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PHYSICAL PROCESSES ASSOCIATED WITH CURRENT COLLECTION BY PLASMA CONTACTORS

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Abstract. Recent flight data confirms laboratory observations that the release of neutral gas increases plasma sheath currents. Plasma contactors are devices which release a partially ionized gas in order to enhance the current flow between a spacecraft and the space plasma. Ionization of the expellant gas and the formation of a double layer between the anode plasma and the space plasma are the dominant physical processes. A theory is presented of the interaction between the contactor plasma and the background plasma. The conditions for formation of a double layer between the two plasmas are derived. Double layer formation is shown to be a consequence of the nonlinear response of the plasmas to changes in potential. Numerical calculations based upon this model are compared with laboratory measurements of current collection by hollow cathode-based plasma contactors.

Introduction

Plasma double layers were first reported by Langmuir in 1929. The boundary between two different plasmas frequently takes the form of a double layer. In the laboratory, ionization near an anode forms a localized, dense plasma. The boundary between this anode plasma and any background plasma is visually sharp. The potential drops rapidly at the boundary. Such a localized potential drop requires layers of both positive and negative charge; hence, the term "double layer." Double layers have also been observed in the auroral zone ionosphere (Block, 1978).

Extensive investigations, both theoretical and experimental, have uncovered many properties of the particle distributions and potential structures in double layers (Block, 1978). Most of the theoretical work has focused on the transition region between two semi-infinite half spaces filled with collisionless, Maxwellian plasmas. While some authors have employed the full particle distribution function to describe the plasma charge density (Schamel and Bujarbarua, 1983), most, starting with Langmuir (1929), recognized that, for unmagnetized plasmas, the essential features of the charge density can be modeled in a much simpler fashion (Andrews and Allen, 1971). The theory presented below uses that equilibrium plasmas shield small potential perturbations linearly, while for high potentials,

the shielding decreases. These features are correctly represented in most descriptions of the plasma charge density. This approach is analogous with Van der Waals' theory of simple fluids in which inclusion of approximate expressions for both excluded volume and long range attractive forces are sufficient to describe the first order liquid-gas phase transition.

While previous studies have concentrated on the planar double layer stability conditions, the theory presented here applies more naturally to the case of one plasma expanding spherically into a uniform background plasma. An anode plasma, which was the first system identified as having a double layer by Langmuir (1929), is an example of a spherically expanding plasma. Anode plasmas have been previously been modeled in terms of a spherical double diode (Wei and Wilbur, 1986). The analysis below includes both electrons and ions from both the anode and background plasmas. The advantage for our analysis of the spherically expanding case is the additional parameter, r_{DL}, the radius at which the double layer occurs. The theory below identifies under what conditions a double layer will form and the radius at which it will be located. Only at that radius can a double layer exist and satisfy Poisson's equation.

Planar Double Layers

The equilibrium state of a collisionless, unmagnetized plasma can be described by the Vlasov equation and Poisson's equation,

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}$$
$$\rho(\mathbf{x}) = e\left(\iiint f_i\ (\mathbf{x},\mathbf{v})d\mathbf{v} - \iiint f_e\ (\mathbf{x},\mathbf{v})d\ \mathbf{v}\right) \quad , \qquad (1)$$

where ϕ is the potential, ρ is the charge density, and f_i, f_e the ion and electron distribution functions. The potential is with respect to the unperturbed plasma at a great distance.

In a neutral plasma at equilibrium, small perturbations in potential give rise to Debye shielding. However, as the potential increases, the attracted species is accelerated and the effective shielding decreases. A variety of approximations to this shielding function have been introduced by various authors. The theory presented below is insensitive to the particular functional form of the charge density with respect to potential. Following Andrews and Allen (1971), the charge density is represented by a function of potential. The particular form chosen here,

$$\frac{\rho(\phi)}{\varepsilon_0} = - \frac{\phi}{\lambda_D^2\left(1 + \sqrt{4\pi\left|\frac{\phi}{\theta}\right|^3}\right)} \quad , \tag{2}$$

has been previously used by the authors to successfully calculate ionospheric currents to high-voltage spacecraft (Katz et al., 1981; Mandell and Katz, 1982; Katz et al., 1989). This charge density function, as shown in Figure 1, is highly nonlinear. In the limit of potentials small with respect to the plasma temperature, this expression reduces to Debye shielding.

$$\frac{\rho(\phi)}{\varepsilon_0} \cong - \frac{\phi}{\lambda_D^2} \quad , \quad \phi \to 0 \tag{3}$$

For large potentials, the charge density approaches the one-sided thermal flux of the attracted species divided by the particle velocity.

$$\rho(\phi) \cong - \frac{j_{th}}{v} \quad , \quad \phi \to \infty \tag{4}$$

For two plasmas of equal temperatures and densities, but with reference potentials $\Delta\phi/2$ and $-\Delta\phi/2$, the combined charge density is

$$\rho(\phi) = - \frac{(\phi + \Delta\phi/2)}{1 + \sqrt{4\pi|\phi + \Delta\phi/2|^3}} - \frac{(\phi - \Delta\phi/2)}{1 + \sqrt{4\pi|\phi - \Delta\phi/2|^3}} \tag{5}$$

where λ_D, θ and ε_0 are taken to be unity. Figure 2 shows Equation (5) graphed for four values of $\Delta\phi$, the difference between the reference potentials. For small $\Delta\phi$, the charge density has only one zero. The bulk of the plasma will have potential $\phi=0$; the potential differences are dropped across sheaths at the boundaries. As $\Delta\phi$ increases, the slope of the charge density at $\phi=0$ goes through zero, and two other roots appear. When all three roots are present, the central root is not stable; it corresponds to negative shielding of potential fluctuations. In the limit of very large $\Delta\phi$, the two stable roots approach the two reference potentials.

Integrating Poisson's equation using Equation (5) with large $\Delta\phi$ leads to a double layer as shown in Figure 3. The plasma is quasi-neutral,

$$\nabla^2\phi = -\rho \cong 0 \quad , \tag{6}$$

everywhere but within the double layer. Plasmas whose Debye lengths are small compared with the scale size for potential changes are typically described assuming quasi-neutrality. Quasi-neutrality implies that the potential everywhere is at or near a root of the charge equation. However, regardless of the Debye length, for the boundary potentials in Figure 3 there exists no solution that is quasi-neutral everywhere. A region of large charge density must exist to support the transition between the potentials corresponding to the two roots of Equation (5).

The bifurcation of the charge equation occurs at the critical value of the separation of potentials,

$$\Delta\phi_{critical} \cong 1.43\theta \ ,$$

below which a double layer does not exist. The critical value of potential is numerically just the separation between the minimum and maximum in the charge density expression, Equation (2), since the expression has odd symmetry around the origin. Thus any similar expression for charge density will result in about the same critical value of the potential separation. The stable roots of the charge equation showing the bifurcation are graphed in Figure 4. Andrews and Allen (1971) previously described planar double layers in terms of the integration of Poisson's equation from one stable root to the other stable root. What has been introduced here is a critical value below which there is only a single root, and above which there are multiple roots and, therefore, double layers.

Source Plasma Expansion Into A Background Plasma

Early laboratory observations of double layers were of those that form the boundary of the glowing hemisphere of intense ionization surrounding an anode. Most analyses assumed planar geometry for simplicity. In this section, it is shown how examining the roots of the charge density expression for a spherically expanding plasma and a uniform background plasma can be used to calculate the radius of the double layer. The introduction of the radial dependence, r, in the charge density expression leads to a unique solution for the double layer radius.

A source at radius, r_s, is assumed to generate an ion current, I_i, which falls through a potential, ϕ_s, as it leaves the anode region. Near the anode, the ions are neutralized by Maxwellian electrons. The ambient plasma, $\rho_a(\phi)$, is represented by Equation (2).

$$\rho(r,\phi) = \rho_s^i(r,\phi) + \rho_s^e(\phi) + \rho_a(\phi) \tag{7}$$

$$\rho_s^i(r,\phi) = \frac{I_i}{4\pi r^2 v_i} = \frac{I_i}{4\pi r^2\sqrt{\dfrac{2E_0}{m_i} + \dfrac{2e\left(\phi_s - \phi\right)}{m_i}}} \tag{8}$$

$$\rho_s^e(\phi) = \frac{e^{(\phi-\phi_s)} - e^{-\phi_s}}{1 - e^{-\phi_s}} \times \left(-\rho_s^i(r_s,\phi_s) - \rho_a(\phi_s)\right) \tag{9}$$

These particular analytical expressions reflect the spherical expansion of the ions and that the effect of the source plasma vanishes at very large radius. The ions have initial energy E_0, and mass m_i. The expression for the source electrons is chosen so that the net charge density vanishes at the source potential at the source radius and also vanishes at zero potential at infinite radius.

The dependence of the charge density on potential for three different radii is shown in Figure 5. The parameters chosen are

$$r_s = 1\ ,$$
$$\phi_s = 20\ ,$$
$$\rho_s^i(r_s,\phi_s) = 100\ ,$$
$$\frac{2E_0}{m_i} = 5\ .$$

At small radii, the source plasma dominates and the charge density equation has a single root at a potential very close to the source potential. Far from the source, the background plasma dominates and the charge density equation has a single root very close to the background plasma reference potential. For a limited range of radii, the charge density equation has three roots. The high potential root is associated with the source plasma; the low potential root is associated with the background plasma. The middle root corresponds to negative shielding and is thus physically unstable. Figure 6 shows the roots as a function of the radius. The two physically realizable branches never intersect. Within the transition between the two branches, the plasma must be nonneutral. The thickness of the nonneutral region can easily be estimated from spherical diode theory (Wei and Wilbur, 1986). For plasmas in which the density changes on a scale-length long compared with the Debye length, the nonneutral region will be short compared with the scale length of the density changes. Since the electric fields in the quasi-neutral region are small, the nonneutral region will contain an equal amount of positive and negative charge,

$$\int_{r_1}^{r_2} \rho(r)\ 4\pi r^2 dr \equiv 0 \ , \tag{10}$$

where r_1 and r_2 are the radial boundaries of the double layer. Rigorously, a determination of the location of the double layer requires solving Poisson's equation from the first root, ϕ_1, to the second root, ϕ_2. An approximate location can be obtained by neglecting the spherical terms in the divergence. Using that the magnitude of the electric field is small in the quasi-neutral regions, Poisson's equation can be multiplied by the electric field and integrated by parts to obtain

$$\int_{\phi_1}^{\phi_2} \rho\left(r_{DL}, \phi\right) d\phi = 0 \quad . \tag{11}$$

Equation (11) locates the double layer at the radius, r_{DL}, where the charge density curve has equal areas of positive and negative charge between the two physical roots. This approximate integration of Poisson's equation was previously presented by Andrews and Allen (1971) as a constraint on the charge density in planar double layers. Here, it is used to locate the double layer.

As seen in the planar case, the multiple roots and the double layer exist only for a potential difference greater than a critical value. When the source potential is less then the critical value, the charge density equation has only a single root, and therefore a quasi-neutral solution exists for all radii. For this case, as shown in Figure 7, there is no double layer.

Numerically, the approximate radius found from Equation (11) is typically within a few percent of that found by solving Poisson's equation. That this potential construct corresponds to the physical double layer is illustrated by Figure 8. The solid curve is calculated by solving Poisson's equation using the charge density given by Equation (7) with additional ionization of the background gas, the circles are laboratory measurements of the potential of an anode plasma double layer by Wilbur and Williams (private communication).

Conclusions

Inclusion of the most basic nonlinear features in the charge density expression leads to a description of plasma double layers in terms of roots of that expression. The theory is similar to Van der Waals' description of a fluid, with the double layer corresponding to the liquid-gas phase transition. The theory locates the double layer in agreement with observation.

Much of the analysis presented depends on heuristic expressions for the plasma charge density. They are the weakest part of the arguments presented. The forms are, with the exception of the source ions, similar to those normally used for modeling plasmas in planar geometry. They are clearly restricted to double layers whose thickness is small compared with their radius of curvature. The use of semi-infinite half-plane formulations makes sense if the scattering lengths, whether through collisions or turbulence, are large compared with the double layer thickness and small compared with the system dimensions. Further research is required to develop a more rigorous theory. In particular, extension to a magneto plasma is required to understand ionospheric double layers.

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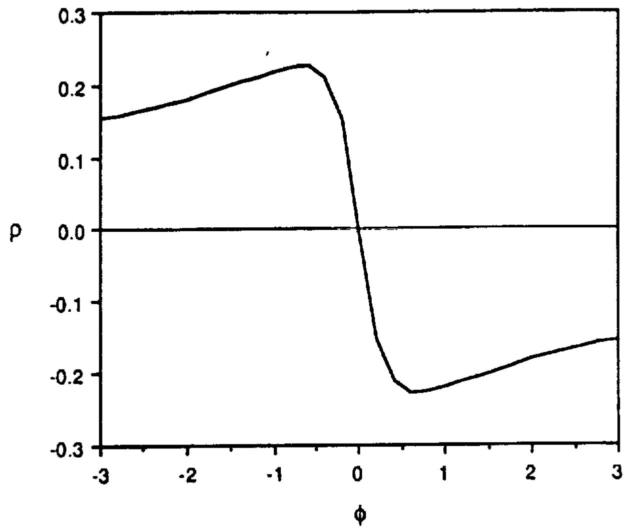

Figure 1. Charge density as a function of potential from Equation (2).

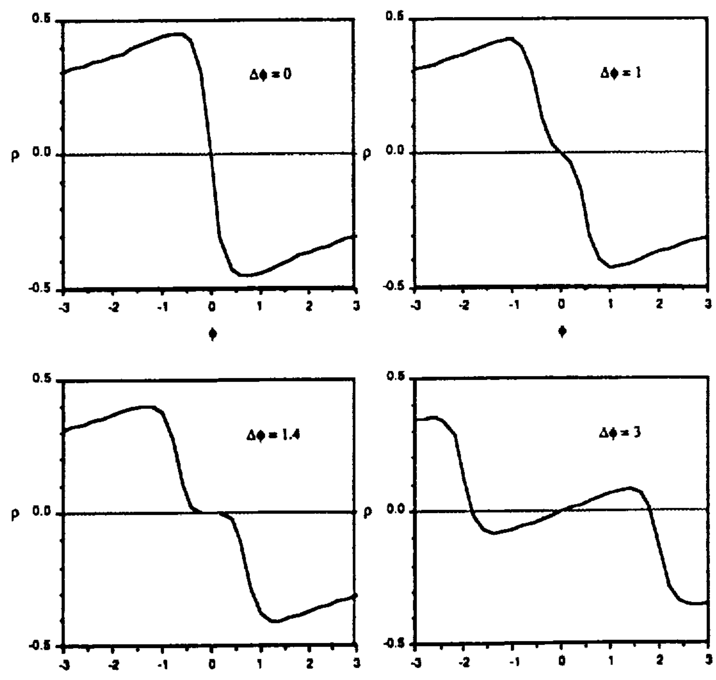

Figure 2. The charge density a function of potential for various differences in the reference potential. The curve for $\Delta\phi = 0$ is just twice that in Figure 1.

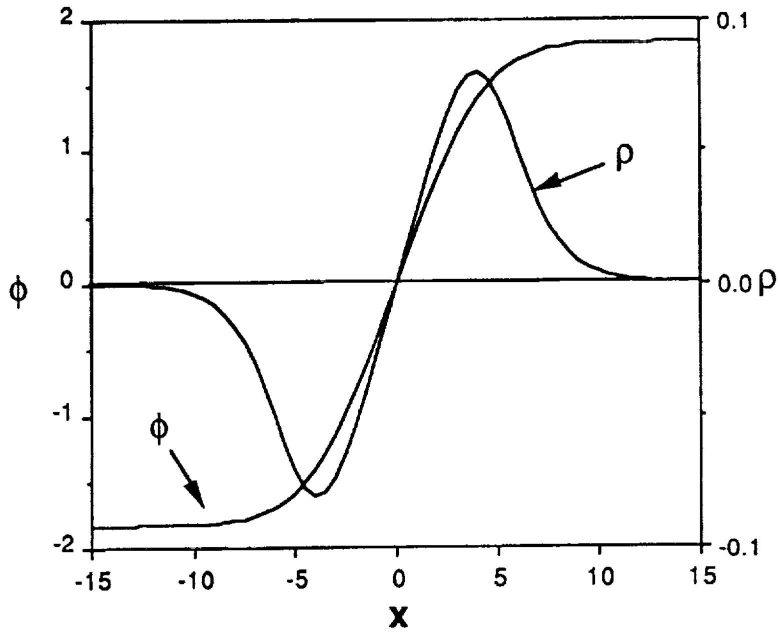

Figure 3. Solution of Poisson's equation with the charge density specified by Equation (5) for $\Delta\phi = 4$. The charge density as a function of position is also graphed and shows the expected double layer structure.

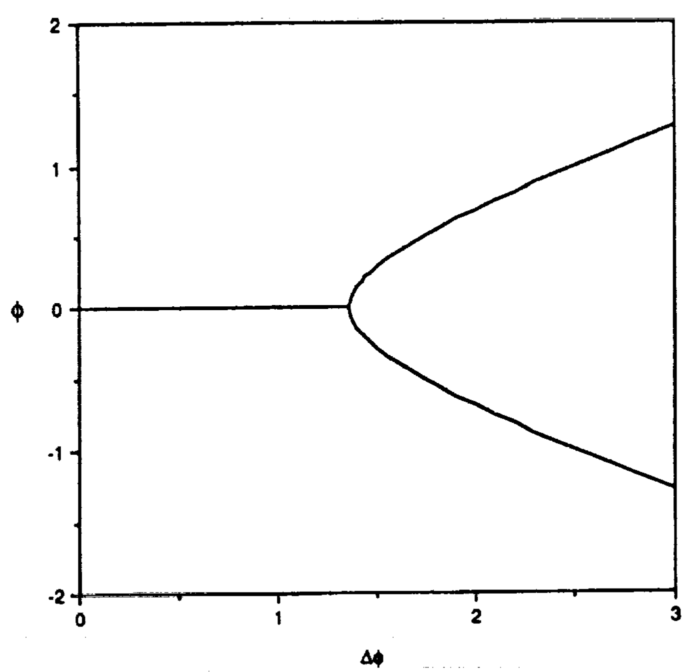

Figure 4. The roots of the charge density expression, Equation (5), plotted as a function of the difference in reference potentials, $\Delta\phi$.

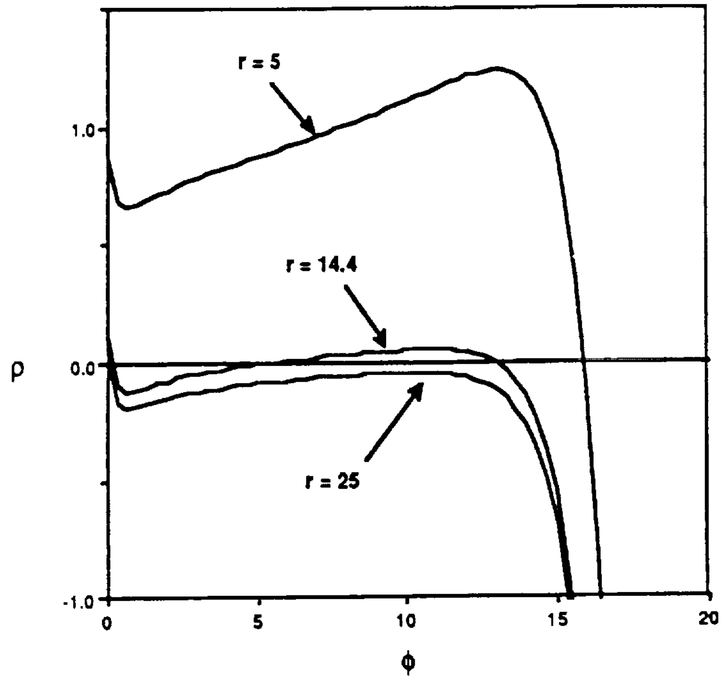

Figure 5. Charge density as function of potential for three radii. The density has a single root for $r = 5$ and $r = 25$, and has three roots for $r = 14.4$

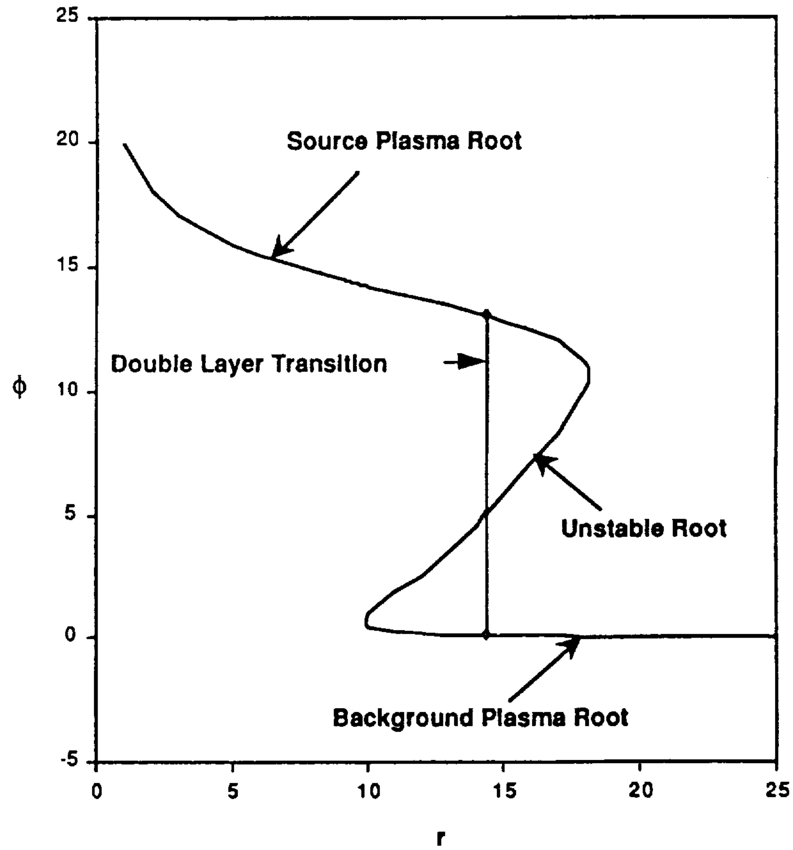

Figure 6. Roots of the charge equation for the spherical expansion of one plasma into a uniform background plasma. The double layer transition is located using Equation (11).

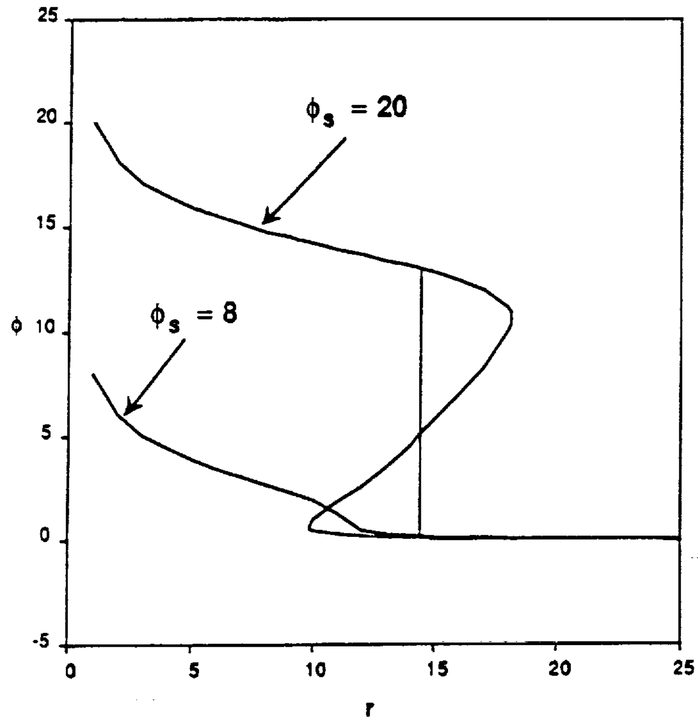

Figure 7. Roots of the charge equation for two different cases of the spherical expansion of a plasma into a uniform background plasma. The upper curve is the same as Figure 6, $\phi_{source} = 20$, and has a double layer. The lower curve is for $\phi_{source} = 8$ and doesn't have a double layer transition.

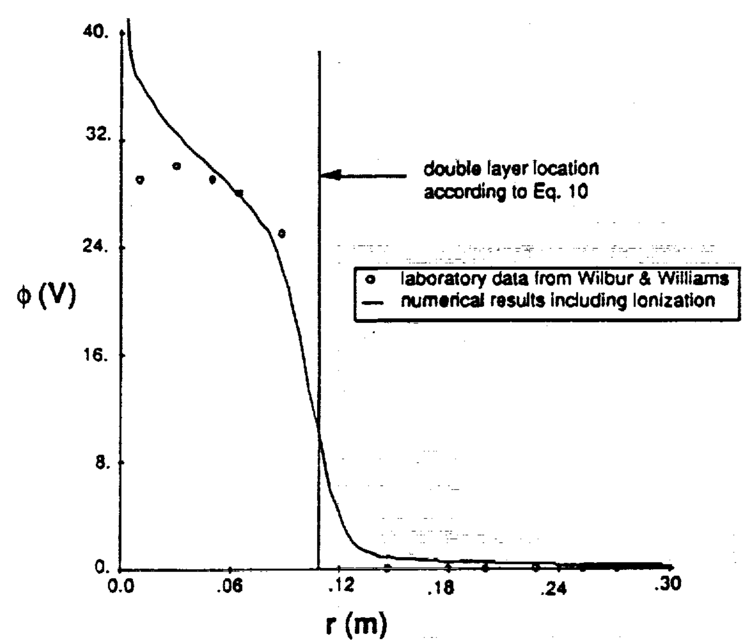

Figure 8. Comparison of numerical results with data for a hollow cathode plasma contactor operating as an electron collector. The location of the double layer according to Equation (11) is seen to agree with both the observation and the numerical, self-consistent solution of Poisson's equation including ionization of the neutral gas.