A STATISTICAL STUDY OF MERGING GALAXIES-THEORY AND OBSERVATIONS

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ABSTRACT. A study of the expected frequency of merging galaxies is conducted, using the impulsive approximation. Results indicate that if we consider mergers involving galaxy pairs without halos in a single crossing time or orbital period, the expected frequency of mergers is two orders of magnitude below the observed value for the present epoch. If we consider mergers invovling several orbital periods or crossing times, the expected frequency goes up by an order of magnitude. Preliminary calculation indicate that if we consider galaxy mergers between pairs with massive halos, the merger is very much hastened.

INTRODUCTION

Observations indicate that the frequency of galaxy mergers (order of magnitude accuracy only) in the present epoch is ~0.3% (Toomre, 1977; Tremaine, 1980). An extrapolation to the past yields a frequency ~5%. Loosely bound pairs of galaxies, that had separated to great distancs in the general cosmic expansion, and have lately fallen together again (for the first or feweth time) in comet-like plunging ellipses, seem to be the most lucrative candidates for mergers. To tackle this problem it is necessary to determine the frequency of merging galaxies on the basis of the collision theory and compare the values so obtained with the observational ones.

In this work we have studied sphere-sphere and disk-sphere collisions. We have assumed that a galaxy and its nearest neighbor are likely to form a bound pair, for frequency determinations. We have not embedded the galaxies in massive halos, except for one case. A detailed paper on merger of galaxies with massive halos will be published soon. The determination of general results and the expected frequency of merging galaxies is greatly facilitated by using the impulsive approximation, as it is a suitable way of studying many collisions with varying parameters. As far as the order-of-magnitude results are concerned, those obtained by this method tally quite well with the results of N-body simulations (Toomre, 1977; Dekel et al., 1980; Alladin and Narasimhan, 1982; and Aguilar and White, 1985). Hence the order of magnitude of the frequency that we obtain by this method will be correct. At any rate, the crucial questions pertain to the order of magnitude of the frequency of merging galaxies.

THEORY

We model the spherical galaxy as a polytrope of index n = 4, and the disk galaxy as an exponential model disk with density distribution, $\sigma(r) = \sigma_c e^{-4r/R_D}$, where $\sigma_c = M_D/(2\pi R_D^2 b)$ is the central density, M_D and R_D being the mass and radius of the disk and b is a constant pertaining to its density distribution. This disk is thickened by a method indicated by Rohlfs and Kreitschmann (1981). A polytropic bulge of index n = 4 is superposed on this disk. The model is discussed in detail in Chatterjee, 1989.

The theory for studying sphere-sphere and disk sphere collisions for mergers taking place in a single orbital period is basically the same as in Chatterjee, 1987 (hereafter referred to as Paper I), except that in the case of disk-sphere collisions the appropriate function $\chi = \chi_{DS}$ is used, which defines the mutual potential energy function for the disk-sphere system (cf. Ballabh, 1975). M_1 and M_2 are the masses and $R_1 = R_2 = R$ their common radii. Mergers of disk-sphere systems are dealt with in detail in Chatterjee 1990a.

To study mergers taking place in several orbital periods we use the same method as outlined in Chatterjee, 1990b. The method is basically a modification of the method used by Alladin, 1965. In the case of disk-sphere collisions the appropriate function $\chi = \chi_{DS}$ is used. If r and θ denote the polar coordinates of the perturber galaxy (Galaxy 1) with respect to the test galaxy (Galaxy 2), in the orbital plane characterizing the relative motion, and μ,ℓ and E, the reduced mass, the angular momentum and the energy of orbital motion, respectively, then the Lagrangian equations for the problem give,

$$dt = dr/[2\mu^{-1}\{E - W(r) - r^2/(2\mu r^2)\}]^{1/2}$$

and $$d\theta = (\ell dr)/[2\mu^{-1}\{E - W(r) - \ell^2/(2\mu r^2)\}]^{1/2} \; (\mu r^2)$$

These equations can be used to determine the orbit and the variation of time along the orbit. The relative orbit is studied numerically by using these equations. At small intervals of r and θ the tidal effects, and the corresponding change in velocity of the stars , is studied by the same method as in Paper I. The change in internal energies of the two galaxies ΔU_1 and ΔU_2 at any time t can be obtained from,

$$V(t) = (dr/dt) = [2\mu^{-1}\{E_i - W(t) - \Delta U_1(t) - \Delta U_2(t)\}]^{1/2}$$

where $E_i = (1/2)\mu V_i^{\,2} - (GM_1M_2/R_2)\ \chi_{si}$

and

$$W(t) = W(r) = W(s) = -(GM_1M_2/R_2)\ \chi_s$$

V_i and E_i are the initial relative velocity and orbital energy of the two galaxies, $W(t) = W(s)$ is the mutual potential energy of the two galaxies at the instantaneous separation $s = r/R$, χ_{s_i} and χ_s are the values of $\chi = \chi_{SS}$ or χ_{DS} (depending on whether it is a sphere-sphere or disk-sphere collision that is under study) (Alladin uses $\psi = s\chi$) at the initial separation $s_i = r_i/R$ and $s = r/R$, respectively.

We study many collisions between galaxies of equal dimensions. In the case of disk-sphere collisions as almost all of the mass of the spherical galaxy is concentrated within 1/3 of its radius, the effective radius of the spherical galaxy is 1/3 that of the disk. In the case of mergers taking place in a single crossing period, each collision is characterized by a value of the distance of closest approach, p, and initial velocity V_i, as discussed in Paper I. In the case of mergers taking place in several orbital periods, the initial separation is taken to play the part of p in frequency determination. In each case the change in velocity due to dynamical friction is calculated for small intervals as the galaxies advance in their relative orbit, and the instantaneous relative velocity so determined is used to continue the integrations of the equations of motion of the two galaxies. Merger takes place when the instantaneous relative velocity, V_i, of the two galaxies equals the velocity of escape between the pair characterized at the instantaneous separation, V_E, as discussed in Paper I. In the case of mergers taking place in a single crossing time the relative velocity at closest approach, V_p is used for frequency determinations, while V_i is taken to play the part of V_p for mergers in several orbital periods.

NUMERICAL RESULTS AND DISCUSSION

General Resuults

The computations of many collisions have been carried out, each collision being characterized by a value of V_p and p (measured in units of the radius of either galaxy) and scaled for different mass ratios of the two galaxies.

For sphere-sphere systems in collision, we find that for central impacts, merger takes place in a single crossing time provided $p \lesssim R_{1/4}$, the radius containing 1/4 of the mass of the victim galaxy, and $V_p \lesssim 1.2\ V_E$. The merger is affected in a single crossing time up to $p \lesssim R_{3/4}$, the radius containing 3/4 of the mass of the victim galaxy; while the merger is effected in more than one orbital period up to $p \sim R$. The merger is effected in $\lesssim$ 2 orbital periods if $R_{3/4} \lesssim p \lesssim 4R_h$ ($4R_h \approx 1/2$ of the radius for polytropic n = 4 model), R_h being the galactic half-mass radius; and in several orbital periods (~5) for $4R_h \lesssim p \lesssim R$. In general beyond $p \sim 4R_h$ the dynamical friction process becomes quite ineffective and the merger is effected extremely slowly. The relative mass of the perturber with respect to the victim galaxy does not seem to affect the merger times markedly, unless the former has less than about 1/5 the mass of the latter. For $p \lesssim 4R_h$, the parabolic perturber orbit is circularized in the first orbital period, and the subsequent orbit effects the merger; while if p lies between $4R_h$ and R, the circularization is achieved in few orbital periods. Beyond $p \sim R$, mergers do not seem to take place as the dynamical friction lacks the ability of circularizing the parabolic perturber orbit and the same recede to an enormous distance from which it does not return in significant orbital timescales.

In the case of disk-sphere collisions, dynamical friction plays a very insignificant role, compared to sphere-sphere collisions, due to the small thickness of the victim disk and mergers do not seem to be possible for hyperbolic velocities, which implies $V \lesssim V_E$. The merger is effected in a single crossing time only if the impact is central or very nearly so, and the range of values of p depends strongly on the relative mass of the bulge with respect to the disk. For these reasons, for the purpose of frequency determinations, it seems to be more convenient to determine the range of values of p for which merger is possible in single and several orbital periods together (for a more detailed treatment see Chatterjee, 1990b). We find that in this case mergers are possible up to $p \sim R$. Only if $p \gtrsim R_h$ (for the victim disk), the merger takes place in a much longer time than otherwise. Beyond $p \sim R$ the perturber does not seem to return in significant orbital timescales.

We have studied only one collision with galaxies having massive halos. So we cannot comment on the frequency of merging galaxies with massive halos, but only mention the results of this simulation, as the parameters for this collision have been carefully chosen. The model used for the halo has the asymptotic dependence $M(r) \sim r$, where r is the distance measured from the center of the galaxy, and is discussed in detail by Allen and Martos (1986). This model is excellent for the study of many collisions as an analytical treatment of the halo is elucidated. The bulge component is modeled as a polytrope of index n = 4. A merger of two identical spherical galaxies is studied. The halo contains about 10 times the mass of the bulge component and 10 its radius, R_b. The initial separation of the two galaxies is taken to be $p = 2R_b$ and the initial relative velocity is taken to be $v_i = 2V_E$. Merger takes place in few orbital periods ($\lesssim 3$). The exact number of orbital periods depends upon the value of the separation of the centers of the two galaxies for which merger is defined to be complete. For frequency determinations, whether or not the merger takes place in a significant orbital timescale is the crucial fact.

Brief Comparison with Previous Work

The results of van Albada and van Gorkom (1977), White (1978), Roos and Norman (1979), are summed up in Figure 1 of Aarseth and Fall (1980). Translated in terms of our parameters, this figure is indicative of a high frequency of mergers in the region corresonding to $p \lesssim 0.8 \times 2R_h = 0.216R$, for polytrope n = 4 model, and $V_E \lesssim V_p \lesssim 1.2V_E$. Villumsen (1982) finds that for the distance of closest approach $p = R_i \lesssim 10$ units, corresponding to 0.22 R (as the tidal radius for his model is 45 units), merger is quite frequent; while for $p = R_i \approx 10$ units, merger occurs only for low relative velocities. Alladin and Parthasaraty (1978) indicate that stellar systems, modeled as polytropes of index $n = 4$, in mutual circular orbit, merge in about 15% of an orbital period for $p \lesssim 0.2R$; while for $p \gtrsim 0.2R$, merger takes considerably more time. All these results compare well with our results that merger in a single crossing time ceases, for sphere-sphere pairs, for $p \lesssim R_{3/4} \lesssim 0.2R$ (for polytrope $n = 4$). The results are not very much model dependent.

In the region $R_{3/4} \lesssim p \lesssim R_h$, merger is achieved in a time ~ 2 orbital periods, while it is achieved in several orbital periods in the region $4R_h \lesssim p \lesssim R$. These results compare quite well with the results of Borne (1984). (For more details see Chatterjee, 1990b.) It is interesting to note that Borne includes a relaxation time in his scheme, which we do not, but the resonating stars which hasten the orbital decay of the galaxies in his simulations play a comparatively insignificant part in our scheme.

Frequency Determinations

In dense regions a galaxy and its nearest neighbor can be visualized to form a loosely bound pair (Toomre, 1977; Tremaine, 1980). As mentioned before, the general opinion is that collisions between initially loosely bound pairs lead to the majority of mergers. In dense regions the average distance between galaxies is ~ $10R_a$, where R_a is the average radius of a galaxy (Mitton, 1977; Ogorodnikov, 1965). So though theoretically the parameter p can vary from 0 to ∞, its mean value in regions where mergers are frequency, can be taken as $p_\infty \lesssim 10$ (in units of the radius of a galaxy). We take the total range of variation of V_p to be from 0 to $4V_E$, as collisions with velocity $V_p \gtrsim 4V_E$ seldom occur

(i) Mergers of Spherical Systems in a Single Crossing Time. Central Impacts: $p \lesssim R_{1/4}$, $V_p \lesssim 1.2\ V_E$. The probability in terms of the energy of the collision is given by $P_E = (1.2\ V_E)^3/(4V_E)^3 = 0.027$; and the probability in terms of the impact parameter is given by $P_p = (R_{1/4}p_\infty)^2 = 4.56 \times 10^{-5}$; which gives the probability of mergers in this case as $P_C = P_E \times P_p = 1.2 \times 10^{-6}$. Which gives the expected frequency of such mergers as 1.2×10^{-4} %.

Off Center Impacts: $R_{1/4} \lesssim p \lesssim R_{3/4}$, leading to mergers only if $V_p \lesssim V_E$. The corresponding probabilities in this case are given by $P_E = [V_E/(4V_E)]^3 = 0.015625$, $P_p = (R^2{}_{3/4} - (R^2{}_{1/4})/p^2 = 3.645 \times 10^{-4}$, giving $P_{OC} = P_E \times P_p = 5.7 \times 10^{-6}$. This gives an expected frequency of 5.7×10^{-4} %. Since these are mutually exclusive events, the probability of mergers of spherical systems in an orbital period is, $P = P_C + P_{OC} = 6.9 \times 10^{-6}$, which leads to an expected frequency of such mergers as 6.9×10^{-4} %.

(ii) Mergers of Spherical Systems in Several Orbital Periods. $R_{3/4} \lesssim p \lesssim R$, $V \lesssim V_E$. In this case the respective probabilities are given by $P_E = [V_E/(4V_E]^3 = 0.015625$, $P_p =$

$(R^2_{3/4} - R^2_{1/4})/p^2 = 9.6 \times 10^{-3}$, which gives $P = P_E \times P_p = 1.5 \times 10^{-4}$; which gives the expected frequency of such mergers as 1.5×10^{-2} %.

Hence the expected frequency of mergers of spherical systems is given by 1.57×10^{-2} %.

(iii) Mergers of Disk-Sphere Systems.

$p \lesssim R$, $V_p \lesssim V_E$. In this case the respective probabilities are given by P_E $[V_E/(4V_E)]^3 = 0.015625$, $P_p = (R/p_\infty)^2 = 0.01$, which gives the merger probability as $P = P_E \times P_p = 1.56 \times 10^{-4}$, which gives the expected frequency of such mergers as 1.56×10^{-2} % $\sim 10^{-2}$ %.

CONCLUSIONS

In general we conclude that the expected frequency of merging galaxies (not considering the effect of massive halos) is ~ 0.01%, which is an order of magnitude below the observational value ~0.1 %. The expected frequency of merging galaxies increases by an order of magnitude when we take into account mergers occurring in several orbital periods, as compared to those occurring in a single crossing time. More than 90% of the mergers take place in several orbital periods, and there is an indication that the majority of them take place in a time ~2 to 3 orbital periods. For mergers in a single crossing time, more than 80% of them are due to off-center impacts.

Spherical and disk-sphere systems in merger seem to be equally frequent.

If our single simulation with galaxies embedded in massive halos is at all typical (which in our view it is), then the frequency of galaxy mergers with massive halos will be at least an order of magnitude higher than in the absence of halos (approaching the observational value for the present epoch). We are studying more collisions of type to throw light on this aspect.

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REFERENCES

Aarseth, S. J., and Fall, S. M. 1980, Astrophys. J., **236**, 43.

Aguilar, L. A., and White, S.D.M. 1985, Astrophys. J., **295**, 374.

Alladin, S. M. 1965, Astrophys. J., **141**, 768.

Alladin, S. M., and Narashimhan, K.S.V.S. 1980, Phys. Reports, **92**, 339.

Alladin, S. M., and Parthasarathy, M. 1978, Monthly Notices Roy. Astron. Soc., **184**, 871. (Corrigendum 1979, Monthly Notices Roy. Astron. Soc., **188**, 303.)

Allen, C., and Martos, M. A. 1986, Rev. Mex. Astron. Astrof., **13**, 137.

Ballabh, G. M. 1975, Astrophys. Space Science, **38**, 407.

Borne, K. D. 1984, Astrophys. J., **287**, 503.

Chatterjee, T. K. 1987, Astrophys. Space Science **137**, 267.

Chatterjee, T. K. 1989, Astrophys. Space Science (in press, 1990?).

Chatterjee, T. K. 1990a, Astrophys. Space Science, in press.

Chatterjee, T. K. 1990b, Astrophys. Space Science, submitted.

Dekel, A., Lecar, M., and Shaham, J. 1980, Astrophys. J, **241**, 946.

Mitton, S. (ed.) 1977, "The Cambridge Encyclopedia of Astronomy," Jonathan Cape Ltd., London, p. 345.

Ogorodnikov, K. F. 1965, "Dynamics of Stellar Systems," Per., London, p. 100.

Rohlfs, K., and Kreitschmann, J. 1981, Astrophys. Space Science, **79**, 289.

Roos, N., and Norman, C. A. 1979, Astron. Astrophys., **76**, 75.

Toomre, A. 1977, in B. M. Tinsley and R. B. Larson (eds.), "The Evolution of Galaxies and Stellar Populations," Yale Univ. Obs., p. 401.

Tremaine, S. D. 1980, in S. M. Fall and D. Lynden Bell (eds.), "The Structure and Evolution of Normal Galaxies," Univ. of Cambridge, p. 67.

Van Albada, T. S., and van Gorkom, J. H. 1977, Astron. Astrophys., **54**, 121.

Villumsen, J. V., 1982, Monthly Notices Roy. Astron. Soc., **199**, 493.

White, S.D.M. 1978, Monthly Notices Roy. Astron. Soc., **184**, 185.