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CURRENT COLLECTION FROM AN UNMAGNETIZED PLASMA : A TUTORIAL

Elden C. Whipple

Center for Astrophysics and Space Science
University of California at San Diego
La Jolla, CA 92093

Abstract. The current collected by a body in an unmagnetized plasma depends in general on: (1) the properties of the plasma; (2) the properties of the body; and (3) the properties of any neutral species that are present. The important plasma properties are the velocity distributions of the plasma particles at a location remote from the body (at "infinity"), and the Debye length which determines the importance of plasma space charge effects. The important body properties are its surface characteristics, namely the conductivity and secondary yield coefficients. The neutral species affect the current through collisions which impede the flow of current and possibly through ionization of the neutrals which can enhance the current. The technique for calculating the current collected by a body in a plasma will reviewed with special attention given to the distinction between orbit limited and space charge limited regimes, the asymptotic variation of the potential with distance from a body, and the concept of a sheath.

Orbit Limited Currents

Consider a body in a plasma where the Debye length is much larger than the body dimensions so that the potential can be taken to be a Coulomb potential. To simplify the discussion we will consider the body to be a sphere and will first look at how the sphere attracts particles from a monoenergetic beam. Figure 1 shows how the trajectories are bent by the attractive potential distribution. In a spherically symmetric potential distribution there are two constants of the motion, the total energy E and the angular momentum J. As the angular momentum is varied, there is a critical trajectory which just barely grazes the sphere. The impact parameter of this trajectory, r_0, defines the radius of an "effective cross-section" for collection of particles. Any particles with angular momentum (or impact parameter) less than that for the critical trajectory will be collected. Therefore the cross-section for collection and the current to the sphere can be obtained from the expressions for the total energy and angular momentum, as shown in Figure 1.

Note that in the derivation of the expression for the current that no explicit use was made of the inverse square dependence of the potential. Therefore a linear current voltage relation holds for any monotonic attractive potential distribution about a sphere provided that trajectories exist at all energies which come from infinity and are tangent to the surface of the sphere. This linear relation between current and an attractive voltage holds for any particle velocity distribution since any particle velocity distribution can be decomposed into superimposed beams. The condition that trajectories exist at all energies which come from infinity and graze the surface of the attracting body is the defining condition for "orbit limited" currents. Laframboise and Parker (1973) have shown that prolate and oblate spheroids also exhibit orbit limited behavior in the Laplace limit as long as the major-to-minor axis ratios are less than 1.653 and 2.537 respectively.

Orbit limited behavior also holds for any monotonic repelling potential about a convex object since every grazing orbit connects to infinity. However, the current to a repelling object is not linear since the particles in the plasma with energies less than the potential energy of the body will not reach it. For a Maxwellian plasma, the attracted and repelled currents are

$$I = I_o(1 - e\phi/kT), \text{for } e\phi < 0 \tag{1}$$

$$I = I_o exp(-e\phi/kT), \text{for } e\phi > 0 \tag{2}$$

where I_o is the random current to the body when it is at zero potential.

Sheath Limited Currents

When the Debye length in the plasma is on the order of or less than the body dimension, then there may not be any trajectories at a given energy which come from the plasma and are tangent to the surface of the body. This is illustrated in Figure 2 where there is a critical trajectory, defined as the non-impacting trajectory which approaches closest to the body for particles with a given energy. Trajectories with less angular momentum will all impact the body at angles of incidence which are not grazing angles. In such a case the critical trajectory defines an "absorption radius" (or absorption boundary) but this can not be easily used to obtain the current since each energy will in general define a different absorption radius. When this kind of behavior occurs the currents are said to be "sheath-limited".

The problem of obtaining sheath-limited currents is difficult since it involves finding the potential distribution from Poisson's equation which is self-consistent with the space charge. Bernstein and Rabinowitz (1959) first showed how to do this and Laframboise (1966) has applied their method to a Maxwellian plasma to obtain currents to spheres and cylinders for various values of plasma parameters. The method makes use of the "effective potential for radial motion", U(r), defined as follows:

$$E = \frac{1}{2}mv_r^2 + \frac{1}{2}mv_\theta^2 + e\phi(r) = \frac{1}{2}mv_r^2 + \frac{J^2}{2mr^2} + e\phi(r) \tag{3}$$

$$J = mrv_\theta \tag{4}$$

then

$$v_r = \sqrt{\frac{2}{m}[E - U(r)]} \tag{5}$$

where

$$U(r) = e\phi(r) + \frac{J^2}{2mr^2} \tag{6}$$

and where the radial and angular components of velocity are v_r and v_t . The second term in (6) is the repelling "centrifugal potential" which can give rise to potential barriers as shown in Figure 3. When the attractive electrostatic potential is weaker than $(1/r^2\ )$ then a maximum in the effective potential does not exist outside the probe surface. However, when the electrostatic potential is stronger than the inverse square potential, then potential barriers can exist for angular momenta greater than zero. Particle trajectories can be pictured in Figure 3 as horizontal lines of constant total energy which are reflected when they are incident on the effective potential curve for a particular angular momentum J. Barriers in the effective potential will repel particles with positive energies and thus reduce the current. Consequently sheath limited currents are always smaller than the orbit limited currents at a given potential. Orbit limited behavior can be seen to exist whenever the electrostatic potential falls off more weakly than an inverse square potential at every radius.

Figures 4 and 5 illustrate how the various types of trajectories which can occur for a given potential variation can be translated into a picture in the velocity space defined by the energy E and square of the angular momentum (J^2). Moments of the particle velocity distribution such as the particle density and current involve integrals over the distribution function, and the boundaries in the (E, J^2) plane between the different types of trajectories must be used in the limits of these integrals. For example, in Figure 4 trajectories of type 1 are populated by incoming plasma particles, and possibly by outgoing secondary particles from the probe surface. Those of type 2 are plasma particles which do not reach the probe. Type 4 consists of particles trapped in closed orbits about the probe, and type 3 consists of secondary particles which are emitted from and return to the probe. In Figure 5 for the sheath limited case, trapped particles do not exist, and type 3 trajectories are for particles which are repelled by a potential barrier and return to the plasma. This type of analysis has been used to calculate the current, space charge, and potential distribution about probes where both plasma particles and secondary particles contributed (Chang and Bienkowski, 1970; Schroder 1973; Tunaley and Jones, 1973; Whipple 1976; Parker, 1976).

Figure 6 shows currents obtained for inverse power law potential variations (Parker and Whipple, 1967). These potentials are not self-consistent but they illustrate nicely how the current decreases as the power n increases. Note how there is only one, linear curve for $n <= 2$. Figure 7 from Laframboise (1966) shows self-consistent currents to a sphere for various values of the probe radius to Debye length ratio.

When the particle velocity distribution is not isotropic, it may still be a reasonable approximation to use a spherically symmetric potential in order to calculate the current. Godard (1975) has used the potential distributions obtained by Laframboise (1966) for a stationary body to calculate the currents for a drifting Maxwellian plasma. These results, shown for a sphere in Figure 8, are appropriate for a positive ion currents to an attractive spherical satellite moving through the ionospheric plasma. Note especially that the current can in some cases initially decrease as the speed ratio of the body increases from zero. This effect is significant in calculating the "gyrophase drift" of a charged dust grain in a magnetic field (Northrop et al., 1989).

The Concept of a Sheath Edge

Intuitively, a sheath is the region close to a charged body where most of the potential drop occurs and where there is significant space charge. The concept of a "sheath edge" is useful because it defines a surface where the potential is close to the plasma potential and where the current can be estimated and equated (or related to) the total current to the body. The concept of a sheath is most useful when the body potential is high and when the Debye length is small compared to the body size. The sheath edge is usually defined as the place where the potential is $(kT/2e)$ so that outside this surface a quasi-neutral solution can be used for the potential. Swift and Schwar (1970) have reviewed work based on the concept of a finite thickness sheath.

The most important application of the sheath concept to current collection is the Langmuir-Blodgett (1923, 1924) derivation of the familiar (3/2) power law for the collected current:

$$I = C(V)^{3/2} \tag{7}$$

Angular momentum effects are neglected in this derivation. It is assumed that the particles are all emitted from one electrode with either zero or very small radial velocities, and that the particles follow the electric field lines to the collector. With these assumptions it is possible to relate the charge density to the current by means of the continuity equation. When the inner electrode is taken to be the collector, then the outer electrode position can be interpreted as the edge of the sheath for applications where a single collector is placed in a plasma.

The three-halves power law in equation (7) may seem to contradict the earlier statement that the maximum current drawn by a body is the orbit-limited current which is linear with voltage. However, the derivation of the current in (7) is for a given ratio of emitter and collector radii. This ratio is contained in the constant C in (7). When the sheath edge around a body in a plasma is taken to be the emitter, the current increases as the potential on the body is increased because the sheath grows larger. The way in which the sheath radius can be estimated for various regimes in space has been discussed in some detail by Parker (1980).

Asymptotic Potential Variation

In the distant plasma far from a spherical body the electrostatic potential varies asymptotically as

$$V = C/r^2 \tag{8}$$

where V is the potential, r the radial distance, and C a constant.

This behavior is obtained from the so-called "plasma solution", where the asymptotic forms of the ion and electron densities are obtained in terms of the local potential and distance, and then quasi-neutrality of the plasma is invoked. Both the ion and electron densities involve terms depending on the potential such as the Boltzmann factor, and solid angle factors depending on the distance, $(1 - r_p^2/r^2)$, where r_p is the radius of the body. In the limit as r becomes large, the potential enters the density terms linearly and this gives the first-order asymptotic variation of the potential as $(1/r^2)$.

In a numerical scheme for obtaining the potential distribution from Poisson's equation where a floating condition is necessary as a boundary condition at a finite distance, then this inverse square potential is the appropriate one to use. Laframboise (1966) has discussed the application of this condition and has given examples of calculations showing how the accuracy of the solutions depends on the distance of the boundary. Parker and Sullivan (1974) have also used this condition.

The value of C in (8) depends on the assumed plasma conditions. Various authors have obtained different expressions (Bernstein and Rabinowitz, 1958; Lam, 1965; Chang and Biekowski, 1970).

Present Issues Involving Current Collection

Finally, we list some of the issues involving current collection which are receiving attention at the present time. These issues have arisen in context of active space experiments where large potentials may occur or where large structures may be used:

1. What determines the current for large attractive potentials? Large potentials have been envisaged for high-power solar arrays. They also can occur when energetic charged particle beams are emitted.

2. Large potentials on spacecraft may involve dipole configurations with overlapping sheaths. What are the collected currents in such configurations? Katz et al. (1989) have recently calculated the current through a dipolar sheath and found good agreement with data.

3. The presence of neutral gas (from the neutral atmosphere, vehicle venting or outgassing, etc.) provides opportunities for ionization and therefore large currents. How can this effect be calculated?

4. Application to tether configurations: in large extended geometries, the spacecraft and tether form a circuit element with the current loop being completed through the plasma. How does the current flow in the plasma to complete the circuit?

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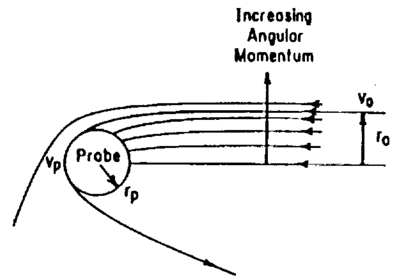
2 Dynamic Constants of Motion:

$$\begin{cases} E = \frac{1}{2}mv_o^2 = \frac{1}{2}mv_p^2 + e\phi_p \\ J = mv_o r_o = mv_p r_p \end{cases}$$



"Effective" Cross-section $= \pi r_o^2 = \frac{\pi J^2}{m^2 v_o^2} = \frac{\pi m^2 r_p^2 v_p^2}{m^2 v_o^2} = \pi r_p^2 (\frac{v_p^2}{v_o^2})$

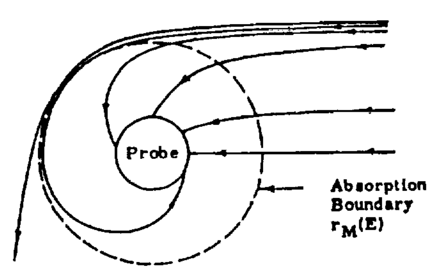
But $(v_p^2/v_o^2) = (\frac{1}{2}mv_p^2/\frac{1}{2}mv_o^2) = \frac{E - e\phi_p}{E} = 1 - \frac{e\phi_p}{E}$

Cross-section $= \pi r_p^2 [1 - \frac{e\phi_p}{E}]$

Current from mono-energetic beam is $I_o[1 - \frac{e\phi_p}{E}] \Longrightarrow$ for MB f(v) $\quad I = I_o[1 - \frac{e\phi_p}{kT}]$

1. Behavior of particles in an attrctive Coulomb potential.

A More General Attractive Potential

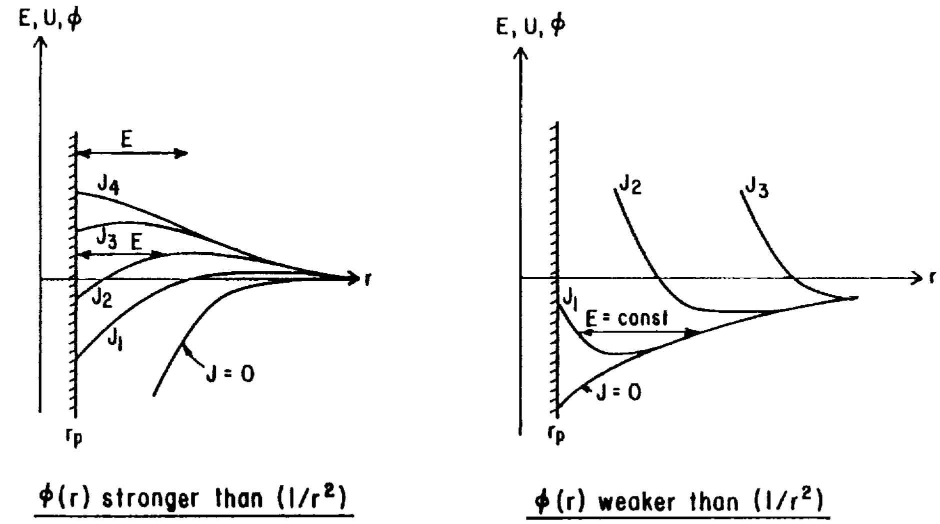


There may not be any trajectories (for a given energy) which come from ∞ and are tangent to the probe.

The last (least J) non-impacting trajectory, for a given energy, defines an ''absorption boundary'' for that energy.

2. Behavior of particles in a more general attractive potential.

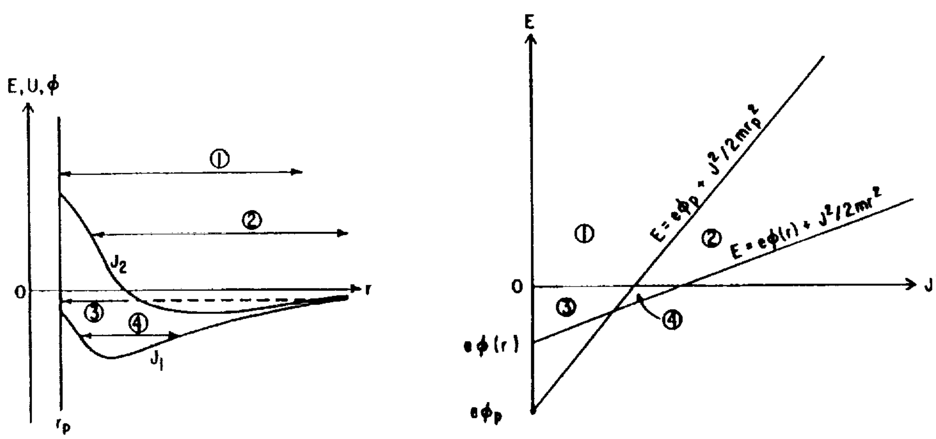
EFFECTIVE POTENTIAL: $U(r) = e\phi(r) + \dfrac{J^2}{2mr^2}$



3. The effective potential U(r) for radial motion.

Moments (density, flux) are integrals in velocity (E, J^2) space.

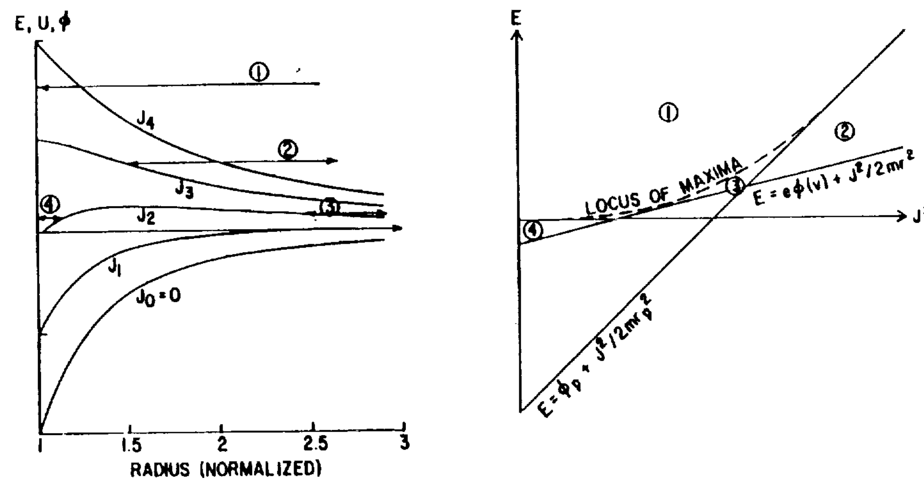
$$M \sim \iint f(E, J^2)\, dE\, dJ^2$$



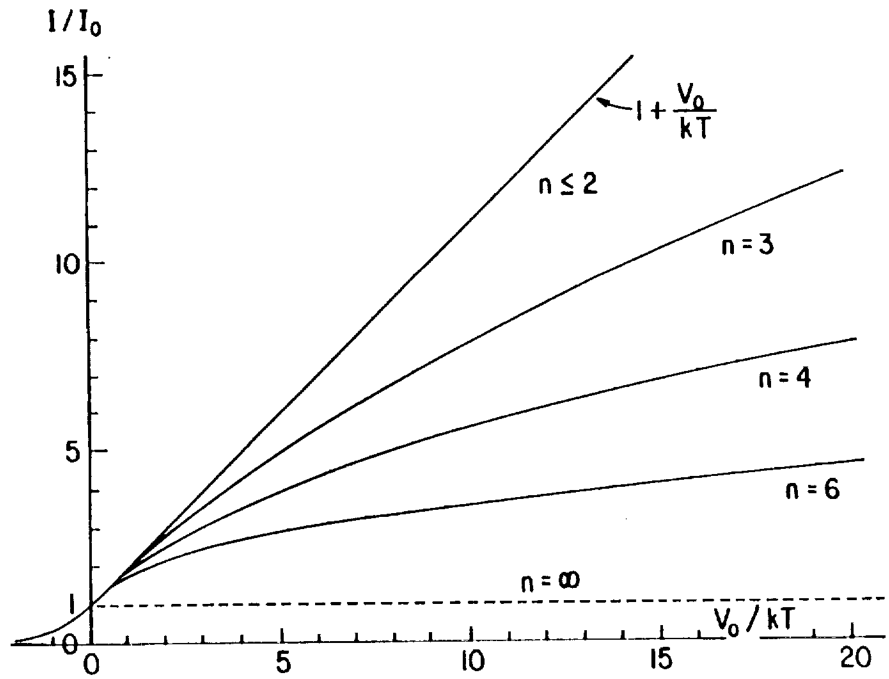
POSSIBLE TRAJECTORIES FOR THE ORBIT-LIMITED CASE

4. Classification of trajectories for orbit-limited trajectories.

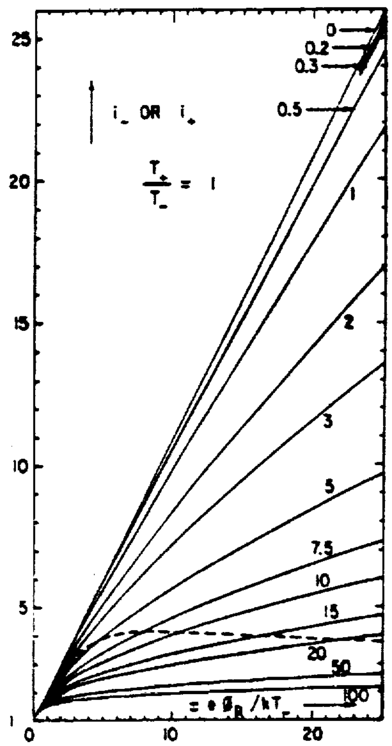
POSSIBLE TRAJECTORIES FOR THE SHEATH-LIMITED CASE



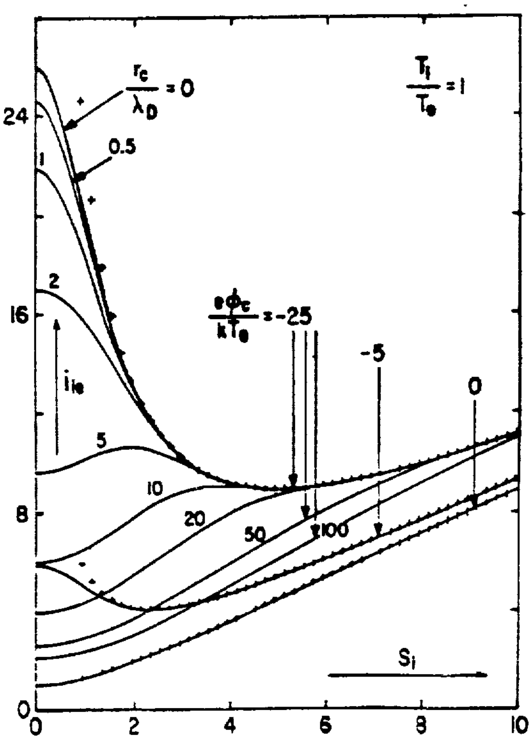
5. Classification of trajectories for sheath-limited trajectories.



6. Current-voltage curves for an inverse power law potential with a Maxwellian velocity distribution (from Parker and Whipple, 1967).



7. Ion or electron current vs. probe potential for various ratios of probe radius to ion or electron Debye length; dotted curve shows trapped-orbit boundary (from Laframboise, 1966).



8. Ion current vs. ion speed ratio with ratios of probe radius to Debye length and probe potential to electron temperature as parameters. The crosses represent the asymptotic solution (from Godard, 1975).