JPL Publication 90-47

[illegible] 47

p 2

An Ohmic Model for Charge Transport in a Semiconductor

Larry D. Edmonds

November 15, 1990

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

JPL Publication 90-47

An Ohmic Model for Charge Transport in a Semiconductor

Larry D. Edmonds

November 15, 1990

NASA
National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (NASA) under the NASA Microelectronics Radiation Effects Ground Test Program.

Abstract

This analysis applies to a quasi-neutral region of uniformly doped semiconductor material. The objective is to solve for the current density in terms of the carrier density and the electric potential boundary values. It is shown that the combined effects of drift and diffusion can be calculated by assuming the current density to obey Ohm's law, but with modified electric potential boundary values.

CONTENTS

1. Introduction

Some problems involving carrier transport in a semiconductor can be solved by hand calculations. A particular case occurs when the geometry and boundary values are simple, potential drops are somehow known, and excess charge carriers are produced in high density (e.g., by an energetic heavy ion). This problem is simple because the carrier density is governed by the ambipolar diffusion equation (discussed in section 2), which contains only one unknown function and is solvable (by hand calculation) for sufficiently simple geometries. However, as pointed out in section 2, ambipolar diffusion describes only the carrier density function, not carrier flow. To solve for the current, additional analysis is needed and the required additional analysis is the subject of this publication. It will be shown that the remaining analysis is very similar to V = I R. Under certain conditions, called strong conduction (defined in section 7), the remaining analysis literally is V = I R.

This method of analyzing carrier transport is not new. In previous work (e.g., Ref. 1), the carrier density was assumed to be governed by ambipolar diffusion, for the purpose of calculating R, and then V = I R was used to calculate current. Although this method has been used in the past, theoretical justification was not given. This publication gives the theoretical justification applicable to strong conduction and shows how the method should be modified for weak conduction (defined in section 7). A few cylindrically symmetric problems involving ion tracks were solved by combining the analysis given here with the assumption of ambipolar diffusion and the results agreed well with predictions from the cylindrical coordinate version of PISCES.

The current density in a quasi-neutral region of uniformly doped semiconductor material is analyzed. Junctions and other structures are represented by boundary conditions, and the objective is to relate current density to the boundary conditions. Such a relationship is an equation of state for the "device" (the quasi-neutral region) analogous to V = I R for a lumped resistor. It will be seen that the relationship is not only analogous to, but also very similar to V = I R.

2. Review

For reference convenience, it is helpful to have some familiar results handy. Therefore, a brief review is given. The reader should consult any standard textbook for a more thorough discussion. Some symbols are defined below:

n_o, p_o = equilibrium electron and hole densities, respectively.

N, P = excess electron and hole densities, respectively.

D_n,D_p =diffusion constants for electrons and holes, respectively.

μ_n, μ_p = mobilities for electrons and holes, respectively.

τ = carrier lifetime (assumed the same for electrons as holes).

V_T = thermal voltage (about 0.026 volts at room temperature).

q = elementary charge.

$\mathbf{J}_n$, $\mathbf{J}_p$ = electron and hole current densities, respectively.

U = electric potential.

ϵ = dielectric constant.

Throughout this publication, equilibrium carrier densities, diffusion constants, mobilities, and carrier lifetime are treated as constants. Also, $D = V_T \mu$ for electrons and holes.

The governing equations are:

$$\mathbf{J}_p = q\, D_p\, [-\mathbf{grad}\, P - (P + p_o)\, \mathbf{grad}\, U/V_T]$$

$$\mathbf{J}_n = q\, D_n\, [\mathbf{grad}\, N - (N + n_o)\, \mathbf{grad}\, U/V_T]$$

$$\mathbf{div}\, \mathbf{J}_p = -q\, (1/\tau + \delta/\delta t)\, P$$

$$\mathbf{div}\, \mathbf{J}_n = q\, (1/\tau + \delta/\delta t)\, N$$

$$-\epsilon\, \mathbf{div}\, \mathbf{grad}\, U = q\, (P - N). \tag{1}$$

The standard quasi-neutral approximation is obtained by regarding ϵ as sufficiently small compared to other relevant constants so that the solutions to the equations can be approximated by the solutions obtained in the limiting case as ϵ approaches zero. In this limit, (1) becomes P = N and substituting into the other equations gives

$$\mathbf{J}_p = q\ D_p\ [-\mathbf{grad}\ P - (P + p_o)\ \mathbf{grad}\ U/V_T] \tag{2a}$$

$$\mathbf{J}_n = q\ D_n\ [\mathbf{grad}\ P - (P + n_o)\ \mathbf{grad}\ U/V_T] \tag{2b}$$

$$\mathbf{div\ grad}\ P + \mathbf{grad}\ P \cdot \mathbf{grad}\ U/V_T + (P + p_o)\ \mathbf{div\ grad}\ U/V_T = (1/\tau + \delta/\delta t)\ P/D_p \tag{3a}$$

$$\mathbf{div\ grad}\ P - \mathbf{grad}\ P \cdot \mathbf{grad}\ U/V_T - (P + n_o)\ \mathbf{div\ grad}\ U/V_T = (1/\tau + \delta/\delta t)\ P/D_n\ . \tag{3b}$$

In this limit, (1) is replaced with $P = N$ and cannot be used to solve for U, but (3) is a closed system of equations and is used to solve for both P and U.

At high injection levels, the equations in (3) can be combined to produce the ambipolar equation

$$\mathbf{div\ grad}\ P = (1/\tau + \delta/\delta t)\ P/D^* \quad [P >> \max(n_o,\ p_o)] \tag{4}$$

where the ambipolar diffusion constant, D^*, is given by

$$1/D^* = (1/D_p + 1/D_n)/2\ . \tag{5}$$

The ambipolar condition (4) is not needed in the theory to follow, but it does help to make the problem more solvable, as discussed later, and applies to some important situations. A few statements should be made to clear up a misconception regarding ambipolar diffusion. Some people visualize the process as one in which the carriers interact strongly with each other, through their own fields, and their motion does not respond to applied fields. In reality, it is only the carrier density function that does not respond to applied fields; carrier flow is very responsive (this will be seen in the Ohmic model derived later). Note that it is possible to have a large flow and still have the density change slowly (characteristic of ambipolar diffusion) if the flow has a small divergence (physically, a small divergence means that carriers moving out of a volume element are replaced by other carriers moving in). Therefore, it is possible for the flow to respond strongly to applied fields, while the density does not. Furthermore, electrons and holes are not required to move together to avoid charge separation, as some people think. They can move very differently and still avoid charge separation as long as the total (electron plus hole) current has a zero divergence. This situation (electrons moving differently than holes) can occur unless prohibited by boundary conditions or by extreme symmetry (e.g., spherical symmetry), where specifying the divergence of a vector field uniquely determines the vector field. Although the phrase does not sound like it, "ambipolar diffusion" describes only the carrier density function, not carrier flow.

3. Solving for Potential in Terms of Carrier Density

Suppose P was somehow known and we want to solve for U in terms of P. If the exact P was known [in this context, "exact" means exactly satisfies (3)], either equation in (3) or any linear combination of the equations could be used (if we knew how to solve it) and the same solution for U will result. But suppose some approximation is used for P that is not exactly consistent with (3). In this case, different linear combinations of (3) can produce different solutions for U, and some can be better than others in the sense that errors in U are less sensitive to errors in P. For example, if the equations in (3) are added, we get

$$(p_o - n_o)\ \mathbf{div\ grad}\ U/2V_T = (1/\tau + \delta/\delta t)\ P/D^* - \mathbf{div\ grad}\ P. \qquad (6)$$

If the approximation (4) is used, the result is

$$\mathbf{div\ grad}\ U = 0.$$

The problem here is that while the right side of (6) is small in some sense when the approximation (4) applies, the coefficient to U on the left side of (6) is also small (compared to P) and dividing by this coefficient to solve for U can produce large errors.

Because some equations are better than others, it is natural to look for the best equation. Determination of the best equation requires a lengthy analysis involving norms and/or variations, but a good equation is easy to identify. In a good equation, the coefficient of U will contain P in order to avoid the problem produced by (6). The coefficient should also contain at least the majority carrier equilibrium density. This is obviously needed for low injection-level conditions, but it might also be needed for high injection-level conditions. The reason is that U is strongly influenced by boundary conditions and, unless $P \gg \max(n_o, p_o)$ everywhere, this influence may propagate through regions of low injection levels. Boundary conditions cannot be properly built into the solution unless the governing equation is valid all of the way to the boundary surface.

One equation derivable from (3) and having all of the required properties is the linear combination

$$\mathbf{div}\ \mathbf{J}_T = 0 \tag{7}$$

where

$$\mathbf{J}_T = \mathbf{J}_n + \mathbf{J}_p \ . \tag{8}$$

Equation (7) can also be written as

$$(P + A)\ \mathbf{div\ grad}\ U/V_T + \mathbf{grad}\ P \cdot \mathbf{grad}\ U/V_T = B\ \mathbf{div\ grad}\ P \tag{9}$$

where

$$A = (D_p\, p_o + D_n\, n_o)/(D_n + D_p)$$
$$= (\mu_p\, p_o + \mu_n\, n_o)/(\mu_n + \mu_p) \qquad (10)$$

$$B = (D_n - D_p)/(D_n + D_p) = (\mu_n - \mu_p)/(\mu_n + \mu_p)\ . \qquad (11)$$

4. Solution for U in Terms of U_H and P

The objective of this section is to solve for U in terms of P and another function, U_H, defined later, subject to boundary conditions.

The boundary surface is partitioned into two sections, A_{INS} and A_{OTH}. A_{INS} is the union of all insulated sections ($\mathbf{J}_T$ has zero normal component) and A_{OTH} is the union of all other sections. Boundary conditions for U are taken to be of the Dirichlet type on A_{OTH}, i.e.,

$$U(\mathbf{x},t) = v(\mathbf{x},t) \qquad \text{on } A_{OTH} \qquad (12)$$

for some boundary value function v. By combining (2) and (8) with the condition that $\mathbf{J}_T$ has a zero normal component on A_{INS}, we have

$$\mathbf{grad}\ U/V_T \cdot \mathbf{n} = B\ \mathbf{grad}\ P \cdot \mathbf{n}/(P + A) \qquad \text{on } A_{INS} \qquad (13)$$

where $\mathbf{n}$ is the normal unit vector.

Note that U can be expressed as the sum of any particular solution, U_P, to (9) plus an appropriate solution, U_H, to the homogeneous equation

$$(P + A)\ \mathbf{div\ grad}\ U_H/V_T + \mathbf{grad}\ P \cdot \mathbf{grad}\ U_H/V_T = 0\ . \tag{14}$$

One particular solution, which can be verified by substitution, is

$$U_P/V_T = B\ \ln[(P + A)/A]\ . \tag{15}$$

Therefore

$$U = U_H + U_P \tag{16}$$

where U_H satisfies (14) and some appropriate boundary conditions. Evaluating (16) on A_{OTH} and A_{INS} while using (13) gives

$$U_H = v - B\ V_T\ \ln[(P + A)/A] \qquad \text{on } A_{OTH} \tag{17}$$

$$\mathbf{grad}\ U_H \cdot \mathbf{n} = 0 \qquad \text{on } A_{INS}\ . \tag{18}$$

5. Expressing J_T in Terms of U_H

Combining (2), (8), (10) and (11) gives

$$\mathbf{J}_T = q\ (D_n + D_p)\ [B\ \mathbf{grad}\ P - (P + A)\ \mathbf{grad}\ U/V_T]$$

and using (15) and (16) gives

$$\mathbf{J}_T = -\ q\ [D_n\ (P + n_o) + D_p\ (P + p_o)]\ \mathbf{grad}\ U_H/V_T\ . \qquad (19)$$

6. The Ohmic Model

The governing equations can be written as

$$U_H = v - B\ V_T\ \ln[(P + A)/A] \qquad \text{on } A_{OTH} \qquad (20)$$

$$\mathbf{grad}\ U_H \cdot \mathbf{n} = 0 \qquad \text{on } A_{INS} \qquad (21)$$

$$\mathbf{div}\ (\sigma\ \mathbf{grad}\ U_H) = 0 \qquad \text{on interior} \qquad (22)$$

$$\mathbf{J}_T = -\ \sigma\ \mathbf{grad}\ U_H \qquad (23)$$

where σ is given by

$$\sigma = q\ [\mu_n\ (P + n_o) + \mu_p\ (P + p_o)]\ . \qquad (24)$$

Now suppose the medium was not a semiconductor, but instead an Ohmic material (i.e., it satisfies Ohm's law) with conductivity σ, but with the applied voltage on the boundary given by the right side of (20) instead of v. To solve for the current, we would solve precisely the same equations (20) through (23). Therefore, to solve for the current in a uniformly doped quasi-neutral semiconductor, including both drift and diffusion, we can pretend that it is an Ohmic material, but modify the boundary conditions by including the logarithmic term in (20). The system of equations, (20) through (24), will be called the Ohmic model.

Note that the ambipolar condition, (4), is not needed and the equations are equally valid for low, medium, and high injection-level conditions. But, unless the ambipolar equation applies (or P is given or can be solved some other way), U_H and P must be treated as simultaneous unknowns. If the ambipolar equation applies, P can be solved first, and then U_H. Another solvable problem consists of low injection-level conditions, so that σ can be treated as a constant and P shows up only in the boundary condition, (20), and does not have to be solved in the interior. Such a problem can be solved for U_H and J_T, but to solve for J_n or J_p it is still necessary to solve for P because it is necessary to solve for U.

The Ohmic model can be expressed in integrated form for two terminal structures when U and P are both constant on both terminals. Let A_{OTH} consist of two parts, A_1 and A_2, and let $v=V_1$ on

A_1 and $v=V_2$ on A_2, where V_1 and V_2 are constant in $\mathbf{x}$. Let $P(\mathbf{x},t)=P_1$ on A_1 and $P(\mathbf{x},t)=P_2$ on A_2, where P_1 and P_2 are also constant in $\mathbf{x}$. The integrated form of (20) through (24) is

$$\mathbf{delta}\ U_H = I\ R \qquad (25)$$

where I is the total current integrated over either A_1 or A_2, R is the total resistance calculated in terms of the conductivity as if the medium were Ohmic, and

$$\mathbf{delta}\ U_H = V_2 - V_1 + B\ V_T\ \ln[(P_1 + A)/(P_2 + A)]\ . \qquad (26)$$

7. Weak and Strong Conduction

A two terminal structure will be called strongly conducting if the logarithmic term in (26) can be neglected. Otherwise, it will be called weakly conducting. A strong conduction problem most closely resembles a conventional Ohmic problem because even the boundary conditions do not need to be modified.

Weak conduction implies that diffusion is an important mode of charge transport, but strong conduction does not imply that diffusion is unimportant. For example, if A_1 and A_2 are metallic contacts (sinks for excess carriers), the logarithmic term is zero. However, a large diffusion current can still occur if a large excess carrier density exists (e.g., via photon absorption) in the medium. Strong conduction implies that U and U_H satisfy the same boundary conditions on A_1 and A_2, but they need not be equal (and, therefore, total current need not equal conduction current) because they do not satisfy the same equation on the interior. U_H satisfies (22), but U does not. In particular, it is not permissible to replace **grad** U_H with **grad** U in (23) because the two quantities can be significantly different. During strong conduction, U cannot be substituted for U_H indiscriminately, but the substitution can be made in the integrated equation (25).

An example of strong conduction is a typical funneling process. In such a process, an ion track partially, or completely, shorts a junction so that some, or all, of the power supply voltage (used to reverse bias the junction) is dropped across the device substrate. The electron mobility in silicon is roughly twice the hole mobility, so B is roughly one third. Assuming the substrate doping is at least $10^{15}/\text{cm}^3$ and the maximum carrier density in the ion track does not exceed $10^{20}/\text{cm}^3$, the logarithmic term in (26) is less than or equal to about 4 V_T or about one tenth of a volt. If the potential drop across the substrate is at least one volt, neglecting the logarithmic term will produce less than 10% error in the boundary values for U_H. To the extent that R can be solved exactly, the calculated current will also have less than 10% error.

8. Some Simple Implications

It is interesting to look at some simple special cases to verify that familiar equations can be retrieved from the Ohmic model.

In the first example, A_1 is an electrode contact and A_2 is a highly reverse-biased junction which acts like a sink for minority carriers. Suppose there is an excess of carriers somewhere in the substrate via initial conditions or irradiation and the objective is to calculate the total current that flows into the junction (sink). We are not told the potential boundary values, but we are told that there is virtually no majority current at the junction (implying no injection from the other side). The potential boundary values will be whatever it takes to block the majority carrier current at A_2. To be definite, suppose the material is n type. To determine the boundary conditions from the given information, that $\mathbf{J}_n=0$ on A_2, use (2b) to get

$$(P + n_o)\ \mathbf{grad}\ U/V_T = \mathbf{grad}\ P \qquad \text{on } A_2$$

and (15) and (16) give

$$n_o\ \mathbf{grad}\ U_H/V_T = (D_p/D_n)\ \mathbf{grad}\ P \qquad \text{on } A_2$$

where we have used $P_2=0$. It is more accurate to use $P_2=-p_o$, but the error in the calculated current is on the order of the reverse saturation current which is neglected in this analysis. Combining the above result with (23) gives

$$J_T = - q \, D_p \, \mathbf{grad} \, P \qquad \text{on } A_2$$

which is the same result obtained from a more elementary analysis that simply looks at the minority carrier diffusion current.

In the next example, A_1 and A_2 are metallic contacts (sinks for excess carriers, i.e., $P_1=P_2=0$), and are both grounded ($V_1=V_2=0$), but there may be an excess carrier density in the interior (e.g., through irradiation). These conditions imply that there is no U_H drop across the device. The conclusion, from (25), is $I=0$. The physical explanation is that electric fields were set up to produce drift currents in opposition to the diffusion currents.

In the last example, the circuit is open ($I=0$). From (25) and (26) we conclude that the right side of (26) is zero. For low injection levels, a first-order approximation for the logarithm gives

$$V_2 - V_1 = V_T \, (P_2 - P_1) \, (D_n - D_p)/(D_p \, p_o + D_n \, n_o)$$

which is the Dember potential for low injection levels [2]. This result is not new, but note how easily it can be derived from the Ohmic model.

9. Terminal Voltage and Current

The potentials and current in (25) refer to the medium, or substrate, side of the surfaces or contacts A_1 and A_2. If the capacitance between A_1 and A_2 is significant, it is necessary to make a distinction between the substrate current, I, and the terminal current, I_T (the current through the lead to the substrate contact). These currents can be different because of a changing surface charge density at the contacts. An easy way to deal with this is to represent the substrate with a parallel resistor and capacitor depicting the capacitance between A_1 and A_2.

There may be an equilibrium contact potential between the contacts and substrate, so that the potential difference across the substrate is not equal to the terminal voltage. The terminal voltage can be represented by including voltage sources, representing equilibrium contact potentials, in the equivalent circuit. Under equilibrium conditions these potentials subtract out, but may not do so under non-equilibrium conditions. For example, if A_1 is a metallic electrode and A_2 is a p-n junction, an ideal voltage source might represent the contact potential at A_1; but at A_2 it is better to use a charged capacitor (combined with some other circuit elements needed to give a more complete representation of a p-n junction). In equilibrium, the capacitor voltage balances with the voltage source at A_1, so that neither is visible to the outside world. However, if the capacitor discharges (e.g., through photoelectrons and holes), the contact potential at A_1 can be seen at the device terminals. Including contact potential is an alternative to making a distinction between actual and applied potential. When contact potentials are included, U is the actual potential.

10. Accuracy of the Ohmic Model

The Ohmic model is an exact result of eqs. (2) and (3), which, in turn, are derived from the assumption of quasi-neutrality. The validity of this approximation is discussed by a number of investigators (e.g., Refs. 2, 3, and 4) and, therefore, need not be discussed in detail here. The equations are expected to be accurate for the uniformly doped quasi-neutral regions of typical silicon devices.

In a practical application, the Ohmic model is likely to be supplemented with an additional approximation (e.g., the ambipolar diffusion equation), so that the carrier density can be treated as known--this is an alternative to supplementing (20) through (24) with another equation, taken from (3), and solving simultaneous equations. It is left up to the investigator to determine the suitability, for the application, of any additional approximations.

11. Conclusion

The analysis applies to a quasi-neutral region of uniformly doped semiconductor material and shows that the total current can be solved by assuming the current density to obey Ohm's law, but with the boundary conditions modified by including the logarithmic term in (20).

References

[1] C. Hu, "Alpha-Particle-Induced Field and Enhanced Collection of Carriers," IEEE Electron Device Letters, Vol. EDL-3, No. 2, pp. 31-34, February 1982.

[2] Oldwig von Roos, "A Note on Photocurrents in Extrinsic Semiconductors," Solid State Electronics, Vol. 22, pp. 229-232, 1979.

[3] S. Wang, Solid State Electronics, McGraw-Hill, pp. 284-286, 1966.

[4] J.M. Feldman, The Physics and Circuit Properties of Transistors, Wiley & Sons, pp. 243-249, 1972.

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. JPL Pub 90-47

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
An Ohmic Model for Charge Transport in a Semiconductor

5. Report Date
November 15, 1990

6. Performing Organization Code

7. Author(s) Larry Edmonds

8. Performing Organization Report No.
JPL Pub. 90-47

9. Performing Organization Name and Address
JET PROPULSION LABORATORY
California Institute of Technology
4800 Oak Grove Drive
Pasadena, California 91109

10. Work Unit No.

11. Contract or Grant No.
NAS7-918

13. Type of Report and Period Covered
JPL Publication

12. Sponsoring Agency Name and Address
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This analysis applies to a quasi-neutral region of uniformly doped semiconductor material. The objective is to solve for the current density in terms of the carrier density and the electric potential boundary values. It is shown that the combined effects of drift and diffusion can be calculated by assuming the current density to obey Ohm's law, but with modified electric potential boundary values.

17. Key Words (Selected by Author(s))
Electronics and electrical engineering

18. Distribution Statement
Unlimited/unclassified

19. Security Classif. (of this report)
Unclassified

20. Security Classif. (of this page)
Unclassified

21. No. of Pages
23

22. Price

JPL 0184 R 9/83