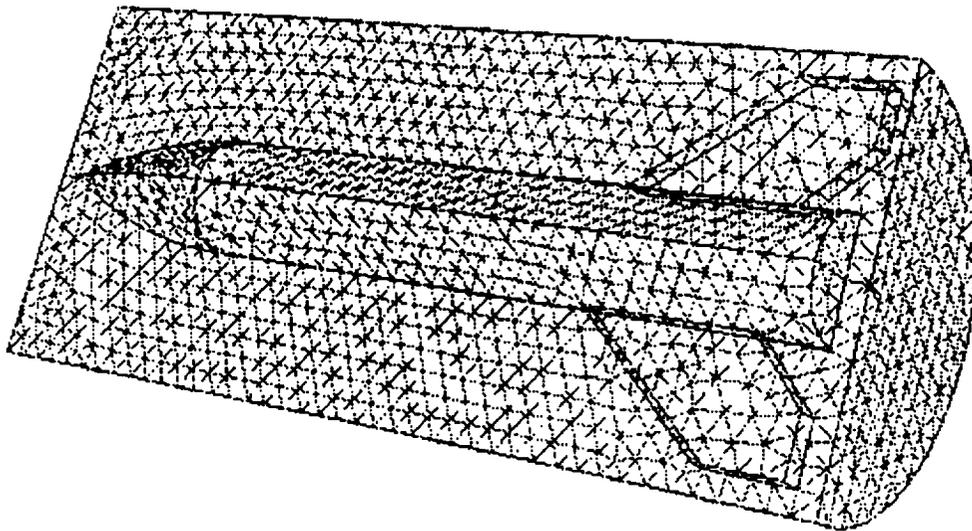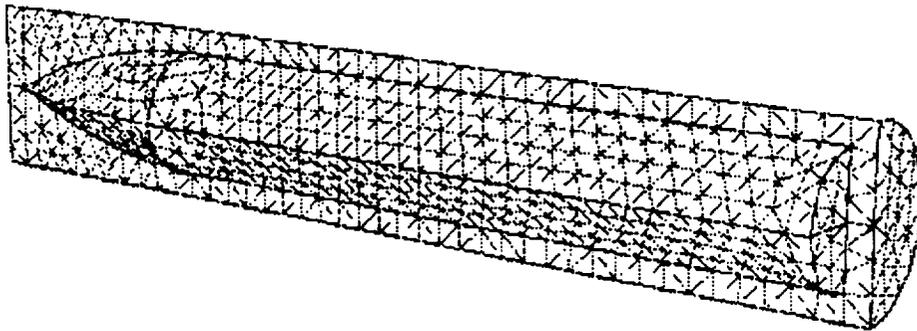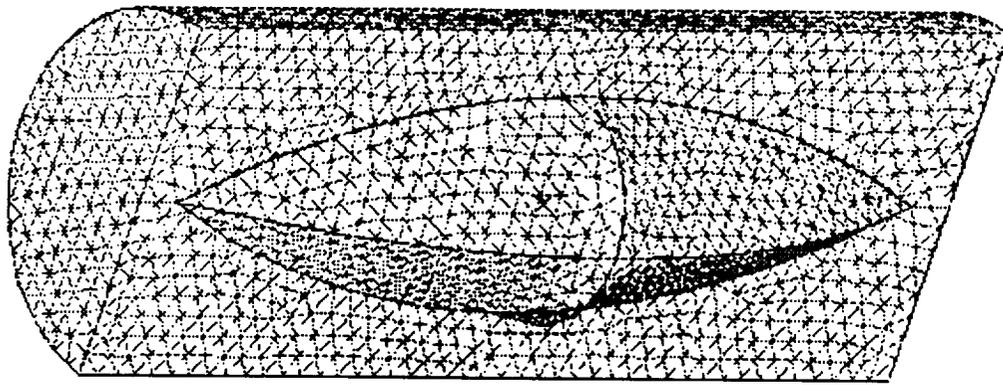
Figure 8: 3D meshes for an ogive and missile-like structures.
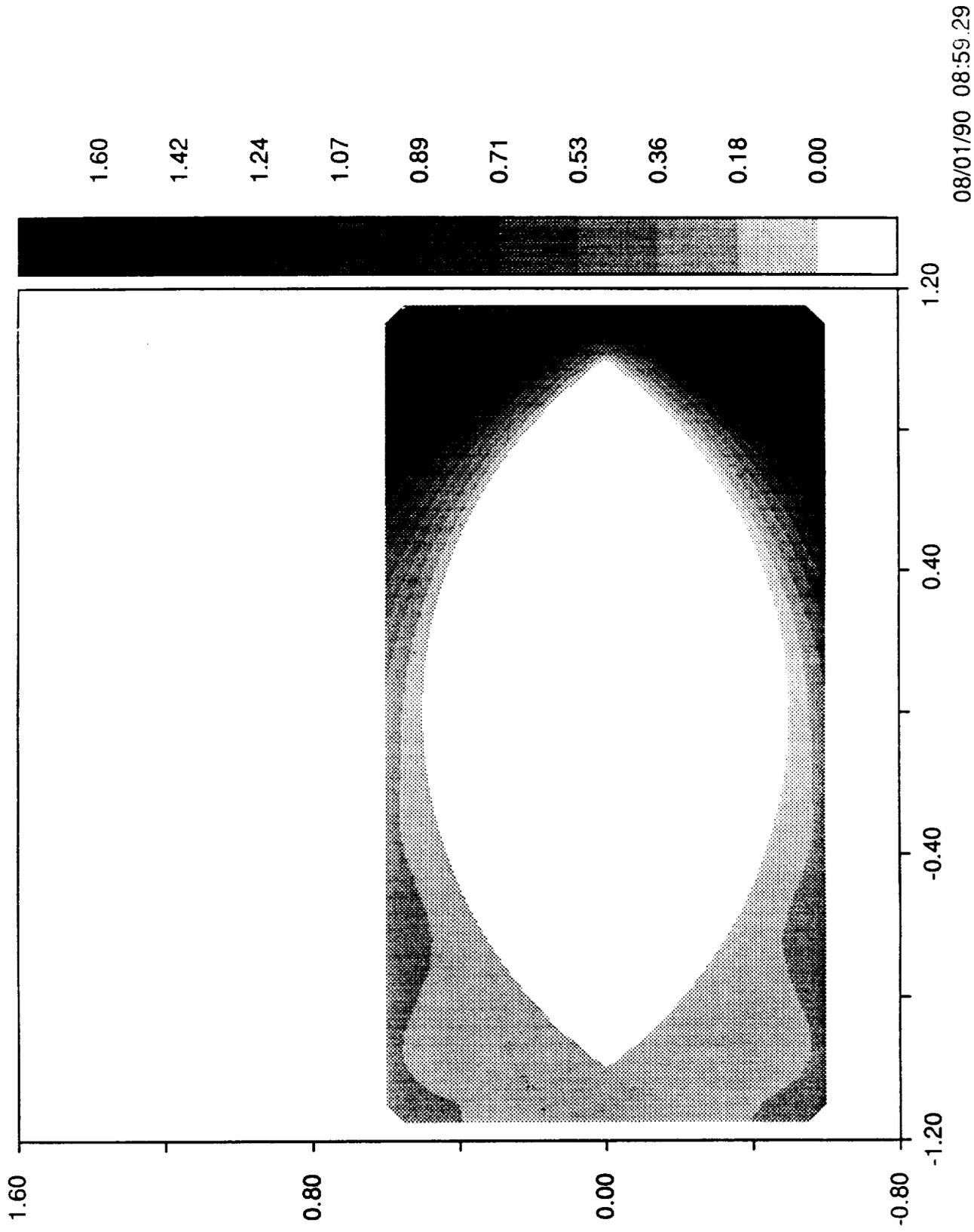
Figure 9:   Gray level plot of the field distibution near a PEC ogive with edge-on incidence.

08/01/90 08:59.29
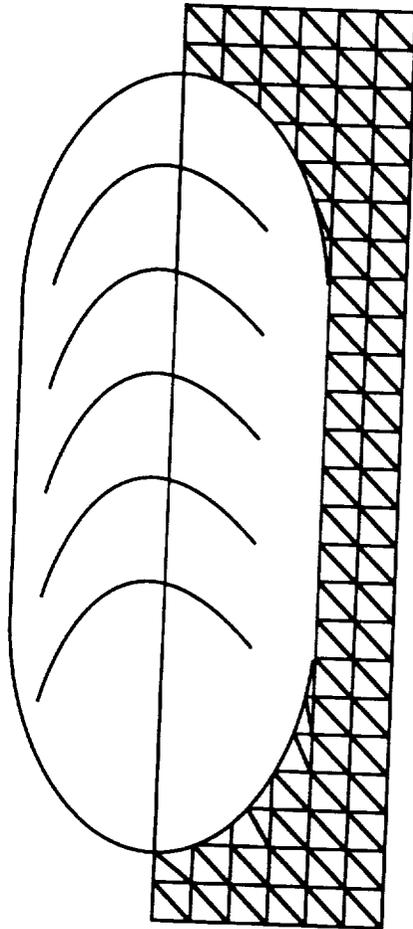
Figure 10: Body of revolution surrounded with a rectangular mesh.

Task Title: Analytical Solutions with Generalized
Impedance Boundary Conditions (GIBC)

Investigators: Mark A. Ricoy and John L. Volakis

Period Covered: February 1990 - September 1990