NASA
Technical Memorandum 103621

AVSCOM
Technical Report 90-C-010

Straddle Design of Spiral Bevel and Hypoid Pinions and Gears

Faydor L. Litvin, Chihping Kuan, and Jonathan Kieffer
University of Illinois at Chicago
Chicago, Illinois

Robert Bossler
Lucas Western Inc.
City of Industry, California

and

Robert F. Handschuh
Propulsion Directorate
U.S. Army Aviation Systems Command
NASA Lewis Research Center
Cleveland, Ohio

US ARMY
AVIATION
SYSTEMS COMMAND
AVIATION R&T ACTIVITY

(NASA-TM-103621) STRADDLE DESIGN OF SPIRAL BEVEL AND HYPOID PINIONS AND GEARS (Illinois Univ.) 12 p CSCL 13H

N91-14602

Unclas
G3/37 0317952

STRADDLE DESIGN OF SPIRAL BEVEL AND HYPOID PINIONS AND GEARS

Faydor L. Litvin, Chihping Kuan, and Jonathan Kieffer
University of Illinois at Chicago
Chicago, Illinois 60680

Robert Bossler
Lucas Western Inc.
City of Industry, California

and

Robert F. Handschuh
Propulsion Directorate
U.S. Army Aviation Systems Command
NASA Lewis Research Center
Cleveland, Ohio 44135-3191

E-5467

SUMMARY

The design of spiral bevel and hypoid gears that have a shaft extended from both sides of the cone apex (straddle design) is considered. A main difficulty of such a design is determining the length and diameter of the shaft that might be undercut by the head cutter during gear tooth generation. A method that determines the free space available for the gear shaft is proposed. The approach avoids collision between the shaft being designed and the head cutter during tooth generation. The approach is illustrated with a numerical example.

INTRODUCTION

The term "straddle design" means that the gear will be provided with a shaft that extends from both sides of the toothed face of the gear for bearing support. An example of a spiral bevel gear with such a shaft is shown in figure 1. The straddle design is preferred for heavily loaded gear trains, since it enables the load to be split between shaft-mounted bearings on both sides of the gear face. The straddle design requires the gear and the shaft to be made as one piece. A design obstacle is the interference (collision) of the shaft with the head cutter during tooth generation.

The valuable experience of Boeing Vertol Company in straddle design has been described by R.J. Drago (ref. 1), who proposed a graphical method for determining the dimensions of an integral shaft for spiral bevel gears.

In this report a method is proposed for detecting collision of the head cutter with the shaft being designed. The procedure determines the optimal length u of the shaft and its radius r (fig. 1). The solution is based on determining a space in the plane of parameters r and u that is free from interference with the head cutter. A computer program has been developed that uses the basic machine tool settings for gear generation as input. The computer program results are the diameter r and length u for the lines that limit the space free from undercutting.

BASIC MACHINE TOOL SETTINGS

In order to begin to solve this problem, the kinematics of the tooth generation process must be modeled. The basic ideas from which this analytical procedure is developed can be found in reference 2. The process of tooth generation will now be described. The following coordinate systems will be considered:

(1) The coordinate system $S_0(X_0,Y_0,Z_0)$, which is rigidly connected to the cutting machine (figs. 2 and 3)
(2) The coordinate system S_c, which is rigidly connected to the cradle that during tooth generation rotates about the Z_0-axis with angular velocity $\omega^{(c)}$
(3) The auxiliary coordinate system S_p, which is rigidly connected to S_0 and the gear being generated and rotates about X_p with angular velocity $\omega^{(p)}$
(4) The coordinate system S_t, which is rigidly connected to the head cutter (fig. 4) (note that the head cutter is mounted on the cradle and rotates with it about the Z_0-axis (fig. 3); in addition, the head cutter rotates about the Z_t-axis, but this rotation only provides the required velocity for cutting and is not related to tooth generation)
(5) An additional coordinate system S_b (fig. 2), which is rigidly connected to the cradle coordinate system S_c and is used to describe the tilt of the head cutter with respect to the cradle

Coordinate systems S_t and S_b initially coincide with each other. Their orientation with respect to S_c is determined with the swivel angle j (fig. 2). The tilt of the head cutter is accomplished by rotating S_t (and the head cutter) about the Y_b-axis by the tilt angle i (fig. 4(b)).

The gear settings (fig. 3) are determined with the following parameters: E_m (the machine offset); γ_m (the machine root angle); ΔB (the sliding base); and ΔA (the machine center-to-back distance) (refs. 4 and 5).

The head cutter settings (fig. 2) are determined with the following parameters: S_R (radial setting); q (cradle angle); j (swivel angle); i (tilt angle, not shown in fig. 2). Parameter q is a variable, since the cradle rotates during tooth generation but is constant for Formate[1] cut gears.

COLLISION OF HEAD CUTTER WITH STRADDLE GEAR SHAFT BEING DESIGNED

A circle of radius r_c on the head cutter (fig. 4) generates a surface in the coordinate system S_p while the cradle with the head cutter rotates about the Z_0-axis (fig. 2). This circle may be represented in coordinate system S_t in parametric form as follows (fig. 4):

$$\mathbf{r}_t = r_c[\cos\theta_t \quad \sin\theta_t \quad 0 \quad 1]^T \tag{1}$$

The transposed column matrix represents the homogeneous coordinates of a point on circle r_c.

1Gleason Works trademark.

The generated surface is represented in coordinate system S_p by the matrix equation

$$r_p^{(1)}(\theta_t,q) = [M_{pn}][M_{n0}][M_{0c}][M_{cb}][M_{bt}]r_t(\theta_t) \tag{2}$$

Matrix $[M_{n0}]$ describes coordinate transformation from S_0 to an auxiliary coordinate system S_n (fig. 3) whose axes are parallel to the axes of S_0.

The superscript in $r_p^{(1)}(\theta_t,q)$ designates surface number 1 (the head cutter surface) that is represented in S_p. The surface of the shaft in S_p will be designated number 2. Here

$$[M_{bt}] = \begin{bmatrix} \cos i & 0 & \sin i & 0 \\ 0 & 1 & 0 & 0 \\ -\sin i & 0 & \cos i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$[M_{cb}] = \begin{bmatrix} \cos(90^\circ - \delta) & \cos(180^\circ - \delta) & 0 & S_R \\ \cos\delta & \cos(90^\circ - \delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\sin j & -\cos j & 0 & S_R \\ \cos j & -\sin j & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where $\delta = 2\pi - j$

$$[M_{0c}] = \begin{bmatrix} \cos q & \sin q & 0 & 0 \\ -\sin q & \cos q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$[M_{n0}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & E_m \\ 0 & 0 & 1 & -\Delta B \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$[M_{pn}] = \begin{bmatrix} \cos\gamma_m & 0 & \sin\gamma_m & -\Delta A \\ 0 & 1 & 0 & 0 \\ -\sin\gamma_m & 0 & \cos\gamma_m & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

By using equations (1) to (7), the head cutter surface Σ_1 can be represented as

$$\left.\begin{array}{l} X_p^{(1)} = f_1(\theta_t, q) \\ Y_p^{(1)} = f_2(\theta_t, q) \\ Z_p^{(1)} = f_3(\theta_t, q) \end{array}\right\} \quad (8)$$

Now consider a family of coaxial cylinders in coordinate system S_p of radius r_i (fig. 5) whose equations are represented by

$$\left.\begin{array}{l} X_p^{(2)} = u_s \\ Y_p^{(2)} = r_i \cos \theta_s \\ Z_p^{(2)} = r_i \sin \theta_s \end{array}\right\} \quad (9)$$

These coaxial cylinders represent gear shafts of radius r_i.

Positive and negative signs on u_s correspond to the projection of current cylinder point M on the positive or negative axis X_p. The origin of coordinate system S_p coincides with the intersection of the shortest distance with the pinion axis (for a hypoid gear drive) or with the apex of the pitch cone (for a spiral bevel gear drive).

The collision of the head cutter surface Σ_1 with the shaft surface Σ_2 means that the surfaces have a point in common. Thus, the collision can be represented by the vector equation

$$\mathbf{r}_p^{(1)}(\theta_t, q) = \mathbf{r}_p^{(2)}(\theta_s, u_s) \quad (10)$$

which yields

$$\left.\begin{array}{l} f_1(\theta_t, q) = u_s \\ f_2(\theta_t, q) = r_i \cos \theta_s \\ f_3(\theta_t, q) = r_i \sin \theta_s \end{array}\right\} \quad (11)$$

The goal is to determine a subspace in the space of parameters (r_i, u_s) that is free from collision of Σ_1 and Σ_2. The determination of such a subspace is based on the following computational procedure:

<u>Step 1</u>: Parameter q, as mentioned previously is a variable parameter, and its range of variation must be determined. Considering that q_0 is the initial value of q, the limiting values of q, q_1 and q_2, can be determined from the equations

$$\left.\begin{array}{l} q_1 = q_0 - \Delta q \\ q_2 = q_0 + \Delta q \end{array}\right\} \quad (12)$$

where

$$\Delta q \geq \frac{\pi}{N_1} \sin \gamma_p \tag{13}$$

γ_p is the pitch angle of the gear, and N_1 is the number of gear teeth.

Step 2: A certain value for q must be chosen in the range $q_1 \leq q \leq q_2$ and then the value of θ_t varied in the range $0 \leq \theta_t \leq 2\pi$. The direct computation allows determination of $r_i \cos \theta_s$ and $r_i \sin \theta_s$ from equation (11). Then r_i is obtained by taking into account that

$$\left[\left(r_i \cos \theta_s\right)^2 + \left(r_i \sin \theta_s\right)^2\right]^{1/2} = r_i \tag{14}$$

Equation (10) is used to determine the values of u_s that correspond to θ_s.

Step 3: The described computational procedure permits a numerical solution of a family of closed curves $r_i[u_s, q^{(i)}]$ (fig. 6), where $q^{(i)}$ are the fixed values of q. A current point N on each curve of the family of curves in determined with the couple $\left[q^{(i)}, \theta_t^{(j)}\right]$, where $q^{(i)}$ is a constant and $\theta_t^{(j)}$ is a variable. The subspace in the coordinate system (u_s, r_i) that is free from the family of curves determines the values of r_i and u_s that can be chosen by the designer to avoid collision between the shaft and the head cutter. Recall that the origin of S_p coincides with the intersection of the shortest distance with the pinion axis (hypoid) or with the apex of the pitch cone (spiral bevel). A computer program for determining the subspace that is free from collision between the head cutter and the shaft has been developed.

LIMITING LINES OF SPACE r_s, u_s

The limiting lines of the space r_s, u_s are shown in figure 7 for a hypoid pinion. Segments L_1 and L_2 of the limiting lines in figure 7 represent the envelopes of the family of closed curves. Segment L_3 belongs to the curve with the initial and final value of q. Segment L_4 (fig. 8) represents the limiting line that corresponds to tangency of the bottom of the head cutter (fig. 4) with the set of cylinders of radii r_i (fig. 5). Those contact lines are shown in figure 8.

The limiting lines for the earlier discussion is shown in figure 9. These lines determine the border of the free space where the extended shaft of the pinion can be located. Figure 9 indicates that a shaft of 19 mm radius can have a length of 40 mm.

The derivation of envelopes L_1 and L_2 is based on the following ideas: Consider that the family of planar curves is represented in the shape of parameters (u,r) by the equation

$$\mathbf{R} = u(\theta_t, q)\mathbf{e}_u + r(\theta_t, q)\mathbf{e}_r \tag{15}$$

where q is the parameter of the family. The conditions of envelope existence are represented by the equation (ref. 2)

$$\mathbf{N} \cdot \frac{\partial \mathbf{R}}{\partial q} = f(\theta_t, q) = 0 \qquad (16)$$

Here

$$\mathbf{N} = \begin{bmatrix} \frac{\partial r}{\partial \theta_t} & -\frac{\partial u}{\partial \theta_t} \end{bmatrix}^T \qquad (17)$$

is the normal to a curve of the family. The tangent $\mathbf{T}$ to the curve is represented by

$$\mathbf{T} = \begin{bmatrix} \frac{\partial u}{\partial \theta_t} & \frac{\partial r}{\partial \theta_t} \end{bmatrix}^T \qquad (18)$$

$$\frac{\partial \mathbf{R}}{\partial q} = \begin{bmatrix} \frac{\partial u}{\partial q} & \frac{\partial r}{\partial q} \end{bmatrix}^T \qquad (19)$$

Equations (15) and (16) considered simultaneously represent the envelope of the family of curves that limits the space (u,r).

The derivation of L_4 is based on the idea that the bottom of the head cutter (it is a plane) can generate in (u,r) an envelope of a family of planes. Consider that the bottom of the head cutter is a ring and each circle of the ring can be represented by its radius r_i and a parameter θ_i. The envelope generated by the family of rings is represented in (u,r) by the equations

$$\left.\begin{aligned} u &= u(r_i, \theta_i) \\ r &= r(r_i, \theta_i) \\ r_i \sin \theta_i &= S_R \cos j \end{aligned}\right\} \qquad (20)$$

The respective values of θ_i can be obtained by varying r_i. As a reminder, the matrix $[M_{0t}]$, which describes the coordinate transformation from S_t to S_0, must be used to derive the functions $u(r_i, \theta_i)$ and $r(r_i, \theta_i)$. Then the points for the limiting line L_4 may be selected.

CONCLUDING REMARKS

A methodology has been proposed that enables spiral bevel or hypoid gear teeth with straddle design to be generated without collision between the head cutter and the shaft. This methodology permits the design of a shaft that extends toward or beyond the gear apex within a given envelope. Equations are used to determine the surface generated by the edge of the head cutter. A zone of safe straddle shaft design can then be determined.

ACKNOWLEDGMENT

This research was partially supported by the Gleason Memorial Fund.

APPENDIX - BASIC PINION MACHINE TOOL SETTINGS

The data used in the example of this report for determining the free space where the gear shaft can be located are given in the following table:

Hypoid gear	Concave side	Convex side
Basic tilt angle, deg	17.260000	16.086000
Swivel angle, deg	-31.266000	-48.216700
Machine root angle, deg	357.5160	357.5830
Cradle angle, deg	81.21620	72.21670
Radial setting, mm	108.9340	111.8520
Sliding base, mm	19.10001	27.64999
Machine center-to-back distance, mm	-2.650000	3.840000
Blank offset, mm	27.11000	31.83000
Cutting ratio	0.2535497	0.2371070
Cutter point radius, mm	112.7750	113.4100
Cutter blade angle, deg	14.000000	31.000000

REFERENCES

1. Drago, R.J.: On the Design and Manufacture of Integral Spiral Bevel Gears for the CH-47D Helicopter. Presented at the Aerospace Gearing Committee Meeting, AGMA, Lake Tahoe, NV, Mar. 1979.

2. Litvin, F.L.: Theory of Gearing. NASA RP-1212, 1989.

3. Litvin, F.L.; Zhang, Y.; Lundy, M.; and Heine, C.: Determination of Settings of a Tilted Head Cutter for Generation of Hypoid and Spiral Bevel Gears. ASME J. Mech. Transmissions Automation Des., vol. 1, Dec. 1988, pp. 495-500.

4. Litvin, F.L.; and Lee H.-T.: Generation and Tooth Contact Analysis of Spiral Bevel Gears With Predesigned Parabolic Functions of Transmission Errors. NASA CR-4259, 1989.

5. Litvin, F.L., et al.: Hypoid Gear Drive With Face-Milled Teeth: Conditions of Pinion Nonundercutting and Fillet Generation. Proceedings of the 1989 Fall Technical Meeting of the American Gear Manufacturers Association, AGMA, Alexandria, VA, 1989, Paper 7.

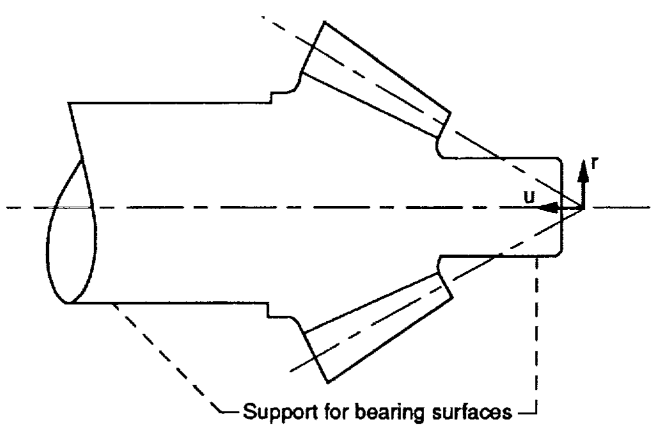


Figure 1.—Straddle-design spiral bevel gear showing parameters r and u.

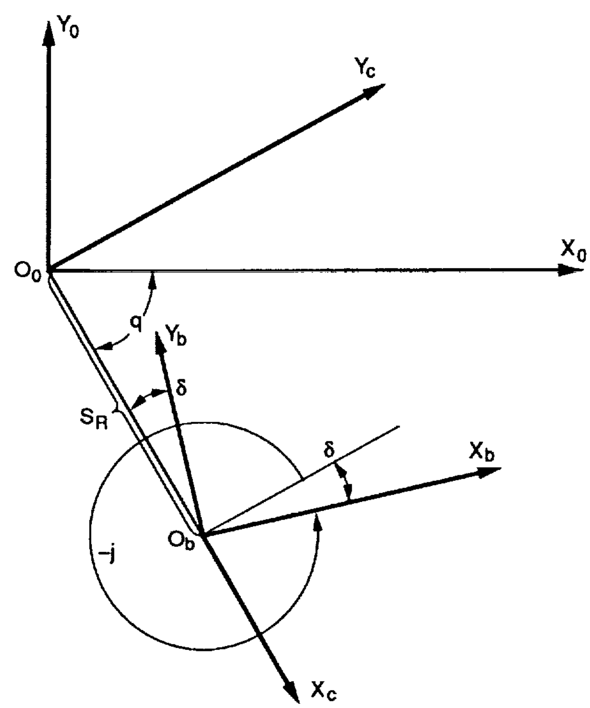
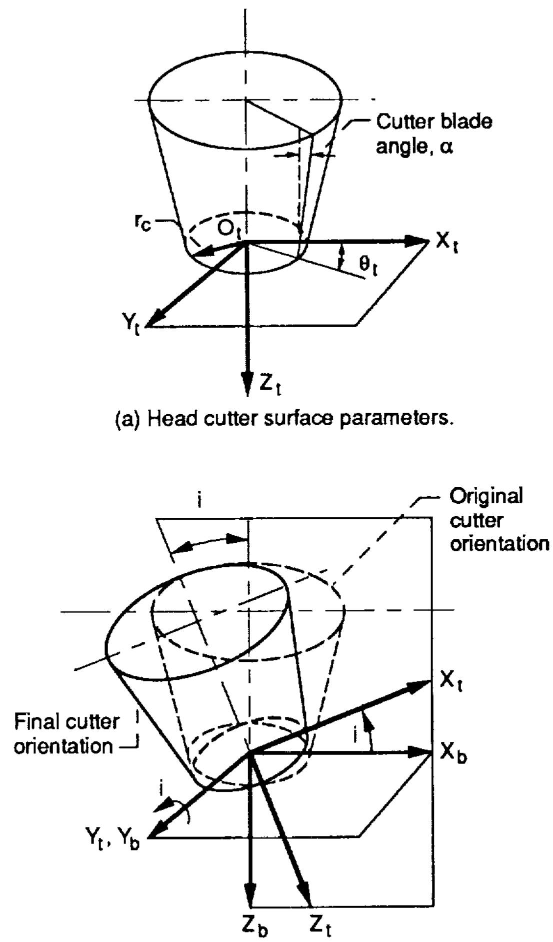


Figure 2.—Cutting machine and cradle coordinate systems.



(a) Head cutter surface parameters.

(b) Coordinate systems for head cutter tilt.

Figure 4.—Pinion head cutter surface.

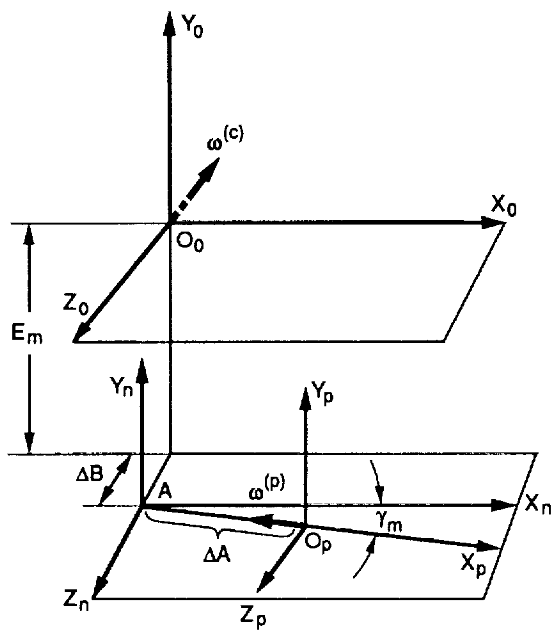


Figure 3.—Angular velocities of cradle and pinion.

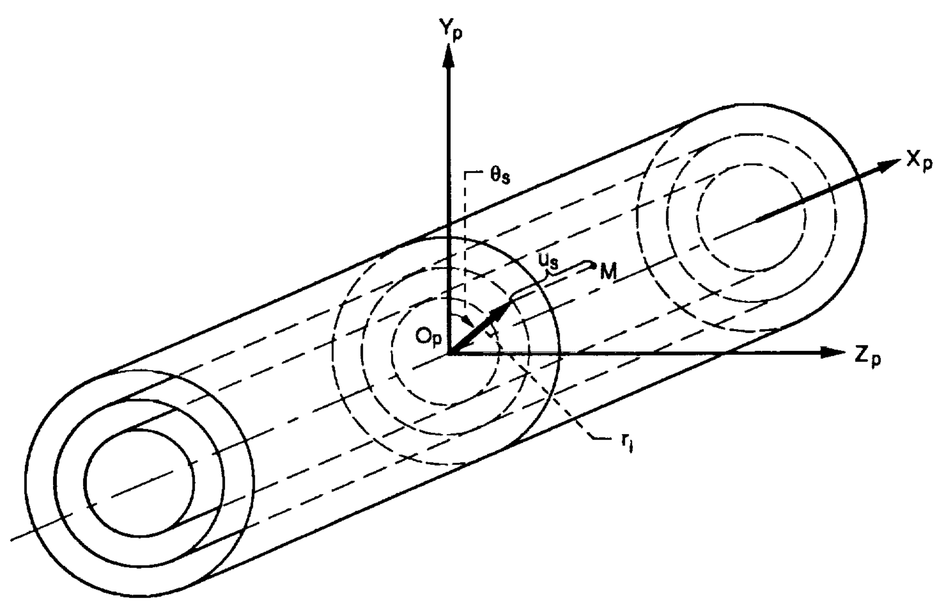


Figure 5.—Family of coaxial cylinders.

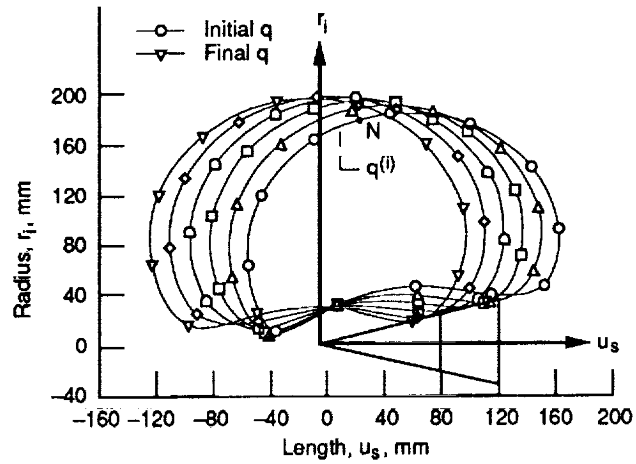


Figure 6.—Family of closed curves.

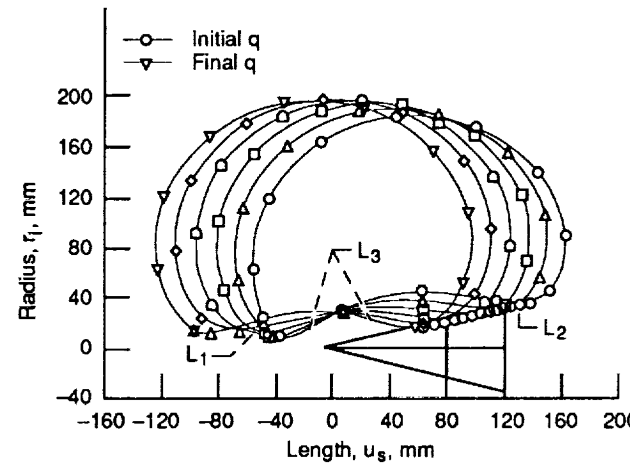


Figure 7.—Limiting lines and envelopes.

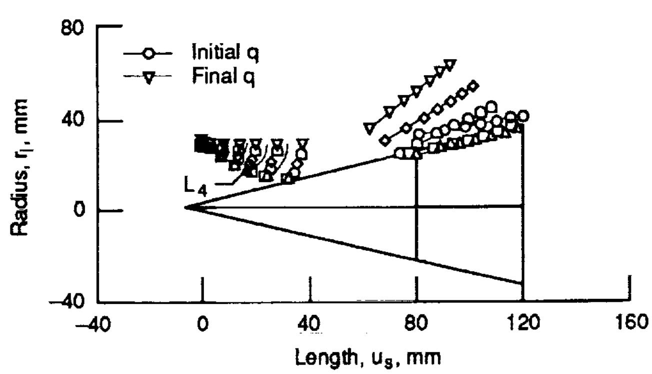


Figure 8.—Limiting lines, L_4.

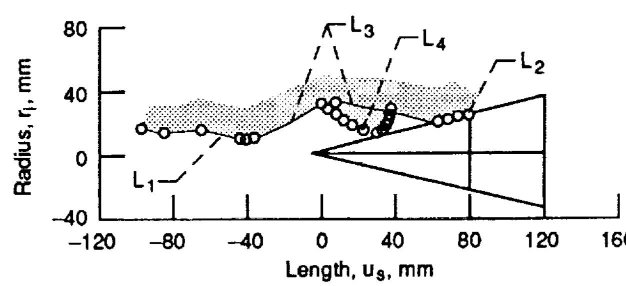


Figure 9.—Border of free space.

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-103621 AVSCOM TR 90-C-010		

4. Title and Subtitle	5. Report Date
Straddle Design of Spiral Bevel and Hypoid Pinions and Gears	November 1990
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
Faydor L. Litvin, Chihping Kuan, Jonathan Kieffer, Robert Bossler, and Robert F. Handschuh	E-5467
	10. Work Unit No. 505-63-51 1L162211A47A
9. Performing Organization Name and Address NASA Lewis Research Center Cleveland, Ohio 44135-3191 and Propulsion Directorate U.S. Army Aviation Systems Command Cleveland, Ohio 44135-3191	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001 and U.S. Army Aviation Systems Command St. Louis, Mo. 63120-1798	14. Sponsoring Agency Code

15. Supplementary Notes

Faydor L. Litvin, Chihping Kuan, and Jonathan Kieffer, University of Illinois at Chicago, Chicago, Illinois 60680. Robert Bossler, Lucas Western Inc., City of Industry, California; and Robert F. Handschuh, Propulsion Directorate, U.S. Army Aviation Systems Command.

16. Abstract

The design of spiral bevel and hypoid gears that have a shaft extended from both sides of the cone apex (straddle design) is considered. A main difficulty of such a design is determining the length and diameter of the shaft that might be undercut by the head cutter during gear tooth generation. A method that determines the free space available for the gear shaft is proposed. The approach avoids collision between the shaft being designed and the head cutter during tooth generation. The approach is illustrated with a numerical example.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Gears; Spiral bevel gears; Gear geometry; Gear manufacture; Straddle gear design	Unclassified – Unlimited Subject Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	12	A03

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161